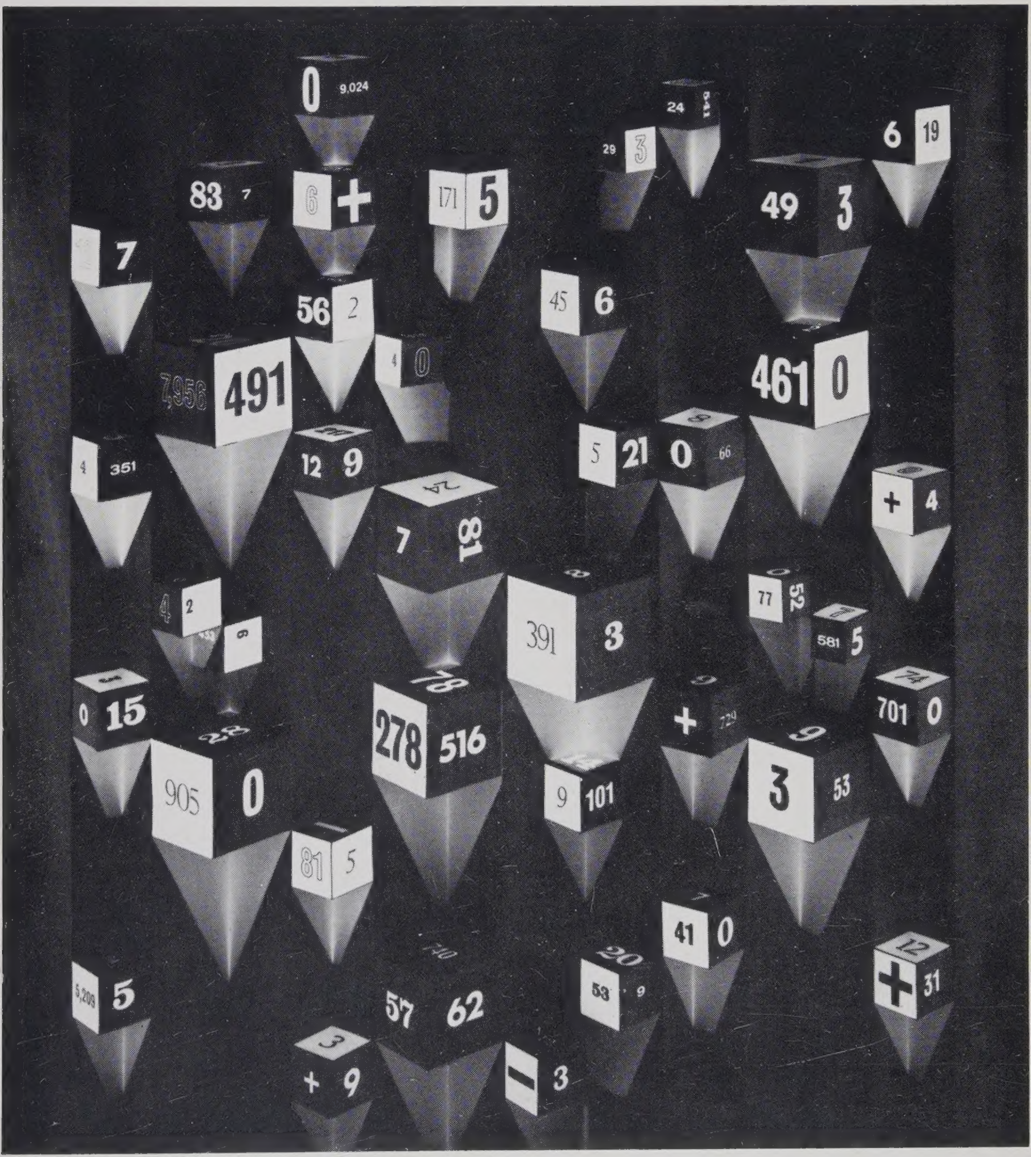
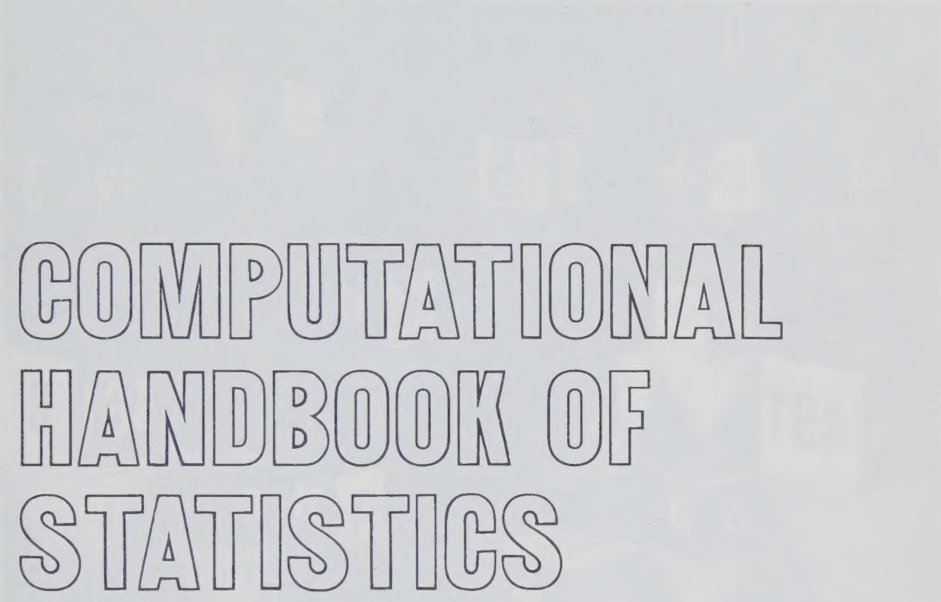
COMPUTATIONAL HANDBOOK OF STATISTICS

JAMES L. BRUNING B. L. KINTZ

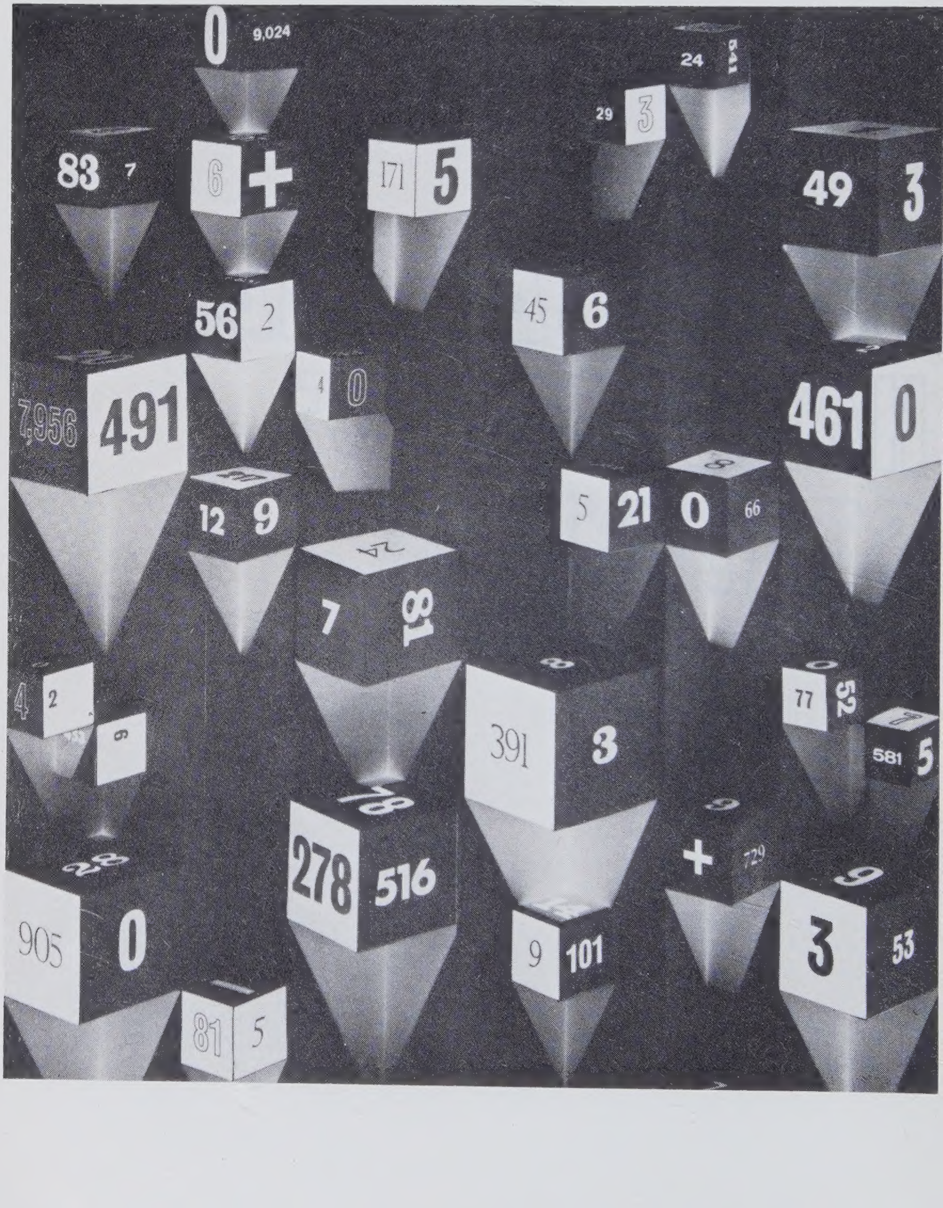


Digitized by the Internet Archive
in 2024

https://archive.org/details/neurologicalment0000lpie



COMPUTATIONAL HANDBOOK OF STATISTICS


0
9,024
24
541
29
3
83
7
6
+
171
5
49
3
56
2
45
6
7,956
491
4
0
461
0
5
21
0
66
12
9
24
7
81
4
2
6
391
3
77
52
581
5
78
278
516
+
729
905
0
9
101
3
53
81
5

COMPUTATIONAL HANDBOOK OF STATISTICS

JAMES L. BRUNING
Ohio University

B. L. KINTZ
Western Washington State College

Scott, Foresman and Company

Library of Congress Catalog Card No. 68-19607.

Copyright © 1968 by Scott, Foresman and Company, Glenview, Illinois 60025.
All Rights Reserved.

Regional offices of Scott, Foresman and Company are located in Atlanta, Dallas, Glenview, Palo Alto, and Oakland, N.J.

PREFACE

Most statistics textbooks concentrate on theoretical discussions and mathematical proofs of the various concepts presented. The result of this approach is that students often have little understanding of how actually to apply statistical tests to their experimental findings. The intent of this book is to reverse this approach and to present statistical concepts and tests *as they are applied.*

Our experience has indicated that carefully worked and clearly described examples can be easily generalized to a wide variety of situations. Consequently, the method of presentation employed throughout this book is based primarily on the technique of teaching by example. The mathematical proofs and theoretical discussions that appear in most textbooks are, for the most part, excluded. What has been substituted is a step-by-step analysis of the computational procedures needed to test the significance of experimental findings. Each test or analysis is illustrated by a clearly worked example, and all computational steps and procedures are expressed verbally rather than in mathematical symbols. Thus, while the analyses range from the simplest *t*-test or correlation to the most complex analysis of variance or multiple correlation, almost no previous mathematical knowledge is required to use this book effectively.

Although the amount of theoretical discussion presented in this book is minimal, the assumptions and basic computational formulas for the various analyses are presented either at the beginning of each section or are incorporated into the example used to demonstrate the computations. These are included, in all but one part of the book, to aid the user in determining the relationship between theory and computational procedure and also the relevance of each analysis to his particular problem. In each of the sections in Part 2, on analysis of variance, formulas are replaced for the most part by a selective bibliography of current statistics texts. The reasons for this are twofold. First, the symbolic notation for these analyses varies widely; and second, the presentation of this material in terms of formulas and symbols would have consumed more time and space than was deemed appropriate in a text of this type.

Although most of the examples in the book deal with the analysis of experimental findings in the fields of the behavioral and social sciences, this text is equally appropriate for courses in education, business, medical research, or any other area where statistical analysis of data is required. In addition, because of its broad range of coverage, it is also appropriate for courses at both the undergraduate and graduate levels.

Since this book requires little mathematical or formal statistical background, it should prove invaluable to instructors in beginning laboratory courses where students typically have completed only the most basic statistics course. Use of this book will permit the design and analysis of more advanced experiments where additional variables are manipulated or repeated measures taken. Our experience has been that the student need only be told which analysis he should apply to his data. From that point, nearly all individuals can follow the step-by-step presentation to completion of the entire analysis.

It has also been our experience that the same use can be made of this text by assistants with limited statistical backgrounds, who are assigned to carry out the analysis of an experiment. With the aid of this handbook, even the most inexperienced assistant can compute highly complex analyses with little or no additional supervision.

One other important use of this book has been noted by individuals interested in computer programing. Since the computational steps of each analysis are indicated separately, they represent a detailed flow chart of the instructions that must be included in a computer program. To illustrate this application, two sample programs, written from the computational steps presented in Sections 2.9 and 2.10, are presented in Appendix M. These programs represent the most complex problems that might be encountered in the programing of analyses of variance. In both instances, the basic approach was simply to translate the verbal steps outlined in this book into the FORTRAN language system.

It would be impossible to thank everyone who has made either a direct or indirect contribution to this book. There are, however, several individuals whose influence warrants special recognition. First, we would like to express our appreciation to Dr. Dee W. Norton, whose methods of teaching "Design and Analysis of Experiments" led to our interest in this area. Dr. G. S. Reynolds and Dr. Louis Aikman read and critically evaluated the manuscript. Their comments were most helpful in making needed changes and improvements. Our publisher-author relationship has been all that could be desired, and we would like to extend thanks to Mr. Bruce Harlow, Mrs. Maud Benjamin, Mrs. Marguerite Clark, and Mr. John J. Miller, all of Scott, Foresman, for their interest, help, and encouragement.

We are indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Oliver & Boyd Ltd., Edinburgh, for permission to reprint Tables III, V, and VI from their book *Statistical Tables for Biological, Agricultural and Medical Research.*

Special thanks are extended to Messrs. Morris Morgret and Ronald Mihalick, for their permission to include the sample programs found in Appendix M. Also, our thanks to Dr. Paul Becker, who suggested the inclusion of these programs and whose suggestions for improvement of the manuscript were greatly appreciated. And to our wives, Marlene and Maria, a special statement of appreciation, not only for typing and editorial help but also for general encouragement in every phase of the manuscript development.

JAMES L. BRUNING
Ohio University

B. L. KINTZ
Western Washington State College

CONTENTS

PART 1 **Organizing Data and Some Simple Computations** 1

1.1 Blocking and Tabling of Data 2
1.2 Range and Standard Deviation 4
1.3 Standard Error of the Mean 6
1.4 The *t*-Test for a Difference Between a Sample Mean and the Population Mean 7
1.5 The *t*-Test for a Difference Between Two Independent Means 9
1.6 The *t*-Test for Related Measures 12

PART 2 **Analysis of Variance** 16

2.1 Completely Randomized Design 22
2.2 Factorial Design: Two Factors 25
2.3 Factorial Design: Three Factors 30
2.4 Treatments-by-Levels Design 38
2.5 Treatments-by-Subjects, or Repeated-Measures, Design 43
2.6 Treatments-by-Treatments-by-Subjects Design 47
2.7 Two-Factor Mixed Design: Repeated Measures on One Factor 54
2.8 Three-Factor Mixed Design: Repeated Measures on One Factor 61
2.9 Three-Factor Mixed Design: Repeated Measures on Two Factors 72
2.10 Latin Square Design: Simple 84
2.11 Latin Square Design: Complex 92

PART 3 Supplemental Computations for Analysis of Variance 106

3.1 Test for Difference Between Variances of Two Independent Samples (Test for Homogeneity of Independent Variances) 107
3.2 Test for Difference Between Variances of Two Related Samples (Test for Homogeneity of Related Variances) 109
3.3 Test for Differences Among Several Independent Variances (F-Maximum Test for Homogeneity of Variances) 110
3.4 The t-Test for Differences Among Several Means 112
3.5 Duncan's Multiple-Range Test for Nearly Equal *ns* 115
3.6 F-Tests for Simple Effects 117
3.7 Use of Orthogonal Components in Tests for Trend 123

PART 4 Correlation and Related Topics 150

4.1 Pearson Product-Moment Correlation 152
4.2 Spearman Rank-Order Correlation (*rho*) 156
4.3 Kendall Rank-Order Correlation (*tau*) 159
4.4 Point-Biserial Correlation 163
4.5 The Correlation Ratio (*eta*) 166
4.6 Partial Correlation: Three Variables 168
4.7 Partial Rank-Order Correlation (Using Kendall's *tau*) 170
4.8 Multiple Correlation: Three Variables 171
4.9 Simple Analysis of Covariance: One Treatment Variable 173
4.10 Factorial Analysis of Covariance: Two Treatment Variables 177
4.11 Reliability of Measurement: The Test As a Whole (Test-Retest, Parallel Forms, and Split Halves) 187
4.12 Reliability of Measurement: The Individual Items (Kuder-Richardson and Hoyt) 188
4.13 Test for Difference Between Independent Correlations 191
4.14 Test for Difference Between Dependent Correlations 193

PART 5 **Nonparametric Tests, Miscellaneous Tests of Significance, and Indices of Relationship** **195**

5.1 Test for Significance of a Proportion 197
5.2 Test for Significance of Difference Between Two Proportions 199
5.3 The Mann-Whitney U-Test for Differences Between Independent Samples 201
5.4 A Sign Test (Wilcoxon) for Differences Between Related Samples 205
5.5 Simple Chi-Square and the *Phi* Coefficient 207
5.6 Complex Chi-Square and the Contingency Coefficient (C) 209

APPENDIXES **215**

A Normal-Curve Areas 216
B Critical Values of "Student's" t Statistic 218
C Centile Values of the Chi-Square Statistic 220
D Per Cent Points in the F Distribution 222
E Fisher's z-Transformation Function for Pearson's r Correlation Coefficient 226
F Critical Values of Pearson's r Correlation Coefficient for Five Alpha Significance Levels 228
G Critical Values of the U Statistic of the Mann-Whitney Test 230
H Critical Values for Hartley's Maximum F-Ratio Significance Test for Homogeneity of Variances 234
I Significant Studentized Ranges for Duncan's New Multiple-Range Test 236
J Critical Values of Wilcoxon's T Statistic for Matched-Pairs Signed-Ranks Test 242
K Coefficients for Orthogonal Polynomials 243
L Squares and Square Roots 244
M Sample Computer Programs for Analysis of Variance 254

TO THE USER

As explained in the Preface, this book approaches statistics from the applied, rather than the theoretical, point of view. It is our hope that the extensive use of examples and the step-by-step presentation of the computational procedures will not only provide models to be followed in computing analyses but will also help the user to gain true insights into how each segment or step fits into the final analysis. It is also our hope that by presenting nearly all the material in verbal rather than symbolic form, this book will help dispel the fears that many have of statistics. At the same time, we have included as aids to the user the basic assumptions and computational formulas for each analysis. These are either presented at the beginning of each section or are incorporated into the computational example, except in Part 2. In each of the sections in this part, on analysis of variance, a selective bibliography of current statistical texts is presented instead.

The following example demonstrates the methods of approach and presentation that are used throughout this book.

EXAMPLE

Assume that a physical education teacher has measured the heights of five boys in one of his classes. He now wishes to compute the average, or mean, height of this group.

The mathematical formula for computation of the mean is

$$\bar{X} = \frac{\sum X}{N}$$

where $\bar{X}$ = mean of the scores
$\sum X$ = sum of the scores
N = number of scores

Step 1. First, table the scores as follows. No particular order is necessary.

Subject	Height (inches)
S_1	60
S_2	53
S_3	57
S_4	52
S_5	58

Step 2. Add all the scores to get the sum for the group.

$$60 + 53 + 57 + 52 + 58 = \mathbf{280}$$

Step 3. Computation of the mean: Divide the value of Step 2 by the number of scores (heights) which were added. (In the present example, there were 5 scores.)

$$\bar{X} = \frac{280}{5} = \mathbf{56\ inches}$$

While it is sometimes possible to use short cuts and combine steps (as in the above example), we recommend that to avoid error and to more fully understand the meaning of each step, the user closely follow the steps we have outlined. During initial study or computation, we also recommend that the user label each step since this greatly facilitates the correction of errors and review of points that were not clear.

PART 1

Organizing Data and Some Simple Computations

Research involves the making of systematic observations, organizing these observations in some meaningful fashion, and then, usually on the basis of statistical tests, drawing conclusions. Raw data—observations in their original form—are usually a mass of numbers or symbols that have little or no meaning. To allow even the most gross kinds of interpretations, the data must be organized in some fashion. Probably the simplest organization is the classification of data into categories. In this instance, a count is usually made of the observations that fall into one class as opposed to another. A good example is an observation of a crowd that is later classified according to the number of males versus females present. Statistical procedures to aid in the interpretation of data collected and organized in this fashion are *usually* (but not always) of the nonparametric type. These statistical tests are discussed in the last part of this book.

The form of data organization that is of interest in the first four parts of this book involves the use of measures that are recorded for each of several observations. These measures almost always imply some notion of varying magnitude or amount. This type of classification differs from the first in that it implies varying amounts of an observation rather than simply indicating whether or not a particular observation was made. In the following sections, organization and computation of measures will be considered in relation to various simple statistical tests that experimenters use as aids in drawing conclusions regarding the outcome of their research.

Organizing Data (Sections 1.1, 1.2, and 1.3)

These first three sections discuss some of the basic procedures for organizing data and putting it in an understandable form. While it is recognized that there are many other procedures for organizing data (such as frequency distributions, etc.), the ones presented in these chapters are those used most often.

Some Simple Computations (Sections 1.4, 1.5, and 1.6)

These three sections are concerned primarily with demonstrating the basic uses of the t-test. These include the t for testing the difference between a sample and a population mean, the difference between two sample means, and the t for related measures.

SECTION 1.1

Blocking and Tabling of Data

In most experiments involving the collection and analysis of data, the data are grouped, or blocked, to reduce variability and to make the scores more manageable for statistical analysis. The most commonly used methods are to take the mean, median, or mode of several scores. The value obtained is then treated as a single score.

For example, if the number of correct responses on each of fifteen trials was the measure recorded, the raw data for Subject 1 might appear as follows:

Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Correct responses for S_1	1	4	2	6	1	3	5	9	7	9	4	9	14	10	14

Since there are fifteen trials, the most reasonable grouping would probably be to combine each consecutive set of three or five trials. If only one score were desired, the entire fifteen trials would be combined. The size of the group, or block, of data is entirely arbitrary.

The most common technique is to take the mean of each successive set of trials. In the above example, assume that the experimenter decides to block by taking the mean of each successive five scores. Thus, the fifteen scores of S_1 would be reduced to three scores, as follows:

Trial Block	1	2	3
Mean of 5 trials for S_1	2.8	6.6	10.2

If an overall mean were desired, the experimenter would simply add all the correct responses and divide by the number of trials given. For S_1 this would be 98/15, which equals 6.53.

Less frequently, the median or mode is used to block raw data. Use of these measures usually occurs only if the data within each set of scores are severely skewed. In the present example, the median would equal the middle score of each block of trials after the scores within each block had been rank ordered. Using the data presented above, the median of trials one through five would be 2, since there are two scores smaller than 2 (1, 1) and two larger (4, 6). The same type of computation would be made for trials six through ten and eleven through fifteen.

Trial Block	1	2	3
Median of 5 trials for S_1	2	7	10

The overall median in this example would be 6.

The mode is simply the score that occurs most often. When applied to blocks of five trials, the scores for S_1 become:

Trial Block	1	2	3
Mode of 5 trials for S_1	1	9	14

The overall mode would be 9, since that score occurs three times during the fifteen trials.

After the data have been blocked, they must be tabled so that statistical analyses can be undertaken. While the specific method of tabling depends upon the statistical analyses to be used, the most common technique involves simply identifying each subject and then entering his score or scores in a row to the right. For example, using the mean scores in the above example:

	Trial Block		
Subject	1	2	3
S_1	2.8	6.6	10.2
S_2	1.4	1.8	4.3
S_3	2.1	6.7	6.4

While it is apparent that different methods of blocking scores can result in different measures being recorded for each subject, the major point to note is that whenever raw data are blocked in some fashion, the statistical analyses may be based entirely on the blocked scores. In the above example, all analyses may be computed as though only three trials had been given or, if an overall score were used, as if only one score had been recorded. It is also important to note that for purposes of determining degrees of freedom (*df*), numbers of scores in a table, group means, etc., all computations and numerical counts are based on the blocked scores and not on the raw data.

SECTION 1.2

Range and Standard Deviation

By grouping and arranging scores, it is possible to get a fairly good understanding of the data that have been collected. In addition to grouping, however, it is usually desirable to obtain some idea of the dispersion of the scores. The simplest such measure is the *range*. This value is computed very simply since it is equal to the difference between the largest and smallest scores in the distribution. For example, in the sample data presented below, the tallest child is 72 inches and the shortest is 43 inches; the range, then, is 29 inches.

Although the range does give some information regarding the dispersion of the scores, its usefulness is limited. By far the most common and useful measure of dispersion is the *standard deviation* (s.d.). There are two ways to compute the standard deviation: directly, using difference scores; and indirectly, using the computational formula. As is shown in nearly all introductory textbooks, these two methods are actually the same.

Since the method that employs the computational formula is more generally applicable, it will be presented here.

EXAMPLE

This example is drawn from the area of developmental psychology. Assume that an experimenter is interested in obtaining information about the heights of twelve-year-olds. To do this, he collects a sample of data (the heights of twenty randomly selected twelve-year-olds) and proceeds to compute the standard deviation.

The mathematical formula for the standard deviation is

$$\text{s.d.} = \sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N-1}}$$

where $\sum X^2$ = the sum of the squared score values
$(\sum X)^2$ = the square of the sum of all the scores
N = the total number of scores used in the computation

Step 1. Table the data as follows. No particular order is necessary.

Subject	Height (inches)
S_1	64
S_2	48
S_3	55
S_4	68
S_5	72
S_6	59
S_7	57
S_8	61
S_9	63
S_{10}	60
S_{11}	60
S_{12}	43
S_{13}	67
S_{14}	70
S_{15}	65
S_{16}	55
S_{17}	56
S_{18}	64
S_{19}	61
S_{20}	60

Step 2. Add all the scores. (*Note*: If you are using a calculator, you can do Step 2 and Step 3 at the same time.)

$$64 + 48 + \cdots + 60 = \mathbf{1208}$$

Step 3. Square all the scores and add the squared values.

$$64^2 + 48^2 + \cdots + 60^2 = \mathbf{73{,}894}$$

Step 4. Square the sum obtained in Step 2, and divide this value by the number of scores that were added to obtain the sum. The resultant value is called the *correction term.*

$$\frac{1208^2}{20} = \frac{1{,}459{,}264}{20} = \mathbf{72{,}963}$$

Step 5. Subtract the value obtained in Step 4 from the sum in Step 3.

$$73{,}894 - 72{,}963 = \mathbf{931}$$

Step 6. Divide the value obtained in Step 5 by $N - 1$.[1] (In this example, it would be $20 - 1 = 19$.) The resultant value is called the *variance.*

$$\frac{931}{19} = \mathbf{49}$$

Step 7. Take the square root of the value obtained in Step 6. This value is the *standard deviation.*

$$\sqrt{49} = \mathbf{7\ in.}$$

SECTION 1.3

Standard Error of the Mean

The standard error of the mean is extremely simple to compute once the steps outlined in Section 1.2 have been completed. All that must be done is to follow Steps 1 through 7 of Section 1.2 and then divide the value obtained in Step 7 by the square root of the size of the sample.

In the example used in Section 1.2, the value obtained in Step 7 was 7 inches. Since there were 20 subjects in the sample, simply divide 7 by the square root of 20 to obtain the standard error of the mean.

$$\frac{7}{\sqrt{20}} = \frac{7}{4.47} = \mathbf{1.57}$$

---

[1] This term is almost universally used since the division by $N - 1$ gives the unbiased *estimate* of the population variance rather than the *actual* variance of the sample. If the actual sample variance is desired, divide by N rather than $N - 1$. Step 7 is the same, regardless.

SECTION 1.4

The t-Test for a Difference Between a Sample Mean and the Population Mean

One of the most commonly used tests of significance is the t-test. One use of this test assumes that the mean for some population is known. Knowing this population value, the experimenter can then determine whether the sample he has chosen is significantly different from the population mean.

EXAMPLE

This example uses the data presented in Section 1.2. Assume that after collecting measures of the heights of twenty randomly selected twelve-year-olds, the experimenter notices that while the standard deviation is large, the average height of the children in this sample is above the population average for twelve-year-olds. By use of the t-test, he can determine whether this difference is significant.

The basic formula for testing the significance of a difference between a sample mean and population mean is

$$t = \frac{\bar{X} - \mu}{\sqrt{\dfrac{\sum X^2 - \dfrac{(\sum X)^2}{N}}{N(N-1)}}}$$

where $\bar{X}$ = the mean of the sample
μ = the mean of the population
$\sum X^2$ = the sum of the squared score values
$(\sum X)^2$ = the square of the sum of all the scores
N = the number of scores used in the analysis

Step 1. Table the data as follows. No particular order is necessary.

Subject	Height (inches)
S_1	64
S_2	48
S_3	55
S_4	68
S_5	72
S_6	59
S_7	57
S_8	61

Subject	Height (inches)
S_9	63
S_{10}	60
S_{11}	60
S_{12}	43
S_{13}	67
S_{14}	70
S_{15}	65
S_{16}	55
S_{17}	56
S_{18}	64
S_{19}	61
S_{20}	60

Step 2. Add all the scores. (*Note*: If you are using a calculator, you can do Step 2 and Step 3 at the same time.)

$$64 + 48 + \cdots + 60 = \mathbf{1208}$$

Step 3. Square all the scores, and add these squared values.

$$64^2 + 48^2 + \cdots + 60^2 = \mathbf{73{,}894}$$

Step 4. Square the sum obtained in Step 2. Then divide this value by the number of scores that were added to get the sum. (In the present example, 20 scores were added.) The resultant value is called the *correction term.*

$$\frac{1208^2}{20} = \frac{1{,}459{,}264}{20} = \mathbf{72{,}963}$$

Step 5. Subtract the result of Step 4 from the sum in Step 3.

$$73{,}894 - 72{,}963 = \mathbf{931}$$

Step 6. Divide the value obtained in Step 5 by $N - 1$. (In the present example, it would be $20 - 1 = 19$.)

$$\frac{931}{19} = \mathbf{49}$$

Step 7. Take the square root of the value obtained in Step 6.

$$\sqrt{49} = \mathbf{7\ in.}$$

Step 8. Divide the value of Step 7 by $\sqrt{N}$. (In the present example, $\sqrt{20} = 4.47$.)

$$\frac{7}{4.47} = \mathbf{1.57}$$

Step 9. Subtract the sample mean (sum in Step 2 divided by N) from the population mean (known from some other source).

$$\frac{1208}{20} = \mathbf{60.4} = \text{sample mean}$$

$$\mathbf{58.0} = \text{population mean read from growth-curve tables}$$

$$58.0 - 60.4 = \mathbf{2.4}$$

(*Note*: For computational purposes, only the absolute difference between the means is important.)

Step 10. Divide the value obtained in Step 9 by the value obtained in Step 8. This yields the t value.

$$t = \frac{2.4}{1.57} = \mathbf{1.53}$$

Step 11. To determine whether the t value is significant, the degrees of freedom must be obtained. For a test of significance of a sample mean and a population mean, the df are equal to $N - 1$. In the present example, this would be $20 - 1 = 19$. The t for significance at the .05 level (see Appendix B) is 2.093. Since the t value in the present example is smaller than 2.093, it is concluded that there is no significant difference between this sample mean and the population mean.

SECTION 1.5

The *t*-Test for a Difference Between Two Independent Means

Probably the most common use of the t-test is to determine whether the performance difference between two groups of subjects is significant. In most experimental situations, the subjects are randomly assigned to the two groups; one of the groups is manipulated experimentally, and the effects of this manipulation are analyzed by comparing the performance of the two groups. However, there are many instances where the groups are already constituted (e.g., males versus females) and the experimenter wishes to determine whether they differ with respect to some other variable (e.g., height, weight, etc.).

EXAMPLE

Assume that a principal wishes to determine whether there is a significant difference between the IQ scores of boys and girls in a particular grade. The score recorded for each student is his score on the Wechsler Intelligence Scale for Children (WISC).

The basic computational formula for the t-test of a difference between two independent means is

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left[\dfrac{\sum X_1^2 - \dfrac{(\sum X_1)^2}{N_1} + \sum X_2^2 - \dfrac{(\sum X_2)^2}{N_2}}{(N_1 + N_2) - 2}\right] \cdot \left[\dfrac{1}{N_1} + \dfrac{1}{N_2}\right]}}$$

where $\bar{X}_1$ = the mean of the first group of scores
$\bar{X}_2$ = the mean of the second group of scores
$\sum X_1^2$ = the sum of the squared score values of the first group
$\sum X_2^2$ = the sum of the squared score values of the second group
$(\sum X_1)^2$ = the square of the sum of the scores in the first group
$(\sum X_2)^2$ = the square of the sum of the scores in the second group
N_1 = the number of scores in the first group
N_2 = the number of scores in the second group

Step 1. Table the data as follows. No particular order within groups is necessary.

Group 1 (Boys)		Group 2 (Girls)	
Subject	Score	Subject	Score
S_1	107	S_{13}	109
S_2	96	S_{14}	94
S_3	88	S_{15}	127
S_4	131	S_{16}	76
S_5	109	S_{17}	115
S_6	84	S_{18}	121
S_7	79	S_{19}	87
S_8	105	S_{20}	92
S_9	108	S_{21}	91
S_{10}	92	S_{22}	98
S_{11}	96	S_{23}	104
S_{12}	101	S_{24}	96
		S_{25}	110
		S_{26}	108

Step 2. Add the scores in Group 1. (*Note*: If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

$$107 + 96 + \cdots + 101 = \mathbf{1196}$$

Step 3. Square every score in Group 1, and add these squared values.

$$107^2 + 96^2 + \cdots + 101^2 = \mathbf{121{,}318}$$

Step 4. Square the value obtained in Step 2. Then divide the squared value by the number of scores on which the sum was based. (In this example, 12 scores were added.)

$$\frac{1196^2}{12} = \frac{1{,}430{,}416}{12} = \mathbf{119{,}201}$$

Step 5. Subtract the value obtained in Step 4 from the sum in Step 3.

$$121{,}318 - 119{,}201 = \mathbf{2117}$$

Step 6. Add the scores in Group 2.

$$109 + 94 + \cdots + 108 = \mathbf{1428}$$

Step 7. Square every score in Group 2, and add these squared values.

$$109^2 + 94^2 + \cdots + 108^2 = \mathbf{148{,}202}$$

Step 8. Square the value obtained in Step 6. Then divide the squared value by the number of scores on which the sum was based. (In the present example, 14 scores were added.)

$$\frac{1428^2}{14} = \frac{2{,}039{,}184}{14} = \mathbf{145{,}656}$$

Step 9. Subtract the value obtained in Step 8 from the sum in Step 7.

$$148{,}202 - 145{,}656 = \mathbf{2546}$$

Step 10. Add the values obtained in Step 5 and Step 9.

$$2117 + 2546 = \mathbf{4663}$$

Step 11. Divide the value obtained in Step 10 by $(n_1 + n_2) - 2$. [In the present example, $(12 + 14) - 2 = 24$.]

$$\frac{4663}{24} = \mathbf{194}$$

Step 12. Multiply the value obtained in Step 11 by $1/n_1 + 1/n_2$. (In the present example, this would equal $1/12 + 1/14$.)

$$194\left(\frac{1}{12} + \frac{1}{14}\right) = 194 \times \frac{13}{84} = \mathbf{30.0}$$

Step 13. Take the square root of the value obtained in Step 12.

$$\sqrt{30.0} = \mathbf{5.48}$$

Step 14. Obtain the mean of Group 1 (sum in Step 2 divided by n_1) and the mean of Group 2 (sum in Step 6 divided by n_2).

$$\frac{1196}{12} = \mathbf{99.67} = \text{mean of Group 1}$$

$$\frac{1428}{14} = \mathbf{102.00} = \text{mean of Group 2}$$

Step 15. Subtract the mean of Group 1 from the mean of Group 2.

$$102.00 - 99.67 = \mathbf{2.33}$$

(*Note*: For computational purposes, only the absolute difference between the means is important.)

Step 16. Divide the value obtained in Step 15 by the value obtained in Step 13. This yields the t value.

$$t = \frac{2.33}{5.48} = \mathbf{.43}$$

Step 17. To determine whether the t value is significant, the degrees of freedom (*df*) must be computed. For a test of significance between two means, the *df* are equal to $(n_1 + n_2) - 2$. In the present example, $(12 + 14) - 2 = 24$. The t value which is significant at the .05 level for 24 *df* (see Appendix B) is equal to 2.064. Since the t value in the present example is less than 2.064, it is concluded that there is no significant difference between the IQ scores of boys and girls in this particular class.

SECTION 1.6

The *t*-Test for Related Measures

A somewhat rarer use of the *t*-test is to determine the significance of a difference between two correlated means. It is most commonly used in this way when two scores are recorded for the same individuals. For instance, IQ scores might be taken at the beginning and end of a special training program to determine if there has been any improvement in test scores. The second use of this test is in the instance where pairs of subjects in two different groups are "matched" on the basis of some variable—like age, sex, race, years of education, etc.—to ensure that the pairs of subjects in each group are the "same" before experimental manipulations are begun.

Points to Consider When Using the *t* for Related Measures

1. Both experimental groups will have the same number of measures since they represent two measures on the same subjects or matched pairs of subjects.
2. If two measures are taken on the same subjects, the treatments-by-subjects analysis (see Section 2.5) is usually more appropriate.

EXAMPLE

This example is taken from the field of education. Assume that the experimenter is interested in determining the effects of a special education program on the intelligence test scores of underprivileged children. His first step is to match several pairs of students on the basis of their Wechsler Intelligence Test scores. One student from each pair is randomly assigned to either the special training group or the control group that receives no special treatment; the remaining student in each pair is assigned to the other group. After six weeks of training, an alternate form of the Wechsler test is given to the students in both groups to determine the effects of the program. The score recorded for each student is his score on this second test. In the present example, the pairs of matched subjects will be represented by S and S'; the test score of each S by X; and the test score of each S' by Y.

There are two formulas that are used for the t for related measures. The first involves computation of a correlation and is given by

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{S_X^2 + S_Y^2 - 2rS_XS_Y}{N}}}$$

where S_X^2 = variance of the X scores
S_Y^2 = variance of the Y scores
r = correlation between X and Y
N = number of pairs of scores

The second formula, which gives *exactly* the same results, is much simpler and will be used to demonstrate the computational steps.

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\dfrac{\sum D^2 - \dfrac{(\sum D)^2}{N}}{N(N-1)}}}$$

where D = difference score between each X and Y pair
N = number of pairs of scores

Step 1. Table the data as follows. No particular order is necessary, except that each pair (S and S') of scores must be in the same relative position.

Group 1 (No Special Treatment)		Group 2 (Special Treatment)	
Subject	Score	Subject	Score
S_1	89	S'_1	94
S_2	86	S'_2	94
S_3	96	S'_3	101
S_4	100	S'_4	105
S_5	94	S'_5	100
S_6	86	S'_6	84
S_7	81	S'_7	81
S_8	93	S'_8	96
S_9	87	S'_9	90
S_{10}	89	S'_{10}	88
S_{11}	110	S'_{11}	115
S_{12}	95	S'_{12}	100
S_{13}	107	S'_{13}	110
S_{14}	96	S'_{14}	102

Step 2. Obtain the difference between each pair of scores.

$$\begin{aligned}
89 - 94 &= -5\\
86 - 94 &= -8\\
96 - 101 &= -5\\
100 - 105 &= -5\\
94 - 100 &= -6\\
86 - 84 &= +2\\
81 - 81 &= 0\\
93 - 96 &= -3\\
87 - 90 &= -3\\
89 - 88 &= +1\\
110 - 115 &= -5\\
95 - 100 &= -5\\
107 - 110 &= -3\\
96 - 102 &= -6
\end{aligned}$$

Step 3. Square all the difference scores recorded in Step 2, and add these squared values.

$$(-5)^2 + (-8)^2 + \cdots + (-6)^2 = 293$$

Step 4. Obtain the *algebraic* sum of the difference scores obtained in Step 2. Square this value, and divide by the number of difference scores recorded.

$$(-5) + (-8) + \cdots + (-6) = -51$$

$$\frac{-51^2}{14} = \frac{2601}{14} = 186$$

Step 5. Subtract the value obtained in Step 4 from the sum in Step 3.

$$293 - 186 = \mathbf{107}$$

Step 6. Divide the value obtained in Step 5 by $N - 1$. (In the present example, this is $14 - 1 = 13$, since N refers to pairs of scores.)

$$\frac{107}{13} = \mathbf{8.23}$$

Step 7. Take the square root of the value obtained in Step 6.

$$\sqrt{8.23} = \mathbf{2.87}$$

Step 8. Divide the value of Step 7 by $\sqrt{N}$. (In the present example, $\sqrt{14} = 3.74$.)

$$\frac{2.87}{3.74} = \mathbf{.767}$$

Step 9. Obtain the mean score of each of the two groups—that is, add all the scores in each group and divide each sum by the number of scores added to obtain it.

$$89 + 86 + \cdots + 96 = \mathbf{1309} = \text{sum for Group 1}$$

$$94 + 94 + \cdots + 102 = \mathbf{1360} = \text{sum for Group 2}$$

$$\frac{1309}{14} = \mathbf{93.50} = \text{mean for Group 1}$$

$$\frac{1360}{14} = \mathbf{97.14} = \text{mean for Group 2}$$

Step 10. Subtract the mean for Group 2 from the mean for Group 1.

$$93.50 - 97.14 = \mathbf{3.64}$$

(*Note*: For computational purposes, only the absolute difference between the means is important.)

Step 11. Divide the value obtained in Step 10 by the value obtained in Step 8. This yields the t value.

$$t = \frac{3.64}{.767} = \mathbf{4.74}$$

Step 12. To determine whether the t value is significant, the degrees of freedom (df) must first be obtained. For the t for related measures, the $df = N - 1$ where N is the number of *pairs* of scores. In the present example, $N - 1 = 13$. From the t tables (see Appendix B), we find that the t value that is significant at the .05 level for 13 df is 2.16. Since the obtained t is larger than 2.16, it is concluded that the special training program improved the IQ test scores.

PART 2

Analysis of Variance

The statistical tests presented in the previous sections are designed to determine the significance of a difference between two groups. In actual practice, however, most experiments involve more than two groups. While it would be possible to test the difference between, say, four experimental groups by using *t*-tests of all possible combinations of two group means, the number of tests necessary to make these comparisons would be large. (The actual number of tests is equal to $N(N - 1)/2$. In the case of an experiment involving four experimental groups, six separate tests would be needed.)

In addition to posing computational problems, many multigroup experiments require that several variables be manipulated at the same time. For example, four groups of subjects might be presented with a learning task and their performance recorded over a series of several trials. The experimenter would very likely be interested not only in overall performance but also in whether the rate of learning of the several groups was the same. To determine this, he needs statistical procedures that are far more complex than the simple, two-group *t* statistic.

In the following sections, several statistical analyses are introduced that permit evaluation of much more complicated experimental designs. To aid the user in determining which analysis is appropriate for a particular experimental design, a descriptive list of all the analyses is presented below. In addition to indicating the type of problem for which each analysis is appropriate, the list also indicates some of the major strengths and weaknesses of each.

Completely Randomized Design (*Section 2.1*)

This design is basically an extension of the *t*-test to experiments involving three or more groups. This design would typically be used, for example, if an experimenter were interested in determining reactions to several different drugs. In this case, subjects would be randomly assigned to the several groups and a different drug administered to each group. After the performance of subjects in all groups had been measured, statistical analyses of the data would be undertaken to determine the differential effects of the drugs.

Factorial Design: Two Factors (*Section 2.2*)

This extension of the completely randomized design permits investigation of one set of variables in combination with some other set. For example, instead of being interested only in the effects of drugs, an experimenter might be interested in determining the effects of drugs in combination with varying amounts of sleep loss. In the simplest case, two drugs, A and B, would be paired with no sleep loss versus twenty-four-hour loss. This would result in the formation of four groups: Group 1—Drug A and no sleep loss; Group 2—Drug A and twenty-four-hour loss; Group 3—Drug B and no sleep loss; Group 4—Drug B and twenty-four-hour loss. Thus, it is possible to determine not only the effects of drugs and sleep loss but also whether the effects of Drugs A and B were enhanced or suppressed by sleep loss. Stated another way, it is possible to determine whether the variables interact.

The experimental procedure for the two-factor design is basically the same as for the completely randomized design. The subjects would be randomly assigned to the four (in this example) groups, sleep loss would be manipulated, and then the drugs would be administered. Again, after measuring performance, statistical analyses of the data would be undertaken.

Factorial Design: Three Factors (*Section 2.3*)

This is a commonly used extension of the two-factor design. The only difference is that a third factor or dimension is added. For example, an experimenter working with Drugs A and B and sleep versus sleep loss might also be interested in determining the effects of these factors in combination with a third factor, alcohol concentration in the blood. If he used only two concentrations, 0.0 and 0.2 per cent, the experimental groups would be constituted as follows: Group 1—Drug A, no sleep loss, no alcohol; Group 2—Drug A, no sleep loss, alcohol; Group 3—Drug A, sleep loss, no alcohol; Group 4—Drug A, sleep loss, alcohol. Groups 5, 6, 7, and 8 would receive the above combinations of sleep loss and alcohol in combination with Drug B. The assignment of subjects, measurement of performance, and analysis of the results are basically the same as in the two-factor design.

Treatments-by-Levels Design (Section 2.4)

This represents a slight variation on the two-factor design discussed above. All the computations needed to analyze the treatments-by-levels design are the same as for the two-factor design; the only difference is in the intent of the design and interpretation of the results. In the treatments-by-levels, one of the factors usually represents some variable already known to affect performance. For example, if it were known that sleep loss generally heightens the effects of drugs, it would probably be wise for the experimenter to include different levels of sleep loss as a variable in his drug experiments. By so doing, variability due to the specific effects of different amounts of sleep loss could be statistically separated from the effects of the drugs.

Treatments-by-Subjects, or Repeated-Measures, Design (Section 2.5)

The treatments-by-subjects design is a variation of the completely randomized design. Instead of using several different groups of subjects with each group receiving a single drug, only one group of subjects would be used and each subject would receive all the drug treatments. If the treatments-by-subjects design were to be used for the drug experiment, the single group of subjects would probably receive one drug, perform the criterion task, and wait until the effects wore off. Then a second drug would be administered, the criterion task would be performed, and another waiting period would ensue. This same procedure would be repeated until every subject in the group had received every drug. Analysis of the findings would be based on the subjects' performance while under the influence of the several drugs. The major advantage of this design over the completely randomized design is that fewer numbers of subjects are required.[1] The major disadvantage is that there might be carry-over effects from one treatment to the next. In the present example, it is possible that exposure to one drug would heighten or lessen sensitivity to other drugs administered later. Also, the subjects might become progressively more proficient at performing the criterion task and show an improvement in performance that would be due to learning and not to the effects of the drugs.

Treatments-by-Treatments-by-Subjects Design (Section 2.6)

This two-factor design bears the same relationship to the treatments-by-subjects design as the factorial bears to the completely randomized design. The treatments-by-treatments-by-subjects design is basically the same as the two-factor factorial design, with one notable exception. Instead of different subjects being assigned to the experimental groups, one group of subjects performs under all experimental conditions. Using the example from the two-factor factorial design, an experiment using the treatments-by-treatments-by-subjects design would probably be conducted as follows. First, all subjects would receive Drug A with no prior sleep loss, perform the criterion task, and then wait for the effects to wear off. After a suitable period, the same subjects would again receive Drug A, but this time the drug would follow a twenty-four-hour sleep loss. After performance measures had been recorded and a sufficient period allowed for the effects of the drug to wear off, the same procedures would be repeated with Drug B. The major advantage of this design over the factorial is that far fewer subjects are required. As in the treatments-by-subjects design, the major disadvantage is that there may be carry-over effects from one treatment to the next.

---

[1] Very often the advantage of increased statistical power is also gained by using a repeated-measures design. The reason for this is that the random variability of a single subject from one measuring period to the next is usually much less than the variability introduced by measuring and comparing different subjects.

Two-Factor Mixed Design: Repeated Measures on One Factor (*Section 2.7*)

This design is essentially a combination of the completely randomized and treatments-by-subjects designs. It involves the assignment of subjects to several experimental groups. However, instead of taking only one measure, as in the completely randomized design, the performance of the subjects would be measured repeatedly to determine the effects of the experimental variables over a prolonged period of time. This recording of repeated measures is, of course, identical with the procedure used in the treatments-by-subjects design.

To further clarify, let us continue with the example used to illustrate the completely randomized design. In the two-factor mixed design, subjects would again be assigned to the several experimental groups, and each group would be administered a different drug. Then, the subjects' performance on the criterion task would be measured several times.

Since measures are recorded over several successive test periods, this design permits (1) comparison of the overall performance of the experimental groups (as in the completely randomized design), (2) evaluation of performance changes from one measuring period to the next (as in the treatments-by-subjects design), and (3) evaluation of the drug effects in relation to the passage of time between measuring periods. This particular design is very widely used in research in the behavioral sciences.

Three-Factor Mixed Design: Repeated Measures on One Factor (*Section 2.8*)

This extension of the two-factor mixed design is essentially a combination of the factorial and repeated-measures designs. In the three-factor mixed design, the experimental groups are formed and subjects are assigned in exactly the same manner as in the factorial design. The basic difference (using the example of the factorial design) is that the effects of the drugs in combination with sleep loss would be measured several times rather than only once. Consequently, this design permits not only the evaluation of the overall experimental effects (as in the factorial design) but also the evaluation of general changes and interactions of the variables over the passage of time (as in the repeated-measures designs).

This particular design is probably the one used most often in behavioral science research. There are several slight variations of the format presented here, the most common of which involves presenting subjects with a task to be learned during a predetermined set of practice trials. In this situation, the repeated measures represent the learning rates of the subjects in the several experimental groups.

Three-Factor Mixed Design: Repeated Measures on Two Factors (Section 2.9)

This design is essentially a combination of the completely randomized and the treatments-by-treatments-by-subjects designs. As in the completely randomized design, subjects are randomly assigned to two or more experimental groups. For example, one group might be subjected to a twenty-four-hour sleep loss just prior to the experiment and the other group to no sleep loss. Each of these different groups would then be treated in the same fashion as in the treatments-by-treatments-by-subjects design. That is, two additional factors (say, Drug A versus Drug B and 0.0 versus 0.2 per cent alcohol) would be combined and presented to each subject in both of the experimental groups. Consequently, all subjects in the sleep-loss and no-sleep-loss groups would receive Drug A with *no* alcohol, perform the criterion task, wait for the effects to dissipate, then receive Drug A *with* alcohol, etc., until all variable combinations had been experienced. The major advantage of the three-factor mixed design over the regular three-factor design discussed above is that many fewer subjects are required to conduct the experiment. Carry-over effects probably represent the most serious drawback.

Latin Square Design: Simple (Section 2.10)

This design is a variation of the treatments-by-subjects, or repeated-measures, design. The basic difference in the Latin square design is that the presentation of the treatments is deliberately counterbalanced. Thus, instead of giving subjects the various drugs in the same order (Drug A, then B, then C, etc.) or randomly assigning the order of presentation of the drugs for each subject, a systematic order of presentations is set up so that order and carry-over effects can be ascertained. Assume that three drugs are to be investigated. If the simple Latin square design were to be used, one third of the subjects would receive Drug A first, then B, then C; a second third would receive Drug C first, then A, then B; and the final third would receive Drug B first, then C, then A. As indicated above, this design permits the analysis not only of the effects of the drugs but also of the order in which they were given.

Latin Square Design: Complex (Section 2.11)

This design can be viewed as a variation of the two-factor mixed design or as an extension of the simple Latin square. As in the two-factor mixed design, subjects would be randomly assigned to different experimental groups (like sleep loss versus no sleep loss). The drugs would then be administered to all subjects in both groups in the same manner as in the simple Latin square. Thus, one third of the subjects in the no-sleep-loss group would receive Drug A, then B, then C; a second third would receive Drug C, then A, then B; and a final third would receive Drug B, then C, then A. The same procedure would be used with the subjects in the sleep-loss group. Thus, this design permits not only the evaluation of the effects of the drugs and of sleep loss but also the evaluation of the effects of the order of drug administration in relation to the other two factors.

In Parts 2 and 3 of this book, only the verbal instructions for computing the analyses and the computed examples are presented. In place of symbols and formulas, the textbook sources of the several analyses are listed at the beginning of each section. The reasons for this are twofold. First, several of the more complex analyses require up to twenty separate computational formulas, plus additional ones for retabling the data. Second, and more important, there is no standard set of terms and symbols to indicate analysis-of-variance operations. These tend to be specific to each text, and translation from one text to another is difficult. To include all the symbols and terms currently in use or to develop a set of our own is beyond the scope of this book.

In Part 3, specific supplemental computations and the uses of multiple comparisons employed in conjunction with analyses of variance are discussed. Because of the relative specificity of these computations, the reader is encouraged to read the introductory comments at the beginning of Part 3 to determine whether a particular test is appropriate to his experimental findings.

SECTION 2.1

Completely Randomized Design

One of the more simple experimental designs that permit comparison of several groups is the *simple randomized*, or *completely randomized*, *design*. This design is known by various names and is treated in the following statistics textbooks.

Carlborg, F. W. *Introduction to Statistics.* Glenview, Ill.: Scott, Foresman and Company, 1968. Chapter 7, pp. 208–223.

Cochran, W. G., and Cox, G. M. *Experimental Designs*, 2nd ed. New York: John Wiley & Sons, Inc., 1957. Chapter 4, pp. 95–97.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 9, pp. 117–125.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education.* Boston: Houghton Mifflin Company, 1953. Chapter 3, pp. 47–66.

McNemar, Q. *Psychological Statistics*, 3rd. ed. New York: John Wiley & Sons, Inc., 1962. Chapter 15, pp. 252–270.

Ray, W. S. *Statistics in Psychological Research.* New York: The Macmillan Company, 1962. Chapter 15, pp. 254–262.

Walker, H. M., and Lev, J. *Statistical Inference.* New York: Holt, Rinehart & Winston, 1953. Chapter 9, pp. 196–216.

Winer, B. J. *Statistical Principles in Experimental Design.* New York: McGraw-Hill Book Company, 1962. Chapter 3, pp. 56–70.

Points to Consider When Using the Completely Randomized Design

1. For each subject in the experimental groups, there can be only *one* score. If many measures are taken for each subject, these measures must be combined in some fashion (added, etc.) so that there is only one score for each subject.
2. Although it is not necessary, it is usually best to have an equal number of subjects in each of the experimental groups. In most experiments, ten to fifteen subjects per group are used.
3. The number of experimental groups that may be compared is arbitrary. However, it is rare that more than four or five groups are compared in one experiment.

EXAMPLE

The example used to demonstrate the computational procedures is drawn from psychology. Assume that the experimenter is interested in determining the effect of shock on the time required to solve a set of difficult problems. Subjects are randomly assigned to four experimental conditions. Subjects in Group 1 receive no shock; Group 2, very-low-intensity shocks; Group 3, medium shocks; and Group 4, high-intensity shocks. The total time required to solve all the problems is the measure recorded for each subject.

Step 1. After the experiment has been conducted and the data collected, table the data as follows.

Group 1 (No Shock)		Group 2 (Low Shock)		Group 3 (Medium Shock)		Group 4 (High Shock)	
Subject	Time (min.)	Subject	Time (min.)	Subject	Time (min.)	Subject	Time (min.)
S_1	10	S_{13}	3	S_{25}	19	S_{37}	23
S_2	7	S_{14}	8	S_{26}	12	S_{38}	14
S_3	9	S_{15}	7	S_{27}	16	S_{39}	16
S_4	8	S_{16}	5	S_{28}	14	S_{40}	18
S_5	15	S_{17}	6	S_{29}	7	S_{41}	12
S_6	3	S_{18}	10	S_{30}	8	S_{42}	13
S_7	8	S_{19}	12	S_{31}	13	S_{43}	16
S_8	9	S_{20}	4	S_{32}	10	S_{44}	17
S_9	11	S_{21}	7	S_{33}	19	S_{45}	19
S_{10}	9	S_{22}	6	S_{34}	9	S_{46}	14
S_{11}	5	S_{23}	5	S_{35}	15	S_{47}	16
S_{12}	17	S_{24}	15	S_{36}	14	S_{48}	17

Step 2. Add the scores in each group to get the sum of *each group.* (*Note:* If you are using a calculator, Steps 2 and 3 can be done at the same time.)

	Group 1	Group 2	Group 3	Group 4
	10	3	19	23
	7	8	12	14
	⋮	⋮	⋮	⋮
Sums:	**111**	**88**	**156**	**195**

Step 3. Square each number in the table, and add these squared values together.

$$10^2 + 7^2 + \cdots + 17^2 + 3^2 + 8^2 + \cdots + 15^2 + 19^2 + 12^2 + \cdots + 14^2 + 23^2 + 14^2 + \cdots + 17^2 = \mathbf{7434}$$

Step 4. Add all the group sums (Step 2) together to obtain a grand total for the entire table.

$$111 + 88 + 156 + 195 = \mathbf{550}$$

Step 5. Square the grand total (Step 4), and divide the squared value by the total number of measures recorded (see Step 1). (In the present example, the latter number is 48, since there are 4 groups of 12 subjects each.) The value obtained is the correction term.

$$\frac{550^2}{48} = \frac{302{,}500}{48} = \mathbf{6302}$$

Step 6. Subtract the correction term (Step 5) from the sum of the squared values obtained in Step 3. The resultant value is the *sum of squares total,* or SS_t.

$$7434 - 6302 = \mathbf{1132}$$

Step 7. Square the sum of each of the groups (Step 2), divide by the number of measures in each group, and then add these values.

$$\frac{111^2}{12} + \frac{88^2}{12} + \frac{156^2}{12} + \frac{195^2}{12} = \frac{82{,}426}{12} = \mathbf{6869}$$

Step 8. Subtract the correction term (Step 5) from the value obtained in Step 7. The resultant value is the *sum of squares between groups,* or SS_b.

$$6869 - 6302 = \mathbf{567}$$

Step 9. Subtract the sum of squares between groups (Step 8) from the sum of squares total (Step 6). This yields the *sum of squares within groups,* or SS_w.

$$1132 - 567 = \mathbf{565}$$

Step 10. All computations based directly on the data have now been completed. However, since the test of significance (F-test) is a ratio of the mean squares, these values must be computed. To compute the needed mean squares, the degrees of freedom (*df*) for each of the components (SS_t, SS_b, and SS_w) must be determined, as follows.

df for SS_t = the total number of measures (see Step 1) minus 1.

$$48 - 1 = \mathbf{47}$$

df for SS_b = the number of experimental groups minus 1.

$$4 - 1 = \mathbf{3}$$

df for SS_w = the total *df* minus the between *df*.

$$47 - 3 = \mathbf{44}$$

Step 11. The mean squares are then computed as SS/df.

$$\text{ms}_t = \frac{\text{SS}_t}{df} \qquad \text{(This value is not needed for the analysis.)}$$

$$\text{ms}_b = \frac{\text{SS}_b}{df} = \frac{\text{(Step 8)}}{3} = \frac{567}{3} = \mathbf{189}$$

$$\text{ms}_w = \frac{\text{SS}_w}{df} = \frac{\text{(Step 9)}}{44} = \frac{565}{44} = \mathbf{12.84}$$

Step 12. The test of significance (F) is equal to ms_b/ms_w.

$$F = \frac{189}{12.84} = \mathbf{14.71}$$

Step 13. Table the final analysis as follows.

Source	SS	df	ms	F	p
Total	1132	47	—	—	—
Between groups	567	3	189	14.71	$<.001$
Within groups	565	44	12.84	—	—

Since the F value (see Appendix D) of 14.71, with df of 3 and 44, would occur by chance less than once in one thousand times, it is concluded that level of shock intensity does affect the time required to solve these problems.

SECTION 2.2

Factorial Design: Two Factors

Many times an experimenter wishes to determine the effects of two variables in combination with each other. For example, instead of investigating the effects of differing list difficulty on performance (memorizing the lists) in one experiment and then doing another experiment to determine the effects of varying room noise on performance, he may well wish to investigate these variables in combination with each other. The two-factor design, which permits such comparisons, is discussed in the following textbooks.

Carlborg, F. W. *Introduction to Statistics.* Glenview, Ill.: Scott, Foresman and Company, 1968. Chapter 7, pp. 242–251.
Cochran, W. G., and Cox, G. M. *Experimental Designs*, 2nd ed. New York: John Wiley & Sons, Inc., 1957. Chapter 5, pp. 148–155.
Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education.* Boston: Houghton Mifflin Company, 1953. Chapter 9, pp. 207–219.
McNemar, Q. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 16, pp. 303–317.
Walker, H. M., and Lev, J. *Statistical Inference.* New York: Holt, Rinehart & Winston, Inc., 1953. Chapter 14, pp. 348–363.
Winer, B. J. *Statistical Principles in Experimental Design.* New York: McGraw-Hill Book Company, 1962. Chapter 5, pp. 140–162, 228–247.

Points to Consider When Using the Two-Factor Factorial Design

1. For each subject in the experimental groups, there can be only *one* score. As in the simple randomized design, if many measures are taken for each subject, these must be combined (added, etc.) so that there is only one score for each subject.
2. It is usually best to have an equal number of subjects in each of the experimental groups. If equality cannot be obtained, the number of subjects in each of the groups should be proportional. In most experiments, ten to fifteen subjects per group are used.
3. The number of treatment groups within each factor that may be compared is arbitrary. However, it is rare that more than four or five groups are included in either of the two factors.

EXAMPLE

The example used to demonstrate the computational procedures is drawn from psychology. Assume that an experimenter is interested in determining the effects of high- versus low-intensity shock on the memorization of a hard versus an easy list of nonsense syllables. Subjects are randomly assigned to four experimental conditions. Subjects in Group 1 receive periodic low-intensity shocks and must memorize an easy list; Group 2, high shock and an easy list; Group 3, low shock and a hard list; and Group 4, high shock and a hard list. The total number of errors made by each subject is the measure recorded.

Step 1. After the experiment has been conducted, table the data as follows. (*Note:* If more than a 2 × 2 factorial—e.g., a 2 × 3, 3 × 3, etc.—is to be analyzed, simply table the additional treatment groups in their appropriate row or column. All other steps are the same.)

Group 1 (Low Shock–Easy List)		Group 2 (High Shock–Easy List)	
Subject	No. of Errors	Subject	No. of Errors
S_1	9	S_7	15
S_2	16	S_8	13
S_3	14	S_9	9
S_4	11	S_{10}	9
S_5	12	S_{11}	8
S_6	8	S_{12}	11
Group 3 (Low Shock–Hard List)		**Group 4 (High Shock–Hard List)**	
S_{13}	10	S_{19}	19
S_{14}	12	S_{20}	16
S_{15}	18	S_{21}	18
S_{16}	16	S_{22}	23
S_{17}	17	S_{23}	14
S_{18}	15	S_{24}	15

Step 2. Add the scores in each group. (*Note:* If you are using a calculator, Steps 2 and 3 can be done at the same time.)

	Group 1	Group 2	Group 3	Group 4
	9	15	10	19
	16	13	12	16
	⋮	⋮	⋮	⋮
Sums:	**70**	**65**	**88**	**105**

Step 3. Square each number in the entire table (Step 1), and add these squared values.

$$9^2 + 16^2 + \cdots + 15^2 + 13^2 + \cdots + 10^2 + 12^2 + \cdots + 19^2 + 16^2 + \cdots + 15^2 = \mathbf{4832}$$

Step 4. Add the sums of all the groups (Step 2) to get the grand sum of the entire table.

$$70 + 65 + 88 + 105 = \mathbf{328}$$

Then, square this value, and divide by the total number of measures recorded in the table (see Step 1). This yields the correction term.

$$\frac{328^2}{24} = \frac{107{,}584}{24} = \mathbf{4483}$$

Step 5. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 4) from the sum of the squared measures (Step 3).

$$4832 - 4483 = \mathbf{349} = SS_t$$

Step 6. Computation of the effects of the first factor (the overall effects of low shock versus high shock): First, add the sums of the two shock groups (see Step 2), disregarding the difficulty-of-list dimension.

$$70 + 88 = \mathbf{158} = \text{sum of the low-shock groups}$$

$$65 + 105 = \mathbf{170} = \text{sum of the high-shock groups}$$

Then, square the above sums, divide by the number of measures on which each of the sums was based, and add the quotients.

$$\frac{158^2}{12} + \frac{170^2}{12} = \frac{53{,}864}{12} = \mathbf{4489}$$

Then, subtract the correction term from the above value.

$$4489 - 4483 = \mathbf{6} = SS_{\text{shock}}$$

Step 7. Computation of the effects of the second factor (the overall effects of hard versus easy list): First, add the sums of the two same-list-difficulty groups (see Step 2), disregarding the shock dimension.

$$70 + 65 = \mathbf{135} = \text{sum of easy-list groups}$$

$$88 + 105 = \mathbf{193} = \text{sum of hard-list groups}$$

Then, square the above sums, divide by the number of measures on which each of the sums was based, and add the quotients.

$$\frac{135^2}{12} + \frac{193^2}{12} = \frac{55{,}474}{12} = \mathbf{4623}$$

Then, subtract the correction term from the above value.

$$4623 - 4483 = \mathbf{140} = SS_{\text{list}}$$

Step 8. Computation of the interactive effects of shock and list difficulty: First, square the sums of each of the experimental groups (Step 2), divide by the number of measures on which each sum was based, and then add the quotients.

$$\frac{70^2}{6} + \frac{65^2}{6} + \frac{88^2}{6} + \frac{105^2}{6} = \frac{27{,}894}{6} = \mathbf{4649}$$

Then, from this value, subtract the correction term (Step 4), SS_{shock} (Step 6), and SS_{list} (Step 7).

$$4649 - 4483 - 6 - 140 = \mathbf{20} = SS_{shock \times list}$$

Step 9. Computation of the error-term sum of squares (SS_{error}): Simply subtract SS_{shock} (Step 6), SS_{list} (Step 7), and $SS_{shock \times list}$ (Step 8) from SS_{total} (Step 5).

$$349 - 6 - 140 - 20 = \mathbf{183} = SS_{error}$$

Step 10. All computations based directly on the data have now been completed. However, since the *F* ratios are ratios of mean squares, the degrees of freedom (*df*) for each of the components must be computed.

df for SS_t = the total number of measures (see Step 1) minus 1.

$$24 - 1 = \mathbf{23}$$

df for SS_{shock} = the number of shock conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for SS_{list} = the number of list conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for $SS_{shock \times list}$ = the *df* for SS_{shock} times the *df* for SS_{list}.

$$1 \times 1 = \mathbf{1}$$

df for SS_{error} = the *df* for SS_t minus the *df* for SS_{shock}, SS_{list}, and $SS_{shock \times list}$.

$$23 - 1 - 1 - 1 = \mathbf{20}$$

Step 11. The mean squares are then computed as SS/*df*.

ms_t (This value is not needed for the analysis.)

$$ms_{shock} = \frac{(\text{Step 6})}{1} = \frac{6}{1} = \mathbf{6}$$

$$ms_{list} = \frac{(\text{Step 7})}{1} = \frac{140}{1} = \mathbf{140}$$

$$ms_{shock \times list} = \frac{(\text{Step 8})}{1} = \frac{20}{1} = \mathbf{20}$$

$$ms_{error} = \frac{(\text{Step 9})}{20} = \frac{183}{20} = \mathbf{9.15}$$

Step 12. The several *F* ratios are then computed as

$$\frac{ms_{shock}}{ms_{error}} \qquad \frac{ms_{list}}{ms_{error}} \qquad \frac{ms_{shock \times list}}{ms_{error}}$$

Step 13. Table the final analysis as follows.

Source	SS	*df*	ms	*F*	*p*
Total	349	23	—	—	—
Shock	6	1	6	.66	n.s.
List	140	1	140	15.30	<.001
Shock × List	20	1	20	2.19	n.s.
Error	183	20	9.15	—	—

Therefore, we conclude (see Appendix D) that shock had no significant effect on performance and that the interaction of shock by list difficulty was also nonsignificant. However, the effect of list difficulty was significant and would be expected to occur by chance less than once in one thousand times.

SECTION 2.3

Factorial Design: Three Factors

This design is a commonly used extension of the two-factor design discussed in Section 2.2. The one basic difference is that instead of determining the effects of two variables in combination with each other, this design permits evaluation of three variables. The three-factor design is discussed in the following textbooks.

Cochran, W. G., and Cox, G. M. *Experimental Designs*, 2nd ed. New York: John Wiley & Sons, Inc., 1957. Chapter 5, pp. 155–156.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 12, pp. 175–210.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 10, p. 243.

McNemar, Q. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 16, pp. 318–338.

Winer, B. J. *Statistical Principles in Experimental Design.* New York: McGraw-Hill Book Company, 1962. Chapter 5, pp. 162–174, 248–263.

Points to Consider When Using the Three-Factor Factorial Design

1. For each subject in the experimental groups, there can be only *one* score. As in the two-factor design, if many measures are taken for each subject, these must be combined (added, etc.) so that there is only one score for each subject.
2. It is usually best to have an equal number of subjects in each experimental group. In most experiments, ten to fifteen subjects per group are used.
3. The number of treatment groups within each factor that may be compared is arbitrary. However, it is rare that more than four or five groups are included in any of the factors.

EXAMPLE

The example used to demonstrate the computational procedures is drawn from the field of psychology and is essentially an extension of the one used in Section 2.2. Assume that the experimenter, in addition to determining the effects of high- versus low-intensity shock on the memorization of easy versus difficult lists, is also concerned with determining the effects of rate of list presentation on learning in the above conditions. Obviously, the experimenter could conduct one two-factor experiment at a fast rate of presentation and another at a slow rate, or he could include rate of presentation as an additional dimension and conduct the entire experiment as one three-factor design.

In the present example, then, the variables are high versus low shock, hard versus easy list, and fast versus slow rate of presentation. Subjects are randomly assigned to one of eight experimental conditions. Subjects in Group 1 receive periodic low-intensity shock and must memorize an easy list presented at a slow rate; Group 2—low-intensity shock, easy list, fast rate; Group 3—low-intensity shock, hard list, slow rate; Group 4—low-intensity shock, hard list, fast rate. Groups 5, 6, 7, and 8 receive the same treatment as Groups 1, 2, 3, and 4, except that high-intensity shocks are administered: Group 5—high-intensity shock, easy list, slow rate; Group 6—high-intensity shock, easy list, fast rate; Group 7—high-intensity shock, hard list, slow rate; Group 8—high-intensity shock, hard list, fast rate.

The total number of errors made by each subject is the measure recorded.

Step 1. After the experiment has been completed, table the data as follows. (*Note:* If other than a 2 × 2 × 2 factorial—e.g., 2 × 2 × 3, 3 × 2 × 3, etc.—is to be analyzed, simply table the additional treatment groups in their appropriate row or column. All other steps are the same.)

Group 1 (Low, Easy, Slow)		Group 2 (Low, Easy, Fast)		Group 3 (Low, Hard, Slow)		Group 4 (Low, Hard, Fast)	
Subject	No. of Errors	Subject	No. of Errors	Subject	No. of Errors	Subject	No. of Errors
S_1	15	S_7	23	S_{13}	11	S_{19}	25
S_2	8	S_8	16	S_{14}	16	S_{20}	27
S_3	9	S_9	17	S_{15}	8	S_{21}	29
S_4	7	S_{10}	17	S_{16}	14	S_{22}	34
S_5	8	S_{11}	14	S_{17}	20	S_{23}	38
S_6	4	S_{12}	11	S_{18}	16	S_{24}	26

Group 5 (High, Easy, Slow)		Group 6 (High, Easy, Fast)		Group 7 (High, Hard, Slow)		Group 8 (High, Hard, Fast)	
S_{25}	14	S_{31}	24	S_{37}	11	S_{43}	39
S_{26}	6	S_{32}	16	S_{38}	17	S_{44}	32
S_{27}	3	S_{33}	10	S_{39}	12	S_{45}	38
S_{28}	5	S_{34}	18	S_{40}	14	S_{46}	39
S_{29}	6	S_{35}	13	S_{41}	21	S_{47}	33
S_{30}	7	S_{36}	17	S_{42}	18	S_{48}	31

Step 2. Add the scores in each group. (*Note:* If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

	Group 1	Group 2	Group 3	Group 4
	15	23	11	25
	8	16	16	27
	⋮	⋮	⋮	⋮
Sums:	**51**	**98**	**85**	**179**

	Group 5	Group 6	Group 7	Group 8
	14	24	11	39
	6	16	17	32
	⋮	⋮	⋮	⋮
Sums:	**41**	**98**	**93**	**212**

Step 3. Square each number in the entire table (Step 1), and add these squared values.

$$15^2 + 8^2 + \cdots + 31^2 = \mathbf{20{,}083}$$

Step 4. Add all the group sums (Step 2) to get the grand sum of the entire table.

$$51 + 98 + 85 + 179 + 41 + 98 + 93 + 212 = \mathbf{857}$$

Then, square the above value and divide by the number of scores added (recorded in Step 1) to obtain the grand sum. (In the present example, 48 scores were added.) This yields the correction term.

$$\frac{857^2}{48} = \frac{734{,}449}{48} = \mathbf{15{,}301}$$

Step 5. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 4) from the sum of the squared measures (Step 3).

$$20{,}083 - 15{,}301 = \mathbf{4782} = SS_t$$

Step 6. Computation of the effects of the first factor (the overall effects of high versus low shock): First, add the scores of the two same-intensity-shock groups (see Step 2), disregarding the difficulty-of-list and rate-of-presentation dimensions.

$$51 + 98 + 85 + 179 = \mathbf{413} = \text{sum of the low-shock groups}$$

$$41 + 98 + 93 + 212 = \mathbf{444} = \text{sum of the high-shock groups}$$

Then, square the above sums, divide by the number of measures on which each of the sums was based, and add the quotients.

$$\frac{413^2}{24} + \frac{444^2}{24} = \frac{367{,}705}{24} = \mathbf{15{,}321}$$

Then, subtract the correction term (Step 4) from the above value.

$$15{,}321 - 15{,}301 = \mathbf{20} = SS_{\text{shock}}$$

Step 7. Computation of the effects of the second factor (the overall effects of hard versus easy list): First, sum the scores of the two same-list-difficulty groups (Step 2), disregarding the shock-intensity and rate-of-presentation dimensions.

$$51 + 98 + 41 + 98 = \mathbf{288} = \text{sum of the easy-list groups}$$

$$85 + 179 + 93 + 212 = \mathbf{569} = \text{sum of the hard-list groups}$$

Then, square the above sums, divide by the number of measures on which each was based, and add the quotients.

$$\frac{288^2}{24} + \frac{569^2}{24} = \frac{406{,}705}{24} = \mathbf{16{,}946}$$

Then, subtract the correction term (Step 4) from the above value.

$$16{,}946 - 15{,}301 = \mathbf{1645} = SS_{\text{list}}$$

Step 8. Computation of the effects of the third factor (the overall effects of fast versus slow rate of presentation): First, sum the scores of the two same-rate-of-presentation groups (Step 2), disregarding the shock-intensity and list-difficulty dimensions.

$$51 + 85 + 41 + 93 = \mathbf{270} = \text{sum of the slow-rate groups}$$

$$98 + 179 + 98 + 212 = \mathbf{587} = \text{sum of the fast-rate groups}$$

Then, square the above sums, divide by the number of measures on which each was based, and add the quotients.

$$\frac{270^2}{24} + \frac{587^2}{24} = \frac{417{,}469}{24} = \mathbf{17{,}394}$$

Then, subtract the correction term from the above value.

$$17{,}394 - 15{,}301 = \mathbf{2093} = SS_{\text{rate}}$$

Step 9. Computation of the interactive effects of the first and second factors (shock × list): First, sum the scores of the groups which have the same pairings of shock intensity and list difficulty, disregarding the rate-of-presentation dimension.

$$51 + 98 = \mathbf{149} = \text{sum of the low-shock, easy-list groups}$$
$$85 + 179 = \mathbf{264} = \text{sum of the low-shock, hard-list groups}$$
$$41 + 98 = \mathbf{139} = \text{sum of the high-shock, easy-list groups}$$
$$93 + 212 = \mathbf{305} = \text{sum of the high-shock, hard-list groups}$$

Then, square the above sums, divide by the number of scores on which each sum was based, and add the quotients.

$$\frac{149^2}{12} + \frac{264^2}{12} + \frac{139^2}{12} + \frac{305^2}{12} = \frac{204{,}243}{12} = \mathbf{17{,}020}$$

Then, from this value, subtract the correction term (Step 4), the SS_{shock} (Step 6), and the SS_{list} (Step 7).

$$17{,}020 - 15{,}301 - 20 - 1645 = \mathbf{54} = SS_{\text{shock} \times \text{list}}$$

Step 10. Computation of the interactive effects of the first and third factors (shock × rate of presentation): First, sum the scores of the groups that have the same pairings of shock intensity and rate of presentation. Disregard the difficulty-of-list dimension.

$$51 + 85 = \mathbf{136} = \text{sum of the low-shock, slow-rate groups}$$
$$98 + 179 = \mathbf{277} = \text{sum of the low-shock, fast-rate groups}$$
$$41 + 93 = \mathbf{134} = \text{sum of the high-shock, slow-rate groups}$$
$$98 + 212 = \mathbf{310} = \text{sum of the high-shock, fast-rate groups}$$

Then, square the above sums, divide by the number of scores on which each is based, and add the quotients.

$$\frac{136^2}{12} + \frac{277^2}{12} + \frac{134^2}{12} + \frac{310^2}{12} = \frac{209{,}281}{12} = \mathbf{17{,}440}$$

Then, from the above value, subtract the correction term (Step 4), the SS_{shock} (Step 6), and the SS_{rate} (Step 8).

$$17{,}440 - 15{,}301 - 20 - 2093 = \mathbf{26} = SS_{shock \times rate}$$

Step 11. Computation of the interactive effects of the second and third factors (difficulty × rate of presentation): First, sum the scores of the groups that have the same pairings of difficulty of material and rate of presentation. Disregard the shock dimension.

$51 + 41 = \mathbf{92}$ = sum of the easy-list, slow-rate groups
$98 + 98 = \mathbf{196}$ = sum of the easy-list, fast-rate groups
$85 + 93 = \mathbf{178}$ = sum of the hard-list, slow-rate groups
$179 + 212 = \mathbf{391}$ = sum of the hard-list, fast-rate groups

Then, square the above sums, divide by the number of scores on which each is based, and add the quotients.

$$\frac{92^2}{12} + \frac{196^2}{12} + \frac{178^2}{12} + \frac{391^2}{12} = \frac{231{,}445}{12} = \mathbf{19{,}287}$$

Then, from the above value, subtract the correction term (Step 4), the SS_{list} (Step 7), and the SS_{rate} (Step 8).

$$19{,}287 - 15{,}301 - 1645 - 2093 = \mathbf{248} = SS_{list \times rate}$$

Step 12. Computation of the interactive effects of the first, second, and third factors (shock × list difficulty × rate of presentation): First, square the sums of each of the experimental groups (Step 2), divide by the number of measures on which each sum was based, and then add the quotients.

$$\frac{51^2}{6} + \frac{98^2}{6} + \frac{85^2}{6} + \frac{179^2}{6} + \frac{41^2}{6} + \frac{98^2}{6} + \frac{93^2}{6} + \frac{212^2}{6} = \frac{116{,}349}{6} = \mathbf{19{,}391}$$

Then, from this value, subtract the correction term (Step 4), the SS_{shock} (Step 6), the SS_{list} (Step 7), the SS_{rate} (Step 8), the $SS_{shock \times list}$ (Step 9), the $SS_{shock \times rate}$ (Step 10), and the $SS_{list \times rate}$ (Step 11).

$$19{,}391 - 15{,}301 - 20 - 1645 - 2093 - 54 - 26 - 248 = \mathbf{4} = SS_{shock \times list \times rate}$$

Step 13. Computation of the error-term sum of squares (SS_{error}): Simply subtract SS_{shock} (Step 6), SS_{list} (Step 7), SS_{rate} (Step 8), $SS_{shock \times list}$ (Step 9), $SS_{shock \times rate}$ (Step 10), $SS_{list \times rate}$ (Step 11), and $SS_{shock \times list \times rate}$ (Step 12) from the SS_{total} (Step 5).

$$4782 - 20 - 1645 - 2093 - 54 - 26 - 248 - 4 = \mathbf{692} = SS_{error}$$

Step 14. All of the computations based directly on the data have been completed. However, since the F ratios are ratios of mean squares, the degrees of freedom (*df*) for each of the components must be computed.

df for $\mathrm{SS_t}$ = the total number of measures (see Step 1) minus 1.

$$48 - 1 = \mathbf{47}$$

df for $\mathrm{SS_{shock}}$ = the number of shock conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for $\mathrm{SS_{list}}$ = the number of list difficulties minus 1.

$$2 - 1 = \mathbf{1}$$

df for $\mathrm{SS_{rate}}$ = the number of rates of presentation minus 1.

$$2 - 1 = \mathbf{1}$$

df for $\mathrm{SS_{shock \times list}}$ = the *df* for shock times the *df* for list.

$$1 \times 1 = \mathbf{1}$$

df for $\mathrm{SS_{shock \times rate}}$ = the *df* for shock times the *df* for rate.

$$1 \times 1 = \mathbf{1}$$

df for $\mathrm{SS_{list \times rate}}$ = the *df* for list times the *df* for rate.

$$1 \times 1 = \mathbf{1}$$

df for $\mathrm{SS_{shock \times list \times rate}}$ = the *df* for shock times the *df* for list times the *df* for rate.

$$1 \times 1 \times 1 = \mathbf{1}$$

df for $\mathrm{SS_{error}}$ = the *df* for $\mathrm{SS_t}$ minus the *df*s for $\mathrm{SS_{shock}}$, $\mathrm{SS_{list}}$, $\mathrm{SS_{rate}}$, $\mathrm{SS_{shock \times list}}$, $\mathrm{SS_{list \times rate}}$, $\mathrm{SS_{shock \times rate}}$, and $\mathrm{SS_{shock \times list \times rate}}$.

$$47 - 1 - 1 - 1 - 1 - 1 - 1 - 1 = \mathbf{40}$$

Step 15. The mean squares are then computed as SS/*df*.

$\mathrm{ms_t}$ (This value is not needed for the analysis.)

$$\mathrm{ms_{shock}} = \frac{\text{(Step 6)}}{1} = \frac{20}{1} = \mathbf{20}$$

$$\mathrm{ms_{list}} = \frac{\text{(Step 7)}}{1} = \frac{1645}{1} = \mathbf{1645}$$

$$\mathrm{ms_{rate}} = \frac{\text{(Step 8)}}{1} = \frac{2093}{1} = \mathbf{2093}$$

$$\text{ms}_{\text{shock} \times \text{list}} = \frac{(\text{Step } 9)}{1} = \frac{54}{1} = 54$$

$$\text{ms}_{\text{shock} \times \text{rate}} = \frac{(\text{Step } 10)}{1} = \frac{26}{1} = 26$$

$$\text{ms}_{\text{list} \times \text{rate}} = \frac{(\text{Step } 11)}{1} = \frac{248}{1} = 248$$

$$\text{ms}_{\text{shock} \times \text{list} \times \text{rate}} = \frac{(\text{Step } 12)}{1} = \frac{4}{1} = 4$$

$$\text{ms}_{\text{error}} = \frac{(\text{Step } 13)}{40} = \frac{692}{40} = 17.30$$

Step 16. The several F ratios are then computed as

$$\frac{\text{ms}_{\text{shock}}}{\text{ms}_{\text{error}}} \qquad \frac{\text{ms}_{\text{shock} \times \text{list}}}{\text{ms}_{\text{error}}}$$

$$\frac{\text{ms}_{\text{list}}}{\text{ms}_{\text{error}}} \qquad \frac{\text{ms}_{\text{shock} \times \text{rate}}}{\text{ms}_{\text{error}}}$$

$$\frac{\text{ms}_{\text{rate}}}{\text{ms}_{\text{error}}} \qquad \frac{\text{ms}_{\text{list} \times \text{rate}}}{\text{ms}_{\text{error}}}$$

$$\frac{\text{ms}_{\text{shock} \times \text{list} \times \text{rate}}}{\text{ms}_{\text{error}}}$$

Step 17. Table the final analysis as follows.

Source	SS	*df*	ms	*F*	*p*
Total	4782	47	—	—	—
Shock	20	1	20	1.16	n.s.
List	1645	1	1645	95.09	<.001
Rate	2093	1	2093	120.98	<.001
Shock × list	54	1	54	3.12	n.s.
Shock × rate	26	1	26	1.50	n.s.
List × rate	248	1	248	14.34	<.001
Shock × list × rate	4	1	4	.23	n.s.
Error	692	40	17.30	—	—

Therefore, it is concluded (see Appendix D) that list difficulty and presentation rate affect error making in a word-memorizing situation and, also, that the effects of list difficulty and presentation rate are interactive.

SECTION 2.4

Treatments-by-Levels Design

Occasionally a researcher is faced with a group of subjects who either come from distinctly different populations or are tested at different times and places. In this case, he might reasonably expect the subjects in the various samples to differ in some systematic fashion. To control for this source of bias and confusion, a variation of the factorial design, the treatments-by-levels design, is appropriate. It is discussed in the following textbooks.[1]

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 12, pp. 215–219.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 5, pp. 121–155; Chapter 7, pp. 172–189.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 5, pp. 213–216.

Points to Consider When Using the Treatments-by-Levels Design

1. For each subject in the experimental groups, there can be only *one* score. As in the factorial designs, if many measures are taken for each subject, these must be combined so that there is only one score for each subject.
2. It is best to have an equal number of subjects in each experimental group and within each level or replication. In most experiments, ten to fifteen subjects per group are used.
3. The number of treatment groups is arbitrary. The same is true of the number of levels or replications employed. However, it is rare that more than four or five groups or levels are employed.

EXAMPLE

One of the most common uses of the treatments-by-levels design occurs in the field of education when children are drawn from various schools that differ markedly in quality and location. For this example, assume that an experimenter wishes to investigate the effectiveness of three different methods of teaching arithmetic. Because he has reason to believe that the success of the teaching methods depends on the overall quality of the school and the type of students enrolled, he decides to select one school from an upper-class neighborhood, another from a middle-class area, and a third from a slum area. In each school, one class is taught by the usual teacher-only method, a second class by a teacher plus teaching machines, and the third by television plus teaching machines. An achievement test is given to each student at the beginning of the term and again at the end. The score recorded for each student is the amount of improvement (in test points) shown.

---

[1] The computational procedures given in this section are the same for all the textbook chapters listed. However, the philosophies underlying the various designs presented in these texts differ and should be considered before interpretations of the results are made.

Step 1. After the experiment has been completed, table the data as follows.

LEVEL 1 (UPPER-CLASS SCHOOL)

Group 1 (Teacher Only)		Group 2 (Teacher + Machines)		Group 3 (Television + Machines)	
Subject	Points	Subject	Points	Subject	Points
S_1	74	S_7	94	S_{13}	87
S_2	79	S_8	96	S_{14}	76
S_3	94	S_9	92	S_{15}	94
S_4	90	S_{10}	84	S_{16}	92
S_5	88	S_{11}	85	S_{17}	98
S_6	87	S_{12}	89	S_{18}	71

LEVEL 2 (MIDDLE-CLASS SCHOOL)

Group 4		Group 5		Group 6	
S_{19}	70	S_{25}	76	S_{31}	70
S_{20}	78	S_{26}	91	S_{32}	86
S_{21}	96	S_{27}	94	S_{33}	91
S_{22}	86	S_{28}	86	S_{34}	74
S_{23}	80	S_{29}	87	S_{35}	88
S_{24}	92	S_{30}	91	S_{36}	77

LEVEL 3 (LOWER-CLASS SCHOOL)

Group 7		Group 8		Group 9	
S_{37}	92	S_{43}	91	S_{49}	70
S_{38}	86	S_{44}	84	S_{50}	86
S_{39}	84	S_{45}	80	S_{51}	81
S_{40}	71	S_{46}	73	S_{52}	71
S_{41}	74	S_{47}	69	S_{53}	65
S_{42}	78	S_{48}	71	S_{54}	63

Step 2. Add the scores in each group. (*Note:* If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

Level 1

	Group 1	Group 2	Group 3
	74	94	87
	79	96	76
	⋮	⋮	⋮
Sums:	**512**	**540**	**518**

Level 2

	Group 4	Group 5	Group 6
	70	76	70
	78	91	86
	⋮	⋮	⋮
Sums:	**502**	**525**	**486**

Level 3

	Group 7	Group 8	Group 9
	92	91	70
	86	84	86
	⋮	⋮	⋮
Sums:	**485**	**468**	**436**

Step 3. Square each number in the entire table (Step 1), and add these squared values.

$$74^2 + 79^2 + \cdots + 63^2 = \mathbf{374{,}776}$$

Step 4. Add the sums of each group (Step 2) to get the grand sum of the entire table.

$$512 + 540 + 518 + 502 + 525 + 486 + 485 + 468 + 436 = \mathbf{4472}$$

Then, square this value, and divide by the total number of measures recorded in the table (see Step 1). This yields the correction term.

$$\frac{4472^2}{54} = \frac{19{,}998{,}784}{54} = \mathbf{370{,}347}$$

Step 5. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 4) from the sum of the squared measures (Step 3).

$$374{,}776 - 370{,}347 = \mathbf{4429} = SS_t$$

Step 6. Computation of the effects of levels (the overall performance of students in the upper- versus middle- versus lower-class schools): First, add the sums of the three levels, disregarding the other dimension, method of teaching (see Step 2).

$512 + 540 + 518 = \mathbf{1570}$ = sum for the upper-class school
$502 + 525 + 486 = \mathbf{1513}$ = sum for the middle-class school
$485 + 468 + 436 = \mathbf{1389}$ = sum for the lower-class school

Then, square the above sums, divide by the number of measures on which each of the sums was based, and add the quotients.

$$\frac{1570^2}{18} + \frac{1513^2}{18} + \frac{1389^2}{18} = \frac{6{,}683{,}390}{18} = \mathbf{371{,}299}$$

Then, subtract the correction term from the above value.

$$371{,}299 - 370{,}347 = \mathbf{952} = \mathrm{SS}_{\text{levels}}$$

Step 7. Computation of the treatment effects (the overall effects of teacher only versus teacher plus machines versus television plus machines): First, add the sums of the three treatment conditions, disregarding the levels dimension (see Step 2).

$512 + 502 + 485 = \mathbf{1499}$ = sum for teacher-only groups
$540 + 525 + 468 = \mathbf{1533}$ = sum for teacher-plus-machines groups
$518 + 486 + 436 = \mathbf{1440}$ = sum for television-plus-machines groups

Then, square the above sums, divide by the number of measures on which each was based, and add the quotients.

$$\frac{1499^2}{18} + \frac{1533^2}{18} + \frac{1440^2}{18} = \frac{6{,}670{,}690}{18} = \mathbf{370{,}594}$$

Then, subtract the correction term from the above value.

$$370{,}594 - 370{,}347 = \mathbf{247} = \mathrm{SS}_{\text{treatments}}$$

Step 8. Computation of the interactive effects of treatments and levels: First, square the sums of each of the experimental groups (Step 2), divide by the number of measures on which each of the sums was based, and then add the quotients.

$$\frac{512^2}{6} + \frac{540^2}{6} + \frac{518^2}{6} + \frac{502^2}{6} + \frac{525^2}{6} + \frac{486^2}{6}$$
$$+ \frac{485^2}{6} + \frac{468^2}{6} + \frac{436^2}{6} = \frac{2{,}230{,}238}{6} = \mathbf{371{,}706}$$

Then, subtract the correction term (Step 4), the $\mathrm{SS}_{\text{levels}}$ (Step 6), and the $\mathrm{SS}_{\text{treatments}}$ (Step 7) from the above value.

$$371{,}706 - 370{,}347 - 952 - 247 = \mathbf{160} = \mathrm{SS}_{\text{treatments} \times \text{levels}}$$

Step 9. Computation of the error-term sum of squares (SS_{error}): Simply subtract SS_{levels} (Step 6), $SS_{treatments}$ (Step 7), and $SS_{treatments \times levels}$ (Step 8) from SS_t (Step 5).

$$4429 - 952 - 247 - 160 = \mathbf{3070} = SS_{error}$$

Step 10. All computations based directly on the data have now been completed. Since the *F* ratios are ratios of mean squares, the degrees of freedom (*df*) must be computed for each of the components.

df for SS_t = the total number of measures (see Step 1) minus 1.

$$54 - 1 = \mathbf{53}$$

df for SS_{levels} = the number of levels used in the experiment minus 1.

$$3 - 1 = \mathbf{2}$$

df for $SS_{treatments}$ = the number of treatment conditions minus 1.

$$3 - 1 = \mathbf{2}$$

df for $SS_{treatments \times levels}$ = *df* for treatments times the *df* for levels.

$$2 \times 2 = \mathbf{4}$$

df for SS_{error} = the *df* for SS_t minus the *df*s for SS_{levels}, $SS_{treatments}$, and $SS_{treatments \times levels}$.

$$53 - 2 - 2 - 4 = \mathbf{45}$$

Step 11. The mean squares are then computed as SS/*df*.

ms_t (This value is not needed for the analysis.)

$$ms_{levels} = \frac{(\text{Step } 6)}{2} = \frac{952}{2} = \mathbf{476}$$

$$ms_{treatments} = \frac{(\text{Step } 7)}{2} = \frac{247}{2} = \mathbf{123.5}$$

$$ms_{treatments \times levels} = \frac{(\text{Step } 8)}{4} = \frac{160}{4} = \mathbf{40}$$

$$ms_{error} = \frac{(\text{Step } 9)}{45} = \frac{3070}{45} = \mathbf{68.22}$$

Step 12. The several *F* ratios are then computed as

$$\frac{ms_{levels}}{ms_{error}} \qquad \frac{ms_{treatments}}{ms_{error}} \qquad \frac{ms_{treatments \times levels}}{ms_{error}}$$

Step 13. Table the final analysis as follows.

Source	SS	*df*	ms	*F*	*p*
Total	4429	53	—	—	—
Levels	952	2	476	6.98	<.005
Treatments	247	2	123.5	1.81	n.s.
Treatments × levels	160	4	40	.58	n.s.
Error	3070	45	68.22	—	—

Therefore, it is concluded (see Appendix D) that while the expected differences in the overall arithmetic achievements of the students in the three schools were present, there were no differences due to the three methods of instruction and that the methods of instruction did not interact with the achievement level of the students.

SECTION 2.5

Treatments-by-Subjects, or Repeated-Measures, Design

Whenever two or more different treatments are given to the same subjects, a very powerful statistical design can be used. This design, which permits analysis of repeated measures on the same individuals, is known by various names and is treated in the following statistics textbooks.

Carlborg, F. W. *Introduction to Statistics*. Glenview, Ill.: Scott, Foresman and Company, 1968. Chapter 8, pp. 271–273.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 14, pp. 224–227.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 6, pp. 156–171.

McNemar, Q. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 16, pp. 294–303.

Ray, W. S. *Statistics in Psychological Research*. New York: The Macmillan Company, 1962. Chapter 15, pp. 262–268.[1]

---

[1] In this text, the design is treated as the "matched-group design." The computations are exactly the same as those in this chapter. The only difference is that instead of repeated measurements on single subjects, different subjects are matched, placed in different experimental groups, and then compared to determine the effects of the treatments.

Walker, H. M., and Lev, J. *Statistical Inference.* New York: Holt, Rinehart & Winston, Inc., 1953. Chapter 9, pp. 216–226.

Winer, B. J. *Statistical Principles in Experimental Design.* New York: McGraw-Hill Book Company, 1962. Chapter 4, pp. 105–124.

Points to Consider When Using the Treatments-by-Subjects Design

1. Each subject is tested under, and a score is entered for, each treatment condition.
2. The number of subjects employed and the number of treatments given each subject is arbitrary. However, most experiments use ten to fifteen subjects, and it is rare that the effects of more than four or five treatments are compared.

EXAMPLE

In addition to analyzing the effects of successive experimental manipulations, this design is often used to indicate and evaluate the stability of measures over a prolonged period. This use will be illustrated in the following example. Assume that an experimenter wishes to know whether the mean scores on an intelligence test remain stable from year to year or whether they differ to a degree larger than could be expected by chance alone. To answer this question, the experimenter chooses twenty subjects, all twelve years old, and obtains an IQ score for each subject. Then, each year after that, for three years, another IQ test is given to the same twenty students. The IQ scores on each of these four tests are the data recorded.

Step 1. At the end of four years, when the data have been collected, they are tabled as follows.

Subject	IQ—Age 12	IQ—Age 13	IQ—Age 14	IQ—Age 15
S_1	98	102	113	108
S_2	104	100	105	111
S_3	126	131	128	136
S_4	74	79	84	81
S_5	86	85	90	88
S_6	92	92	88	95
S_7	98	96	101	103
S_8	110	115	107	118
S_9	118	122	129	120
S_{10}	97	96	104	100
S_{11}	83	85	80	88
S_{12}	124	128	135	138
S_{13}	133	132	130	138
S_{14}	101	112	115	114
S_{15}	112	108	116	114
S_{16}	96	91	95	98
S_{17}	89	96	90	97
S_{18}	95	101	99	104
S_{19}	87	95	94	97
S_{20}	93	99	92	102

Step 2. Add the scores in each treatment (age level in the present example) to get the sum for each treatment. (*Note*: If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

Subject	Age 12	Age 13	Age 14	Age 15
S_1	98	102	113	108
S_2	104	100	105	111
⋮	⋮	⋮	⋮	⋮
Sums:	**2016**	**2065**	**2095**	**2150**

Step 3. Square each number in the table (Step 1), and add these squared values together.

$$98^2 + 104^2 + \cdots + 102^2 = \mathbf{886{,}376}$$

Step 4. Add the sums for each treatment (Step 2) to obtain a grand total for the entire table.

$$2016 + 2065 + 2095 + 2150 = \mathbf{8326}$$

Step 5. Square the grand total (Step 4), and divide by the total number of measures recorded (see Step 1). This yields the correction term.

$$\frac{8326^2}{80} = \mathbf{866{,}528}$$

Step 6. Computation of the total sum of squares (SS_t): Subtract the correction term (Step 5) from the sum of the squared values obtained in Step 3.

$$886{,}376 - 866{,}528 = \mathbf{19{,}848} = SS_t$$

Step 7. Add the four scores recorded for each subject, and write the twenty sums in a column.

Subject	Age 12		Age 13		Age 14		Age 15	Sums
S_1	98	+	102	+	113	+	108	**421**
S_2	104	+	100	+	105	+	111	**420**
⋮	⋮		⋮		⋮		⋮	⋮
S_{20}	93	+	99	+	92	+	102	**386**

Step 8. Square all the sums obtained in Step 7, and add the squares.

$$421^2 + 420^2 + \cdots + 386^2 = \mathbf{3{,}540{,}914}$$

Step 9. Divide the value obtained in Step 8 by the number of scores that were added to get each of the sums in Step 7. (In the present example, 4 numbers were added to get each of the sums.)

$$\frac{3{,}540{,}914}{4} = \mathbf{885{,}228}$$

Step 10. Computation of the sum of squares for subjects (SS_{subjects}): Subtract the correction term (Step 5) from the value obtained in Step 9.

$$885{,}228 - 866{,}528 = \mathbf{18{,}700} = SS_{\text{subjects}}$$

Step 11. Square each of the values obtained in Step 2, and add the squares.

$$2016^2 + 2065^2 + 2095^2 + 2150^2 = \mathbf{17{,}340{,}006}$$

Then, divide by the number of scores on which each sum in Step 2 was based. (In the present example, this would be 20.)

$$\frac{17{,}340{,}006}{20} = \mathbf{867{,}000}$$

Step 12. Computation of the sum of squares for treatments ($SS_{\text{treatments}}$): Subtract the correction term (Step 5) from the value of Step 11.

$$867{,}000 - 866{,}528 = \mathbf{472} = SS_{\text{treatments}}$$

Step 13. Computation of the error-term sum of squares (SS_{error}): Subtract the SS_{subjects} (Step 10) and the $SS_{\text{treatments}}$ (Step 12) from the SS_t (Step 6).

$$19{,}848 - 18{,}700 - 472 = \mathbf{676} = SS_{\text{error}}$$

Step 14. All computations based directly on the data have been completed. However, since the test of significance (*F* ratio) is a ratio of the mean squares, these values must be computed. To do this, the degrees of freedom (*df*) must be determined for each of the components, as follows.

df for SS_t = the total number of measures (see Step 1) minus 1.

$$80 - 1 = \mathbf{79}$$

df for SS_{subjects} = the total number of subjects used in the experiment minus 1.

$$20 - 1 = \mathbf{19}$$

df for $SS_{\text{treatments}}$ = the total number of treatments given each subject minus 1.

$$4 - 1 = \mathbf{3}$$

df for SS_{error} = the total *df* minus the *df* for subjects and the *df* for treatments.

$$79 - 19 - 3 = \mathbf{57}$$

Step 15. The mean squares are then computed as SS/*df*.

ms_{t} (This value is not needed for analysis.)
$\mathrm{ms}_{\mathrm{subjects}}$ (This is also not needed.)

$$\mathrm{ms}_{\mathrm{treatments}} = \frac{(\text{Step 12})}{3} = \frac{472}{3} = \mathbf{157.33}$$

$$\mathrm{ms}_{\mathrm{error}} \quad \frac{(\text{Step 13})}{57} = \frac{676}{57} = \mathbf{11.86}$$

Step 16. Table the analysis as follows. The test of significance (F) is equal to $\mathrm{ms}_{\mathrm{treatment}}/\mathrm{ms}_{\mathrm{error}}$.

Source	SS	*df*	ms	*F*	*p*
Total	19,848	79	—	—	—
Subjects	18,700	19	—	—	—
Treatments	472	3	157.33	13.27	<.001
Error	676	57	11.86	—	—

Since the F value of 13.27, with *df* of 3 and 57, would occur by chance less than once in one thousand times (see Appendix D), it is concluded that the IQ scores were not stable over the four-year period.

SECTION 2.6

Treatments-by-Treatments-by-Subjects Design

The treatments-by-treatments-by-subjects design is a very efficient analysis that can be used when all treatments in a two-factor experiment are administered to each subject. This design is treated in the following textbooks.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 17, pp. 306–309.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 10, pp. 237–238.

McNemar, Q. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 16, pp. 322–329.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 6, pp. 289–290.

Points to Consider When Using the Treatments-by-Treatments-by-Subjects Design

1. Each subject is tested under, and a score is entered for, each treatment condition within both of the factors.
2. The number of subjects employed and the number of treatments given within each factor is arbitrary. However, most experiments use ten to fifteen subjects, and it is rare that the effects of more than four or five treatments within either or both of the factors are compared.

EXAMPLE

Consider a memorization experiment where the experimenter wants to study the relationship between short-term and long-term retention on the one hand and easy and difficult items on the other. The experimenter has every subject learn both easy and difficult items, and he measures the retention of these items over short and long periods of time. In this experiment, the experimenter is not interested in analyzing performance over several trials. Therefore, he will consider only the total number of correct responses. (*Note:* If he were also to consider trials, this would add another dimension and make this a treatments-by-treatments-by-treatments-by-subjects design.) The scores recorded for each subject are the subject's total number of correct responses on the easy and difficult items when he is tested after a short and after a long period of time.

Step 1. Table the data as follows. Remember, each subject performs under each treatment condition within both factors.

	Short-Term Retention		Long-Term Retention	
Subject	Easy Items	Difficult Items	Easy Items	Difficult Items
S_1	83	100	75	40
S_2	100	83	71	41
S_3	100	100	50	43
S_4	89	92	74	47
S_5	100	90	83	62
S_6	100	100	100	70
S_7	92	92	72	44
S_8	100	89	83	62
S_9	50	54	33	38
S_{10}	100	100	72	50

Step 2. Add the scores in each treatment for both factors. (*Note:* If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

Subject	STR–Easy	STR–Difficult	LTR–Easy	LTR–Difficult
S_1	83	100	75	40
S_2	100	83	71	41
⋮	⋮	⋮	⋮	⋮
Sums:	**914**	**900**	**713**	**497**

Step 3. Square each number in the table (Step 1), and add the squares.

$$83^2 + 100^2 + \cdots + 50^2 = \mathbf{248{,}212}$$

Step 4. Add the sums for each treatment for both factors (Step 2) to obtain a grand total for the entire table.

$$914 + 900 + 713 + 497 = \mathbf{3024}$$

Step 5. Square the grand total of Step 4, and divide by the total number of measures recorded in Step 1 (40 in this example). This yields the correction term.

$$\frac{3024^2}{40} = \frac{9{,}144{,}576}{40} = \mathbf{228{,}614.4}$$

Step 6. Subtract the correction term (Step 5) from the sum of the squares obtained in Step 3.

$$248{,}212 - 228{,}614.4 = \mathbf{19{,}597.6} = SS_t$$

Step 7. Add all the scores recorded for each subject in Step 1, and write these sums in a column.

	Short-Term Retention				Long-Term Retention			
Subject	Easy Items		Difficult Items		Easy Items		Difficult Items	Sums
S_1	83	+	100	+	75	+	40	**298**
S_2	100	+	83	+	71	+	41	**295**
⋮	⋮		⋮		⋮		⋮	⋮
S_{10}	100	+	100	+	72	+	50	**322**

Step 8. Square all the sums obtained in Step 7, and add the squares.

$$298^2 + 295^2 + \cdots + 322^2 = \mathbf{937{,}872}$$

Step 9. Divide the value obtained in Step 8 by the number of scores that were added to get each of the sums in Step 7 (4 scores were added in this example).

$$\frac{937{,}872}{4} = \mathbf{234{,}468}$$

Step 10. Subtract the correction term of Step 5 from the value of Step 9.

$$234{,}468 - 228{,}614.4 = \mathbf{5853.6} = \mathrm{SS}_{\mathrm{subjects}}$$

Step 11. Computation of the sum of squares for the first factor (the differential effects of STR versus LTR—$\mathrm{SS}_{\mathrm{retention}}$): First, add the sums of the STR and the LTR treatments (Step 2), disregarding the difficulty-of-items dimension.

$$914 + 900 = \mathbf{1814} = \text{sum of the STR treatment}$$

$$713 + 497 = \mathbf{1210} = \text{sum of the LTR treatment}$$

Then, square the above sums, divide by the number of measures on which each sum was based (20 in this example), and add the quotients.

$$\frac{1814^2}{20} + \frac{1210^2}{20} = \frac{4{,}754{,}696}{20} = \mathbf{237{,}734.8}$$

Then, subtract the correction term (Step 5) from the above value.

$$237{,}734.8 - 228{,}614.4 = \mathbf{9120.4} = \mathrm{SS}_{\mathrm{retention}}$$

Step 12. Computation of the effects of the second factor (the effects of easy versus difficult items—$\mathrm{SS}_{\mathrm{difficulty}}$): First, sum the scores on the easy items and on the difficult items (Step 2), disregarding the retention dimension.

$$914 + 713 = \mathbf{1627} = \text{sum of the easy items}$$

$$900 + 497 = \mathbf{1397} = \text{sum of the difficult items}$$

Then, square the above sums, divide by the number of measures on which each sum was based (20 in this example), and add the quotients.

$$\frac{1627^2}{20} + \frac{1397^2}{20} = \frac{4{,}598{,}738}{20} = \mathbf{229{,}936.9}$$

Then, subtract the correction term (Step 5) from the above value.

$$229{,}936.9 - 228{,}614.4 = \mathbf{1322.5} = \mathrm{SS}_{\mathrm{difficulty}}$$

Step 13. Computation of the interactive effects of the first and second factors ($\mathrm{SS}_{\mathrm{retention} \times \mathrm{difficulty}}$): First, square the sums of each of the experimental treatments (Step 2), divide by the number of measures on which each sum was based (10 in this example), and add the quotients.

$$\frac{914^2}{10} + \frac{900^2}{10} + \frac{713^2}{10} + \frac{497^2}{10} = \frac{2{,}400{,}774}{10} = \mathbf{240{,}077.4}$$

Then, subtract the correction term (Step 5), the sum of squares for retention (Step 11), and the sum of squares for difficulty (Step 12) from the above value.

$$240{,}077.4 - 228{,}614.4 - 9120.4 - 1322.5 = \mathbf{1020.1} = \mathrm{SS}_{\text{retention} \times \text{difficulty}}$$

Step 14. Computation of the error term for retention ($\mathrm{SS}_{\text{error for retention}}$): This requires that the data be retabled as follows. For each subject, add the number of correct responses made to the tests of STR and those of LTR (disregarding the difficulty-of-items dimension).

Subject	STR	LTR
S_1	183 (83 + 100)	115 (75 + 40)
S_2	183 (100 + 83)	112 (71 + 41)
S_3	200	93
S_4	181	121
S_5	190	145
S_6	200	170
S_7	184	116
S_8	189	145
S_9	104	71
S_{10}	200	122

Then, square all the sums in the above table, and add the squares.

$$183^2 + 183^2 + \cdots + 122^2 = \mathbf{489{,}622}$$

Then, divide the above value by the number of scores added to get each of the sums entered in the table in Step 14 (2 in this example).

$$\frac{489{,}622}{2} = \mathbf{244{,}811}$$

Then, from this value, subtract the correction term (Step 5), the sum of squares for subjects (Step 10), and the sum of squares for retention (Step 11).

$$244{,}811 - 228{,}614.4 - 5853.6 - 9120.4 = \mathbf{1222.6} = \mathrm{SS}_{\text{error for retention}}$$

Step 15. Computation of the error term for item difficulty ($\mathrm{SS}_{\text{error for difficulty}}$): Again, the data must first be retabled. For each subject, add the number of correct responses made on the easy and on the difficult items (disregarding the retention conditions).

Subject	Easy	Difficult
S_1	158 (83 + 75)	140 (100 + 40)
S_2	171 (100 + 71)	124 (83 + 41)
S_3	150	143
S_4	163	139
S_5	183	152
S_6	200	170
S_7	164	136
S_8	183	151
S_9	83	92
S_{10}	172	150

Then, square all the sums in the above table, and add the squares.

$$158^2 + 171^2 + \cdots + 150^2 = \mathbf{472{,}632}$$

Then divide this value by the number of scores added to get each of the sums in the above table (2 in this example).

$$\frac{472{,}632}{2} = \mathbf{236{,}316}$$

Then, from this value, subtract the correction term (Step 5), the sum of squares for subjects (Step 10), and the sum of squares for difficulty (Step 12).

$$236{,}316 - 228{,}614.4 - 5853.6 - 1322.5 = \mathbf{525.5} = SS_{\text{error for difficulty}}$$

Step 16. Computation of the error term for retention-by-difficulty interaction ($SS_{\text{error for retention} \times \text{difficulty}}$): From the total sum of squares (Step 6), subtract the sum of squares for subjects (Step 10), sum of squares for retention (Step 11), sum of squares for difficulty (Step 12), sum of squares for difficulty by retention (Step 13), error for retention (Step 14), and error for difficulty (Step 15).

$$19{,}597.6 - 5853.6 - 9120.4 - 1322.5 - 1020.1 - 1222.6 - 525.5$$
$$= \mathbf{532.9} = SS_{\text{error for retention} \times \text{difficulty}}$$

Step 17. All computations based directly on the data have been completed. However, since the test of significance (F ratio) is a ratio of mean squares, the degrees of freedom must be determined for each of the components.

df for SS_t = the total number of measures recorded in Step 1 minus 1.

$$40 - 1 = \mathbf{39}$$

df for SS_{subjects} = the number of subjects used in the experiment minus 1.

$$10 - 1 = \mathbf{9}$$

df for $SS_{\text{retention}}$ = the number of retention conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for $SS_{\text{error for retention}}$ = the *df* for SS_{subjects} times the *df* for $SS_{\text{retention}}$.

$$9 \times 1 = \mathbf{9}$$

df for $SS_{\text{difficulty}}$ = the number of difficulty conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for $SS_{\text{error for difficulty}}$ = the *df* for SS_{subjects} times the *df* for $SS_{\text{difficulty}}$.

$$9 \times 1 = \mathbf{9}$$

df for $SS_{\text{retention} \times \text{difficulty}}$ = the *df* for $SS_{\text{retention}}$ times the *df* for $SS_{\text{difficulty}}$.

$$1 \times 1 = \mathbf{1}$$

df for $SS_{\text{error for retention} \times \text{difficulty}}$ = the *df* for SS_{subjects} times the *df* for $SS_{\text{retention}}$ times the *df* for $SS_{\text{difficulty}}$.

$$9 \times 1 \times 1 = \mathbf{9}$$

Step 18. The desired mean squares are then computed as SS/*df*.

$$ms_{\text{retention}} = \frac{(\text{Step } 11)}{1} = \frac{9120.4}{1} = \mathbf{9120.4}$$

$$ms_{\text{difficulty}} = \frac{(\text{Step } 12)}{1} = \frac{1322.5}{1} = \mathbf{1322.5}$$

$$ms_{\text{difficulty} \times \text{retention}} = \frac{(\text{Step } 13)}{1} = \frac{1020.1}{1} = \mathbf{1020.1}$$

$$ms_{\text{retention error}} = \frac{(\text{Step } 14)}{9} = \frac{1222.6}{9} = \mathbf{135.844}$$

$$ms_{\text{difficulty error}} = \frac{(\text{Step } 15)}{9} = \frac{525.5}{9} = \mathbf{58.389}$$

$$ms_{\text{difficulty} \times \text{retention error}} = \frac{(\text{Step } 16)}{9} = \frac{532.9}{9} = \mathbf{59.211}$$

Step 19. Table the analysis as follows. The tests of significance (*F* ratios) are computed as

$$\frac{ms_{\text{retention}}}{ms_{\text{error for retention}}} \qquad \frac{ms_{\text{difficulty}}}{ms_{\text{error for difficulty}}}$$

$$\frac{ms_{\text{difficulty} \times \text{retention}}}{ms_{\text{error for difficulty} \times \text{retention}}}$$

Source	SS	*df*	ms	*F*	*p*
Total	19,597.6	39	—	—	—
Subjects	5853.6	9	—	—	—
Retention	9120.4	1	9120.4	67.14	<.001
Difficulty	1322.5	1	1322.5	22.65	<.005
Difficulty × retention	1020.1	1	1020.1	17.23	<.005
Error retention	1222.6	9	135.84	—	—
Error difficulty	525.5	9	58.39	—	—
Error retention × difficulty	532.9	9	59.21	—	—

Therefore, it is concluded (see Appendix D) that:

1. The length of the retention interval had a significant effect on the amount retained.
2. The difficulty of the learned material significantly affected the amount retained.
3. The effects of difficulty of material and retention interval interacted to a significant degree.

SECTION 2.7

Two-Factor Mixed Design: Repeated Measures on One Factor

This two-factor mixed design is basically a combination of the completely randomized design and the treatments-by-subjects design. Not only does this design permit comparison of the differences in the overall performance of the subjects in the several experimental groups, but it also permits evaluation of the changes in performance shown by the subjects during the experimental session. It is known by various names and is discussed in the following textbooks.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 14, pp. 227–232.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 13, pp. 266–273.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 7, pp. 302–312.

Points to Consider When Using the Two-Factor Mixed Design

1. If trials or observations are blocked, each block is treated as one score.
2. Although it is not necessary, it is usually best to have an equal number of subjects in each experimental group.
3. The number of experimental groups compared, the number of subjects per group, and the number of trials or observations (scores) recorded for each subject is arbitrary. However, it is rare that more than four or five experimental groups are compared in one experiment and that more than five or six scores are entered and analyzed for each subject. In most experiments, ten to fifteen subjects are assigned to each experimental group.

EXAMPLE

Assume that an experimenter wishes to determine the effects of meaningfulness of material on rate of learning. He randomly assigns subjects to three groups, one of which must learn a low-meaningful list of nonsense syllables, the second a medium-meaningful list, and the third a high-meaningful list. All subjects are given fifteen trials, and the number of correct responses per trial is recorded. To simplify his analysis, he blocks the number of correct responses into three blocks of five trials each. Thus, the scores used in the analysis represent the number of correct responses for each successive block of five trials.

Step 1. After the experiment has been conducted, table the data as follows.

Group 1 (Low-Meaningful Material)

Subject	Trial Block 1	Trial Block 2	Trial Block 3
S_1	3	3	4
S_2	3	4	4
S_3	1	3	4
S_4	1	2	3
S_5	2	3	5
S_6	4	5	6
S_7	4	5	6
S_8	1	4	5
S_9	1	4	5
S_{10}	2	3	3
S_{11}	2	2	4

Group 2 (Medium-Meaningful Material)

S_{12}	2	3	6
S_{13}	1	4	5
S_{14}	3	6	7
S_{15}	2	4	6
S_{16}	1	2	3
S_{17}	4	5	5
S_{18}	1	3	4
S_{19}	1	2	4
S_{20}	2	3	4
S_{21}	3	5	6
S_{22}	4	6	7

Group 3 (High-Meaningful Material)

S_{23}	2	4	8
S_{24}	3	6	3
S_{25}	4	7	7
S_{26}	1	7	4
S_{27}	3	7	12
S_{28}	1	4	2
S_{29}	1	5	3
S_{30}	2	5	6
S_{31}	3	6	6
S_{32}	1	5	12
S_{33}	4	9	7

Step 2. Add the scores in each group for each trial block (hereafter, referred to simply as "trials").

Group 1

Subject	Trial 1	Trial 2	Trial 3
S_1	3	3	4
S_2	3	4	4
⋮	⋮	⋮	⋮
Sums:	**24**	**38**	**49**

Group 2

S_{12}	2	3	6
S_{13}	1	4	5
⋮	⋮	⋮	⋮
Sums:	**24**	**43**	**57**

Group 3

S_{23}	2	4	8
S_{24}	3	6	3
⋮	⋮	⋮	⋮
Sums:	25	65	70

Step 3. Obtain the sum for each group by adding the sums of the individual trials (Step 2).

$$24 + 38 + 49 = 111 = \text{sum of Group 1}$$
$$24 + 43 + 57 = 124 = \text{sum of Group 2}$$
$$25 + 65 + 70 = 160 = \text{sum of Group 3}$$

Step 4. Add the scores for *each* subject in each group. (*Note:* If you are using a calculator, Step 5 can be done at the same time as Step 4.)

Group 1

Subject	Trial 1		Trial 2		Trial 3	Sums
S_1	3	+	3	+	4	10
S_2	3	+	4	+	4	11
⋮	⋮		⋮		⋮	⋮
S_{11}	2	+	2	+	4	8

Group 2

S_{12}	2	+	3	+	6	11
S_{13}	1	+	4	+	5	10
⋮	⋮		⋮		⋮	⋮
S_{22}	4	+	6	+	7	17

Group 3

S_{23}	2	+	4	+	8	14
S_{24}	3	+	6	+	3	12
⋮	⋮		⋮		⋮	⋮
S_{33}	4	+	9	+	7	20

Step 5. Square each score in the entire table (Step 1), and add these squared values to get a grand sum of the squared values.

$$3^2 + 3^2 + \cdots + 7^2 = 2043$$

Step 6. Add the group totals (Step 3) to get the grand sum of the entire table.

$$111 + 124 + 160 = 395$$

Step 7. Square the grand sum (Step 6), and divide it by the total number of measures in the entire table—i.e., the number of subjects times the number of measures per subject (see Step 1). (In the present example, this would be 33 × 3 = 99.) The resultant value is the correction term.

$$\frac{395^2}{99} = \frac{156{,}025}{99} = \mathbf{1576}$$

Step 8. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 7) from the sum of the squared scores (Step 5).

$$2043 - 1576 = \mathbf{467} = SS_t$$

Between-Subjects Effects (Steps 9–11)

Step 9. Computation of the between-subjects sum of squares (SS_b): First, square the sum of each subject's scores (Step 4), and add these squared values.

$$10^2 + 11^2 + \cdots + 20^2 = \mathbf{5271}$$

Then, divide this value by the number of trials given each subject.

$$\frac{5271}{3} = \mathbf{1757}$$

Then, subtract the correction term from the above value.

$$1757 - 1576 = \mathbf{181} = SS_b$$

Step 10. Computation of the effects of the experimental conditions on overall performance (SS_c): (In the present example, these are the meaningfulness effects.) First, square the sum for each experimental group (Step 3), and divide these values by the number of measures in each experimental group. (In the present example, this would be 11 subjects × 3 trials per subject, or 33.) Then, add these quotients.

$$\frac{111^2}{33} + \frac{124^2}{33} + \frac{160^2}{33} = \frac{53{,}297}{33} = \mathbf{1615}$$

Then, subtract the correction term from the above value.

$$1615 - 1576 = \mathbf{39} = SS_c$$

Step 11. Computation of the between-subjects error term (error_b): Simply subtract the SS_c (Step 10) from the SS_b (Step 9).

$$181 - 39 = \mathbf{142} = \text{error}_b$$

Within-Subjects Effects (Steps 12–15)

Step 12. Computation of the within-subjects sum of squares (SS_w): Simply subtract the SS_b (Step 9) from the SS_t (Step 8).

$$467 - 181 = \mathbf{286} = SS_w$$

Step 13. Computation of the sum of squares for trials (SS_{tr}): First, add the sums of Trial 1 (Step 2) for all experimental groups:

$$24 + 24 + 25 = \mathbf{73} = \text{sum for Trial 1}$$

Do the same for Trial 2 and Trial 3.

$$38 + 43 + 65 = \mathbf{146} = \text{sum for Trial 2}$$
$$49 + 57 + 70 = \mathbf{176} = \text{sum for Trial 3}$$

Then, square each of these sums, and divide the products by the number of measures on which each sum was based.

$$\frac{73^2}{33} + \frac{146^2}{33} + \frac{176^2}{33} = \frac{57{,}621}{33} = \mathbf{1746}$$

Then, subtract the correction term from the above value.

$$1746 - 1576 = \mathbf{170} = SS_{tr}$$

Step 14. Computation of the sum of squares for the trials-by-conditions interaction ($SS_{tr \times c}$): First, square the sums of each trial in each of the experimental groups (Step 2), divide by the number of measures on which each sum was based, and add these quotients.

$$\frac{24^2}{11} + \frac{38^2}{11} + \frac{49^2}{11} + \frac{24^2}{11} + \frac{43^2}{11} + \frac{57^2}{11}$$
$$+ \frac{25^2}{11} + \frac{65^2}{11} + \frac{70^2}{11} = \frac{19{,}845}{11} = \mathbf{1804}$$

Then, from this value, subtract the correction term (Step 7), SS_c (Step 10), and SS_{tr} (Step 13).

$$1804 - 1576 - 39 - 170 = \mathbf{19} = SS_{tr \times c}$$

Step 15. Computation of the within-subjects error term (error_w): Simply subtract SS_{tr} (Step 13) and $SS_{tr \times c}$ (Step 14) from the SS_w (Step 12).

$$286 - 170 - 19 = \mathbf{97} = \text{error}_w$$

Step 16. All computations based directly on the data have been completed:

$SS_t = 467$ (Step 8)
$SS_b = 181$ (Step 9)
$SS_c = 39$ (Step 10)
$SS_{error_b} = 142$ (Step 11)
$SS_w = 286$ (Step 12)
$SS_{tr} = 170$ (Step 13)
$SS_{tr \times c} = 19$ (Step 14)
$SS_{error_w} = 97$ (Step 15)

However, since the tests of significance (F-tests) are ratios of *mean* squares, we must compute these values. To do this, the degrees of freedom (df) for each of the components must be determined, as follows.

df for SS_t = the total number of measures recorded minus 1.

40 $$99 - 1 = 98$$

df for SS_b = the total number of subjects minus 1.

10 $$33 - 1 = 32$$

df for SS_c = the total number of experimental groups minus 1.

2 $$3 - 1 = 2$$

df for SS_{error_b} = the df for SS_b minus the df for SS_c.

$$32 - 2 = 30$$

df for SS_w = the df for SS_t minus the df for SS_b.

$$98 - 32 = 66$$

df for SS_{tr} = the number of trials given each subject minus 1.

4 $$3 - 1 = 2$$

df for $SS_{tr \times c}$ = the df for SS_{tr} times the df for SS_c.

$$2 \times 2 = 4$$

df for SS_{error_w} = the df for SS_w minus the df for SS_{tr} minus the df for $SS_{tr \times c}$.

$$66 - 2 - 4 = 60$$

Step 17. The mean squares are then computed as SS/df.

ms_t (This value is not needed.)

ms_b (This value is not needed.)

$$ms_c = \frac{(\text{Step } 10)}{2} = \frac{39}{2} = 19.5$$

$$ms_{error_b} = \frac{(\text{Step } 11)}{30} = \frac{142}{30} = 4.73$$

ms_w (This value is not needed.)

$$ms_{tr} = \frac{(\text{Step } 13)}{2} = \frac{170}{2} = 85$$

$$ms_{tr \times c} = \frac{(\text{Step } 14)}{4} = \frac{19}{4} = 4.75$$

$$ms_{error_w} = \frac{(\text{Step } 15)}{60} = \frac{97}{60} = 1.62$$

Step 18. Table the final analysis as follows. The tests of significance (F-tests) are equal to

$$\frac{\text{ms}_c}{\text{ms}_{\text{error}_b}} \qquad \frac{\text{ms}_{tr}}{\text{ms}_{\text{error}_w}} \qquad \frac{\text{ms}_{tr \times c}}{\text{ms}_{\text{error}_w}}$$

Source	SS	*df*	ms	*F*	*p*
Total	467	98	—	—	—
Between subjects	181	32	—	—	—
Conditions	39	2	19.50	4.12	$<.05$
Error_b	142	30	4.73	—	—
Within subjects	286	66	—	—	—
Trials	170	2	85.00	52.46	$<.001$
Trials × conditions	19	4	4.75	2.93	$<.05$
Error_w	97	60	1.62	—	—

Hence, it is concluded (see Appendix D) that:

1. Meaningfulness of material significantly affected the overall amount learned.
2. Subjects learned as a function of practice (trials).
3. The groups of subjects learned at different rates.

SECTION 2.8

Three-Factor Mixed Design: Repeated Measures on One Factor

The three-factor mixed design is basically a combination of the factorial design and the treatments-by-subjects design. Not only does this design permit examination of the effects of two factors in combination with each other, but it also permits examination of performance variations shown by the subjects during the experimental session. It is known by several names and is discussed in the following textbooks.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 14, pp. 233–239.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 13, pp. 281–284.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 7, pp. 337–344.

Points to Consider When Using the Three-Factor Mixed Design

1. As in the other designs, if trials are blocked, each block is treated as one score.
2. It is usually best to have an equal number of subjects in each of the experimental groups. If equality cannot be obtained, the number of subjects in each group should be proportional. In most experiments, ten to fifteen subjects are assigned per group.
3. The number of experimental groups within each factor is arbitrary. However, it is rare that more than four or five groups are included in either of the two factors. Similarly, it is rare that more than five or six scores are entered and analyzed for each of the subjects.

EXAMPLE

Assume that an experimenter wishes to determine the effects of high versus low self-esteem in combination with failure versus nonfailure instructions on the learning of verbal material. Subjects are randomly assigned to one of four groups. The subjects in Group 1 (high self-esteem/failure) are initially told that they are likely to perform the task very well. After a short practice period, they are then told that they actually performed very poorly and must try harder on the next task, which "counts." Group 2 subjects (high self-esteem/nonfailure) are also initially told that they are likely to perform well. After the practice period, they are told that they did do very well but should not relax their efforts since the next task is the one that "counts." Group 3 subjects (low self-esteem/failure) are initially told that their performance is likely to be very low. Following the practice period, they are told that they did poorly, as expected, and should try harder on the next task. Group 4 subjects (low self-esteem/nonfailure) are also told that their performance is likely to be very low. Following the practice period, they are told that they actually did very well and should try to do as well on the following task.

Following these manipulations of self-esteem and failure/nonfailure, the subjects in all groups are given twenty trials to learn a list of low-meaningful nonsense syllables. For analysis purposes, the experimenter blocks the scores for the twenty trials into four blocks of five trials each.

The number of correct responses within each of these blocks of five trials is the score recorded.

Step 1. After the experiment has been conducted, table the data as follows.

Group 1 (High Self-Esteem/Failure)

Subject	Trial Block 1	Trial Block 2	Trial Block 3	Trial Block 4
S_1	4	6	7	9
S_2	6	7	9	11
S_3	5	8	10	13
S_4	3	4	6	9
S_5	4	6	9	10
S_6	5	5	6	8
S_7	7	8	9	12

Group 2 (High Self-Esteem/Nonfailure)

S_8	5	8	9	12
S_9	3	5	10	14
S_{10}	7	10	12	13
S_{11}	5	8	11	15
S_{12}	4	9	13	15
S_{13}	3	5	9	12
S_{14}	6	7	8	11

Group 3 (Low Self-Esteem/Failure)

S_{15}	3	4	4	5
S_{16}	5	6	8	9
S_{17}	5	6	6	8
S_{18}	5	6	7	8
S_{19}	3	4	4	5
S_{20}	7	7	8	7
S_{21}	6	8	8	8

Group 4 (Low Self-Esteem/Nonfailure)

S_{22}	4	8	10	14
S_{23}	5	10	14	18
S_{24}	3	7	12	18
S_{25}	5	8	11	17
S_{26}	6	11	15	19
S_{27}	6	8	10	15
S_{28}	5	11	17	20

Step 2. Add the scores in each group for each trial block (hereafter referred to only as "trials").

Group 1

Subject	Trial 1	Trial 2	Trial 3	Trial 4
S_1	4	6	7	9
S_2	6	7	9	11
⋮	⋮	⋮	⋮	⋮
Sums:	**34**	**44**	**56**	**72**

Group 2

S_8	5	8	9	12
S_9	3	5	10	14
⋮	⋮	⋮	⋮	⋮
Sums:	**33**	**52**	**72**	**92**

Group 3

S_{15}	3	4	4	5
S_{16}	5	6	8	9
⋮	⋮	⋮	⋮	⋮
Sums:	**34**	**41**	**45**	**50**

Group 4

S_{22}	4	8	10	14
S_{23}	5	10	14	18
⋮	⋮	⋮	⋮	⋮
Sums:	**34**	**63**	**89**	**121**

Step 3. Obtain the sum for each group by adding the sums of the individual trials.

$$
\begin{aligned}
34 + 44 + 56 + 72 &= \mathbf{206} = \text{sum for Group 1}\\
33 + 52 + 72 + 92 &= \mathbf{249} = \text{sum for Group 2}\\
34 + 41 + 45 + 50 &= \mathbf{170} = \text{sum for Group 3}\\
34 + 63 + 89 + 121 &= \mathbf{307} = \text{sum for Group 4}
\end{aligned}
$$

Step 4. Add the scores for *each* subject in each group. (*Note:* If you are using a calculator, Step 5 can be done at the same time as Step 4.)

Group 1

Subject	Trial 1		Trial 2		Trial 3		Trial 4	Sums
S_1	4	+	6	+	7	+	9	**26**
S_2	6	+	7	+	9	+	11	**33**
⋮	⋮		⋮		⋮		⋮	⋮
S_7	7	+	8	+	9	+	12	**36**

Group 2

S_8	5	+	8	+	9	+	12	**34**
S_9	3	+	5	+	10	+	14	**32**
⋮	⋮		⋮		⋮		⋮	⋮
S_{14}	6	+	7	+	8	+	11	**32**

Group 3

S_{15}	3	+	4	+	4	+	5	**16**
S_{16}	5	+	6	+	8	+	9	**28**
⋮	⋮		⋮		⋮		⋮	⋮
S_{21}	6	+	8	+	8	+	8	**30**

Group 4

S_{22}	4	+	8	+	10	+	14	**36**
S_{23}	5	+	10	+	14	+	18	**47**
⋮	⋮		⋮		⋮		⋮	⋮
S_{28}	5	+	11	+	17	+	20	**53**

Step 5. Square each score in the entire table (Step 1), and add these squared values to get the grand sum of the squared numbers.

$$4^2 + 6^2 + \cdots + 20^2 = \mathbf{9412}$$

Step 6. Add the group totals (Step 3) to get the grand sum of the entire table.

$$206 + 249 + 170 + 307 = \mathbf{932}$$

Step 7. Square the grand sum, and divide it by the total number of measures in the entire table—i.e., the number of subjects times the number of measures per subject (see Step 1). The value obtained is the correction term.

$$\frac{932^2}{112} = \mathbf{7755}$$

Step 8. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 7) from the sum of squares (Step 5).

$$9412 - 7755 = \mathbf{1657} = \mathrm{SS}_t$$

Between-Subjects Effects (Steps 9–13)

Step 9. Computation of the between-subjects sum of squares (SS_b): First, square the sum of each subject's scores (Step 4), and add these squared values.

$$26^2 + 33^2 + \cdots + 53^2 = \mathbf{33{,}324}$$

Then, divide this value by the number of trials given to each subject.

$$\frac{33{,}324}{4} = \mathbf{8331}$$

Then, subtract the correction term (Step 7) from the above value.

$$8331 - 7755 = \mathbf{576} = SS_b$$

Step 10. Computation of the effects of the first experimental condition on overall performance (the overall effects of high versus low self-esteem—SS_{esteem}): First, sum the scores of the two self-esteem groups (Step 3), disregarding the failure/nonfailure instructions.

206 = sum for Group 1 (high self-esteem/failure)
+249 = sum for Group 2 (high self-esteem/nonfailure)
455 = sum for all high self-esteem subjects

170 = sum for Group 3 (low self-esteem/failure)
+307 = sum for Group 4 (low self-esteem/nonfailure)
477 = sum for all low self-esteem subjects

Then, square the sums of the two experimental conditions obtained above, and divide each product by the number of measures added to get each of the sums. (In the present example, the divisor is 56 in both cases: 7 subjects per group × 2 groups × 4 trials per subject.) Then, add the quotients.

$$\frac{455^2}{56} + \frac{477^2}{56} = \mathbf{7759}$$

Then, subtract the correction term (Step 7) from the above value.

$$7759 - 7755 = \mathbf{4} = SS_{esteem}$$

Step 11. Computation of the effects of the second experimental condition on overall performance (the effects of failure versus nonfailure—$SS_{failure}$): First, sum the scores of the two failure/nonfailure groups (Step 3), disregarding the high versus low self-esteem.

206 = sum for Group 1 (high self-esteem/failure)
+170 = sum for Group 3 (low self-esteem/failure)
376 = sum for all subjects who "failed"

249 = sum for Group 2 (high self-esteem/nonfailure)
+307 = sum for Group 4 (low self-esteem/nonfailure)
556 = sum for all subjects who did not "fail"

Then, square the sums of these two experimental conditions, and divide the squared values by the number of measures added to get each sum.

(Here, each sum was obtained by adding 56 scores.) Then, add the quotients.

$$\frac{376^2}{56} + \frac{556^2}{56} = \mathbf{8045}$$

Then, subtract the correction term (Step 7) from the above value.

$$8045 - 7755 = \mathbf{290} = SS_{\text{failure}}$$

Step 12. Computation of the interactive effects of the two experimental conditions: (In this example, we are attempting to determine whether the effects of failure or nonfailure instructions on learning are influenced by the self-esteem of the subject—$SS_{\text{esteem} \times \text{failure}}$.) First, square the sum of each of the experimental groups (Step 3); then, divide these products by the number of measures added to get each. (Here, 28 scores were added to get each.) Then add the quotients.

$$\frac{206^2}{28} + \frac{249^2}{28} + \frac{170^2}{28} + \frac{307^2}{28} = \mathbf{8128}$$

Then, from this value, subtract the correction term (Step 7), the SS_{esteem} (Step 10), and the SS_{failure} (Step 11).

$$8128 - 7755 - 4 - 290 = \mathbf{79} = SS_{\text{esteem} \times \text{failure}}$$

Step 13. Computation of the between-subjects error term (error_b): Simply subtract the SS_{esteem} (Step 10), SS_{failure} (Step 11), and $SS_{\text{esteem} \times \text{failure}}$ (Step 12) from the SS_b (Step 9).

$$576 - 4 - 290 - 79 = \mathbf{203} = \text{error}_b$$

Within-Subjects Effects (Steps 14–19)

Step 14. Computation of the within-subjects sum of squares (SS_w): Simply subtract the SS_b (Step 9) from the SS_t (Step 8).

$$1657 - 576 = \mathbf{1081} = SS_w$$

Step 15. Computation of the sum of squares for trials (SS_{trials}): First, add the sums of each trial for *all* experimental groups (Step 2).

$$\begin{aligned} 34 + 33 + 34 + 34 &= \mathbf{135} = \text{sum of Trial 1} \\ 44 + 52 + 41 + 63 &= \mathbf{200} = \text{sum of Trial 2} \\ 56 + 72 + 45 + 89 &= \mathbf{262} = \text{sum of Trial 3} \\ 72 + 92 + 50 + 121 &= \mathbf{335} = \text{sum of Trial 4} \end{aligned}$$

Then, square each of these sums, divide each square by the number of measures on which that sum was based, and add the quotients.

$$\frac{135^2}{28} + \frac{200^2}{28} + \frac{262^2}{28} + \frac{335^2}{28} = \mathbf{8539}$$

Then, subtract the correction term (Step 7) from the above value.

$$8539 - 7755 = \mathbf{784} = \mathrm{SS}_{\mathrm{trials}}$$

Step 16. Computation of the sum of squares for the trials-by-first-condition interaction (here, the trials-by-self-esteem interaction—$\mathrm{SS}_{\mathrm{trials} \times \mathrm{esteem}}$): First, sum the scores of the two self-esteem groups on each trial (Step 2), disregarding the failure/nonfailure instructions.

	Trial 1	Trial 2	Trial 3	Trial 4
Group 1 (high self-esteem/failure)	34	44	56	72
Group 2 (high self-esteem/nonfailure)	33	52	72	92
Sums of all high-self-esteem *S*s =	**67**	**96**	**128**	**164**
Group 3 (low self-esteem/failure)	34	41	45	50
Group 4 (low self-esteem/nonfailure)	34	63	89	121
Sums of all low-self-esteem *S*s =	**68**	**104**	**134**	**171**

Then, square the sums of each trial, divide by the number of measures on which each of the sums was based, and add the quotients together.

$$\frac{67^2}{14} + \frac{96^2}{14} + \frac{128^2}{14} + \frac{164^2}{14} + \frac{68^2}{14} + \frac{104^2}{14} + \frac{134^2}{14} + \frac{171^2}{14} = \mathbf{8544}$$

Then, from this value, subtract the correction term (Step 7), $\mathrm{SS}_{\mathrm{esteem}}$ (Step 10), and $\mathrm{SS}_{\mathrm{trials}}$ (Step 15).

$$8544 - 7755 - 4 - 784 = \mathbf{1} = \mathrm{SS}_{\mathrm{trials} \times \mathrm{esteem}}$$

Step 17. Computation of the sum of squares for the trials-by-second-condition interaction (here, the trials-by-failure/nonfailure interaction—$\mathrm{SS}_{\mathrm{trials} \times \mathrm{failure}}$): First, sum the scores of the two failure/nonfailure groups on each trial (Step 2), disregarding high and low self-esteem.

	Trial 1	Trial 2	Trial 3	Trial 4
Group 1 (high self-esteem/failure)	34	44	56	72
Group 3 (low self-esteem/failure)	34	41	45	50
Sums for all "failure" *S*s =	**68**	**85**	**101**	**122**
Group 2 (high self-esteem/nonfailure)	33	52	72	92
Group 4 (low self-esteem/nonfailure)	34	63	89	121
Sums for all "nonfailure" *S*s =	**67**	**115**	**161**	**213**

Then, square the sums of each trial, divide by the number of measures on which each of the sums was based, and add the quotients together.

$$\frac{68^2}{14}+\frac{85^2}{14}+\frac{101^2}{14}+\frac{122^2}{14}+\frac{67^2}{14}+\frac{115^2}{14}+\frac{161^2}{14}+\frac{213^2}{14}=\mathbf{8996}$$

Then, from this value, subtract the correction term (Step 7), SS_{failure} (Step 11), and SS_{trials} (Step 15).

$$8996-7755-290-784=\mathbf{167}=SS_{\text{trials}\times\text{failure}}$$

Step 18. Computation of the sum of squares for the trials-by-first-condition-by-second-condition interaction (here, the interaction of trials by self-esteem by failure/nonfailure—$SS_{\text{trials}\times\text{esteem}\times\text{failure}}$): First, square the sums of each trial in each of the experimental groups (Step 2). Divide each squared value by the number of measures on which each sum was based, and then add the quotients together.

$$\frac{34^2}{7}+\cdots+\frac{72^2}{7}+\frac{33^2}{7}+\cdots+\frac{92^2}{7}+\frac{34^2}{7}$$

$$+\cdots+\frac{50^2}{7}+\frac{34^2}{7}+\cdots+\frac{121^2}{7}=\frac{63{,}902}{7}=\mathbf{9129}$$

Then, from this value, subtract the correction term (Step 7), SS_{esteem} (Step 10), SS_{failure} (Step 11), $SS_{\text{esteem}\times\text{failure}}$ (Step 12), SS_{trials} (Step 15), $SS_{\text{trials}\times\text{esteem}}$ (Step 16), and $SS_{\text{trials}\times\text{failure}}$ (Step 17).

$$9129-7755-4-290-79-784-1-167$$

$$=\mathbf{49}=SS_{\text{trials}\times\text{esteem}\times\text{failure}}$$

Step 19. Computation of the within-subjects error term (error_w): Subtract SS_{trials} (Step 15), $SS_{\text{trials}\times\text{esteem}}$ (Step 16), $SS_{\text{trials}\times\text{failure}}$ (Step 17), and $SS_{\text{trials}\times\text{esteem}\times\text{failure}}$ (Step 18) from SS_w (Step 14).

$$1081-784-1-167-49=\mathbf{80}=\text{error}_w$$

Step 20. All computations have now been completed:

SS_t = 1657 (Step 8)
SS_b = 576 (Step 9)
SS_{esteem} = 4 (Step 10)
SS_{failure} = 290 (Step 11)
$SS_{\text{esteem}\times\text{failure}}$ = 79 (Step 12)
SS_{error_b} = 203 (Step 13)
SS_w = 1081 (Step 14)
SS_{trials} = 784 (Step 15)
$SS_{\text{trials}\times\text{esteem}}$ = 1 (Step 16)
$SS_{\text{trials}\times\text{failure}}$ = 167 (Step 17)
$SS_{\text{trials}\times\text{esteem}\times\text{failure}}$ = 49 (Step 18)
SS_{error_w} = 80 (Step 19)

However, since the tests of significance (F-tests) are ratios of *mean* squares, we must compute these values. To do this, the degrees of freedom (df) for each of the components must be determined, as follows.

df for SS_t = the total number of measures recorded minus 1.

$$112 - 1 = \mathbf{111}$$

df for SS_b = the total number of subjects minus 1.

$$28 - 1 = \mathbf{27}$$

df for SS_{esteem} = the number of self-esteem groups minus 1.

$$2 - 1 = \mathbf{1}$$

df for SS_{failure} = the number of failure/nonfailure groups minus 1.

$$2 - 1 = \mathbf{1}$$

df for $SS_{\text{esteem} \times \text{failure}}$ = the df for esteem times the df for failure/nonfailure.

$$1 \times 1 = \mathbf{1}$$

df for SS_{error_b} = the df for SS_b minus the dfs for esteem, failure, and esteem by failure.

$$27 - 1 - 1 - 1 = \mathbf{24}$$

df for SS_w = the df for the SS_t minus the df for SS_b.

$$111 - 27 = \mathbf{84}$$

df for SS_{trials} = the number of trials given each subject minus 1.

$$4 - 1 = \mathbf{3}$$

df for $SS_{\text{trials} \times \text{esteem}}$ = the df for SS_{trials} times the df for SS_{esteem}.

$$3 \times 1 = \mathbf{3}$$

df for $SS_{\text{trials} \times \text{failure}}$ = the df for SS_{trials} times the df for SS_{failure}.

$$3 \times 1 = \mathbf{3}$$

df for $SS_{\text{trials} \times \text{esteem} \times \text{failure}}$ = the df for SS_{trials} times the df for SS_{failure} times the df for SS_{esteem}.

$$3 \times 1 \times 1 = \mathbf{3}$$

df for SS_{error_w} = the df for SS_w minus the dfs for SS_{trials}, $SS_{\text{trials} \times \text{esteem}}$, $SS_{\text{trials} \times \text{failure}}$, and $SS_{\text{trials} \times \text{esteem} \times \text{failure}}$.

$$84 - 3 - 3 - 3 - 3 = \mathbf{72}$$

Step 21. The mean squares are then computed as SS/*df*.

ms_t (This value is not needed.)

ms_b (This value is not needed.)

$$ms_{\text{esteem}} = \frac{(\text{Step } 10)}{1} = \frac{4}{1} = \mathbf{4}$$

$$ms_{\text{failure}} = \frac{(\text{Step } 11)}{1} = \frac{290}{1} = \mathbf{290}$$

$$ms_{\text{esteem} \times \text{failure}} = \frac{(\text{Step } 12)}{1} = \frac{79}{1} = \mathbf{79}$$

$$ms_{\text{error}_b} = \frac{(\text{Step } 13)}{24} = \frac{203}{24} = \mathbf{8.46}$$

ms_w (This value is not needed.)

$$ms_{\text{trials}} = \frac{(\text{Step } 15)}{3} = \frac{784}{3} = \mathbf{231.33}$$

$$ms_{\text{trials} \times \text{esteem}} = \frac{(\text{Step } 16)}{3} = \frac{1}{3} = \mathbf{.33}$$

$$ms_{\text{trials} \times \text{failure}} = \frac{(\text{Step } 17)}{3} = \frac{167}{3} = \mathbf{55.67}$$

$$ms_{\text{trials} \times \text{esteem} \times \text{failure}} = \frac{(\text{Step } 18)}{3} = \frac{49}{3} = \mathbf{16.33}$$

$$ms_{\text{error}_w} = \frac{(\text{Step } 19)}{72} = \frac{80}{72} = \mathbf{1.11}$$

Step 22. Table the final analysis as follows. The tests of significance (*F* ratios) are equal to the mean squares of the experimental conditions divided by their appropriate error terms.

Source	SS	*df*	ms	*F*	*p*
Total	1657	111	—	—	—
Between subjects	576	27	—	—	—
High/low esteem	4	1	4	<1	—
Failure/nonfailure	290	1	290	34.27	<.001
Esteem × failure	79	1	79	9.34	<.001
Error_b	203	24	8.46	—	—
Within subjects	1081	84	—	—	—
Trials	784	3	261.33	235.43	<.001
Trials × esteem	1	3	.33	<1	—
Trials × failure	167	3	55.67	50.15	<.001
Trials × esteem × failure	49	3	16.33	14.71	<.001
Error_w	80	72	1.11	—	—

Hence, it is concluded (see Appendix D) that:

1. The type of instructions given the subjects (failure versus nonfailure) affects the overall performance level.
2. The effect of the instructions, however, is dependent upon the self-esteem of the subjects.
3. All subjects did learn as a function of practice (trials).
4. The rate of learning is dependent upon the type of instructions given.
5. The effect of the instructions on rate of learning is dependent upon the self-esteem level of the subjects.

SECTION 2.9

Three-Factor Mixed Design: Repeated Measures on Two Factors

This design is basically a combination of the treatments-by-treatments-by-subjects design and the completely randomized design. Instead of only one group of subjects receiving all the treatment conditions (as in the treatments-by-treatments-by-subjects design), two or more experimental groups are compared (as in the completely randomized design). Thus, this design permits the comparison of several groups' performance and the evaluation of the variations in performance shown by the subjects under all treatment conditions within both factors presented. It is discussed in the following textbooks.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 17, pp. 309–311.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 13, pp. 292–297.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 7, pp. 319–337.

Points to Consider When Using the Three-Factor Mixed Design

1. Each subject in a particular experimental group is tested under each treatment condition within both of the factors.
2. The number of subjects used in each experimental group is arbitrary. Again, while it is not necessary, it is best to have an equal number of subjects in each group. In most experiments, ten to fifteen subjects are assigned to each group.
3. The number of treatments given within each of the factors is arbitrary. However, it is rare that more than four or five treatments within either or both of the factors are compared.

EXAMPLE

Consider a situation where two groups of subjects practice a coordination task using a pursuit rotor apparatus. All subjects in *both* groups practice for four 60-second trials with the rotor moving clockwise. Then all subjects practice for four 60-second trials with the rotor moving counterclockwise. (*Note:* If the experimenter decided to counterbalance and have half the subjects practice under the counterclockwise condition first, this would then be a Latin square design.)

The two experimental groups are formed as follows. One group practices under a schedule where the four practice trials (under both the clockwise and counterclockwise conditions) consist of 60 seconds of practice with 60 seconds of rest between each practice trial. The other group practices for 60 seconds with only 10 seconds of rest between each trial. Thus, the present experiment permits evaluation of (1) the effects of the amount of rest between practice trials, (2) the effects of the direction of rotation, and (3) changes across practice trials. The response measured for all subjects under all treatments is time on target during each 60-second trial.

Step 1. After the data have been collected, they are tabled as follows.

GROUP 1 (60-SECOND PRACTICE/60-SECOND REST)

	Clockwise				Counterclockwise			
Subject	Trial 1	Trial 2	Trial 3	Trial 4	Trial 1	Trial 2	Trial 3	Trial 4
S_1	21	26	35	43	2	9	20	24
S_2	18	28	41	46	5	10	14	15
S_3	25	24	45	48	10	15	21	21
S_4	19	20	38	37	1	3	8	7
S_5	14	18	25	33	4	7	12	20
S_6	17	16	28	29	8	10	18	25
S_7	12	18	29	35	6	13	24	28
S_8	15	17	27	30	1	7	17	26
S_9	20	28	26	29	2	4	16	14
S_{10}	18	20	31	38	9	23	36	39

GROUP 2 (60-SECOND PRACTICE/10-SECOND REST)

S_{11}	15	17	26	30	11	14	17	20
S_{12}	6	9	17	24	15	14	19	22
S_{13}	22	23	28	26	10	18	16	13
S_{14}	18	26	29	32	16	18	15	21
S_{15}	11	16	15	18	2	1	6	9
S_{16}	14	22	22	32	7	11	10	14
S_{17}	19	21	24	29	9	12	11	17
S_{18}	16	17	17	22	13	17	21	20
S_{19}	12	16	25	23	4	9	14	19
S_{20}	10	12	16	17	8	10	15	15

Step 2. Add the scores for each treatment in both experimental groups. (*Note:* If you are using a calculator, Step 2 and Step 3 can be done at the same time.)

GROUP 1

	Clockwise				Counterclockwise			
Subject	Trial 1	Trial 2	Trial 3	Trial 4	Trial 1	Trial 2	Trial 3	Trial 4
S_1	21	26	35	43	2	9	20	24
S_2	18	28	41	46	5	10	14	15
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Sums:	**179**	**215**	**325**	**368**	**48**	**101**	**186**	**219**

GROUP 2

S_{11}	15	17	26	30	11	14	17	20
S_{12}	6	9	17	24	15	14	19	22
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Sums:	**143**	**179**	**219**	**253**	**95**	**124**	**144**	**170**

Step 3. Square each number in the entire table (Step 1), and then add the squares.

$$21^2 + 18^2 + \cdots + 15^2 = \mathbf{69{,}810}$$

Step 4. Add all the sums obtained in Step 2 to get the grand sum for the entire table.

$$179 + \cdots + 48 + \cdots + 143 + \cdots + 95 + \cdots + 170 = \mathbf{2968}$$

Step 5. Square the grand sum obtained in Step 4, and divide it by the total number of measures recorded—i.e., the total number of subjects times the number of measures recorded for each subject (in this example, 20 × 8 = 160). This yields the correction term.

$$\frac{2968^2}{160} = \frac{8{,}809{,}024}{160} = \mathbf{55{,}056.4}$$

Step 6. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 5) from the sum of the squared values in Step 3.

$$69{,}810 - 55{,}056.4 = \mathbf{14{,}753.6} = SS_t$$

Between-Subjects Effects (Steps 7–9)

Step 7. Computation of the effects of the experimental conditions on overall performance (in the present example, the effects of 60- versus 10-second rest periods—SS_{groups}): First, add all the trial sums obtained in Step 2 for Group 1 and for Group 2.

$$179 + 215 + 325 + 368 + 48 + 101 + 186 + 219$$
$$= \mathbf{1641} = \text{sum for Group 1}$$

$$143 + 179 + 219 + 253 + 95 + 124 + 144 + 170$$
$$= \mathbf{1327} = \text{sum for Group 2}$$

Then, square each of these totals, divide each square by the number of scores added to get the total (80 in this example), and add the quotients.

$$\frac{1641^2}{80} + \frac{1327^2}{80} = \frac{4{,}453{,}810}{80} = \mathbf{55{,}672.6}$$

Then, subtract the correction term (Step 5) from the above value.

$$55{,}672.6 - 55{,}056.4 = \mathbf{616.2} = SS_{\text{groups}}$$

Step 8. Computation of the between-subjects sum of squares (SS_b): First, add the scores for each subject in each group.

Group 1

Subject	Trial 1	Trial 2	Trial 3	Trial 4	Trial 1	Trial 2	Trial 3	Trial 4	Sums
S_1	21 +	26 +	35 +	43 +	2 +	9 +	20 +	24	**180**
S_2	18 +	28 +	41 +	46 +	5 +	10 +	14 +	15	**177**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
S_{10}	18 +	20 +	31 +	38 +	9 +	23 +	36 +	39	**214**

Group 2

S_{11}	15 +	17 +	26 +	30 +	11 +	14 +	17 +	20	**150**
S_{12}	6 +	9 +	17 +	24 +	15 +	14 +	19 +	22	**126**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
S_{20}	10 +	12 +	16 +	17 +	8 +	10 +	15 +	15	**103**

Then, square the sum of each subject's scores, and add these squares.

$$180^2 + 177^2 + \cdots + 103^2 = \mathbf{460{,}482}$$

Then, divide the above value by the number of scores which were added to get the sum for each subject. (In the present example, 8 scores were added to get each of the above sums.)

$$\frac{460{,}482}{8} = \mathbf{57{,}560.25}$$

Then, subtract the correction term (Step 5) from the above value.

$$57{,}560.25 - 55{,}056.4 = \mathbf{2503.8} = SS_b$$

Step 9. Computation of the between-subjects error term (SS_{error_b}): Simply subtract the sum of squares for groups (Step 7) from the sum of squares for between subjects (Step 8).

$$2503.8 - 616.2 = \mathbf{1887.6} = SS_{error_b}$$

Within-Subjects Effects (Steps 10–20)

Step 10. Computation of the within-subjects sum of squares (SS_w): Simply subtract the between-subjects sum of squares (Step 8), from the total sum of squares (Step 6).

$$14{,}753.6 - 2503.8 = \mathbf{12{,}249.8} = SS_w$$

Step 11. Computation of the sum of squares for the first within-subjects factor (here, the clockwise versus counterclockwise rotation—$SS_{rotation}$): First, add the sums for the clockwise and for the counterclockwise conditions (Step 2), disregarding the trials and the rest-period dimensions.

$$179 + 215 + 325 + 368 + 143 + 179 + 219 + 253 = \mathbf{1881} = \text{sum for clockwise}$$

$$48 + 101 + 186 + 219 + 95 + 124 + 144 + 170 = \mathbf{1087} = \text{sum for counterclockwise}$$

Then, square each of these sums, and divide each square by the number of scores added to get each sum. (In this example 80 scores were added to obtain each sum.) Then, add the quotients.

$$\frac{1881^2}{80} + \frac{1087^2}{80} = \frac{4{,}719{,}730}{80} = \mathbf{58{,}996.6}$$

Then, subtract the correction term (Step 5) from the above value.

$$58{,}996.6 - 55{,}056.4 = \mathbf{3940.2} = SS_{rotation}$$

Step 12. Computation of the sum of squares for the second within-subjects factor (here, the effects of trials—SS_{trials}): First, sum the scores for each of the four trials (Step 2), disregarding the rotational-direction and rest-period dimensions.

$$\begin{aligned}
179 + 143 + 48 + 95 &= \mathbf{465} = \text{sum for Trial 1}\\
215 + 179 + 101 + 124 &= \mathbf{619} = \text{sum for Trial 2}\\
325 + 219 + 186 + 144 &= \mathbf{874} = \text{sum for Trial 3}\\
368 + 253 + 219 + 170 &= \mathbf{1010} = \text{sum for Trial 4}
\end{aligned}$$

Then, square these sums, and divide each square by the number of scores added to get each of the above sums. (In this example, each of the above sums represents a total of 40 scores, since 10 subjects' scores were added to get each of the sums in Step 2.) Then, add the quotients.

$$\frac{465^2}{40} + \frac{619^2}{40} + \frac{874^2}{40} + \frac{1010^2}{40} = \frac{2{,}383{,}362}{40} = \mathbf{59{,}584.0}$$

Then, subtract the correction term (Step 5) from the above value.

$$59{,}584.0 - 55{,}056.4 = \mathbf{4527.6} = SS_{\text{trials}}$$

Step 13. Computation of the interaction between experimental groups (rest interval) and the first factor (rotational direction)—$SS_{\text{groups} \times \text{rotation}}$: First, add the sums for each treatment for each of the two groups (Step 2). In this example, disregard the trials dimension and obtain the sums for the clockwise treatment in Group 1 and the counterclockwise treatment in Group 1. Do the same for Group 2.

$$\begin{aligned}
179 + 215 + 325 + 368 &= \mathbf{1087} = \text{sum for Group 1, clockwise}\\
143 + 179 + 219 + 253 &= \mathbf{794} = \text{sum for Group 2, clockwise}\\
48 + 101 + 186 + 219 &= \mathbf{554} = \text{sum for Group 1, counterclockwise}\\
95 + 124 + 144 + 170 &= \mathbf{533} = \text{sum for Group 2, counterclockwise}
\end{aligned}$$

Then, square these sums, and divide by the number of scores added to obtain each sum. (In the present example, each of the above sums represents a total of 40 scores.) Then add the quotients.

$$\frac{1087^2}{40} + \frac{794^2}{40} + \frac{554^2}{40} + \frac{533^2}{40} = \frac{2{,}403{,}010}{40} = \mathbf{60{,}075.2}$$

Then, subtract the correction term (Step 5), the sum of squares for groups (Step 7), and the sum of squares for rotational direction (Step 11), from the above value.

$$60{,}075.2 - 55{,}056.4 - 616.2 - 3940.2 = \mathbf{462.4} = SS_{\text{groups} \times \text{rotation}}$$

Step 14. Computation of the interaction between experimental groups (rest interval) and the second factor (trials)—$SS_{\text{groups} \times \text{trials}}$: First, add the sums for each trial for each of the two groups (Step 2). In the present example, disregard the rotational-direction dimension and obtain the sums for Trials 1, 2, 3, and 4 in Group 1. Do the same for Group 2.

179 + 48 = **227** = sum for Group 1, Trial 1
215 + 101 = **316** = sum for Group 1, Trial 2
325 + 186 = **511** = sum for Group 1, Trial 3
368 + 219 = **587** = sum for Group 1, Trial 4
143 + 95 = **238** = sum for Group 2, Trial 1
179 + 124 = **303** = sum for Group 2, Trial 2
219 + 144 = **363** = sum for Group 2, Trial 3
253 + 170 = **423** = sum for Group 2, Trial 4

Then, square each of these sums, and divide by the number of scores added to obtain each sum. (In the present example, each sum represents a total of 20 scores: 10 subjects and 2 scores per subject.) Then, add the quotients.

$$\frac{227^2}{20} + \frac{316^2}{20} + \cdots + \frac{423^2}{20} = \frac{1{,}216{,}226}{20} = \mathbf{60{,}811.3}$$

Then, subtract the correction term (Step 5), the sum of squares for groups (Step 7), and the sum of squares for trials (Step 12) from the above value.

$$60{,}811.3 - 55{,}056.4 - 616.2 - 4527.6 = \mathbf{611.1} = SS_{\text{groups} \times \text{trials}}$$

Step 15. Computation of the interaction between the first factor (rotational direction) and the second factor (trials)—$SS_{\text{rotation} \times \text{trials}}$: First, add the sums for each trial for both rotational-direction conditions (Step 2). In the present example, disregard the groups effects and obtain the sum for Trials 1, 2, 3, and 4 under the clockwise condition. Then do the same for the counterclockwise condition.

179 + 143 = **322** = sum for Trial 1, clockwise
215 + 179 = **394** = sum for Trial 2, clockwise
325 + 219 = **544** = sum for Trial 3, clockwise
368 + 253 = **621** = sum for Trial 4, clockwise
48 + 95 = **143** = sum for Trial 1, counterclockwise
101 + 124 = **225** = sum for Trial 2, counterclockwise
186 + 144 = **330** = sum for Trial 3, counterclockwise
219 + 170 = **389** = sum for Trial 4, counterclockwise

Then, square each of these sums, and divide by the number of scores added to obtain each. (In the present example, each sum represents a total of 20 scores: 20 subjects and 1 score per subject.) Then, add the quotients.

$$\frac{322^2}{20} + \frac{394^2}{20} + \cdots + \frac{389^2}{20} = \frac{1{,}271{,}792}{20} = \mathbf{63{,}589.6}$$

Then, subtract the correction term (Step 5), the sum of squares for rotation (Step 11), and the sum of squares for trials (Step 12) from the above value.

$$63{,}589.6 - 55{,}056.4 - 3940.2 - 4527.6 = \mathbf{65.4} = SS_{\text{rotation} \times \text{trials}}$$

Step 16. Computation of the interaction among groups, rotation, and trials ($SS_{\text{groups} \times \text{rotation} \times \text{trials}}$): First, square each sum obtained in Step 2, and divide each square by the number of scores added to obtain each sum. (In the present example, each sum represents a total of 10 scores.) Then add the quotients.

$$\frac{179^2}{10} + \frac{215^2}{10} + \cdots + \frac{170^2}{10} = \frac{652{,}874}{10} = \mathbf{65{,}287.4}$$

Then, from this value, subtract the correction term (Step 5), the sum of squares for groups (Step 7), the sum of squares for rotation (Step 11), the sum of squares for trials (Step 12), the sum of squares for groups by rotation (Step 13), the sum of squares for groups by trials (Step 14), and the sum of squares for rotation by trials (Step 15).

$$65{,}287.4 - 55{,}056.4 - 616.2 - 3940.2 - 4527.6 - 462.4$$
$$-611.1 - 65.4 = \mathbf{8.1} = SS_{\text{groups} \times \text{rotation} \times \text{trials}}$$

Step 17. Computation of the total error for within subjects (SS_{error_w}): (*Note:* This error term is not used in any of the F-tests, but it is needed for later computations.) Subtract the final values of Steps 11, 12, 13, 14, 15, and 16 from the final value of Step 10.

$$12{,}249.8 - 3940.2 - 4527.6 - 462.4 - 611.1 - 65.4 - 8.1$$
$$= \mathbf{2635.0} = SS_{\text{error}_w}$$

Step 18. Computation of the error term for the rotation and for the rotation-by-groups interaction (SS_{error_1}): This requires that the data be retabled as follows. Sum the scores of each subject for the four trials within each rotational-direction condition.

Group 1

Subject	Clockwise	Counterclockwise
S_1	125 (21 + 26 + 35 + 43)	55 (2 + 9 + 20 + 24)
S_2	133 (18 + 28 + 41 + 46)	44 (5 + 10 + 14 + 15)
⋮	⋮	⋮
S_{10}	107	107

Group 2

Subject	Clockwise	Counterclockwise
S_{11}	88	62
S_{12}	56	70
⋮	⋮	⋮
S_{20}	55	48

Then, square each of these sums, and divide the squares by the number of scores added to obtain each sum. (In this example, each sum is based on 4 scores.) Then, add the quotients.

$$\frac{125^2}{4} + \frac{133^2}{4} + \cdots + \frac{48^2}{4} = \frac{254{,}140}{4} = \mathbf{63{,}535}$$

Then, from this value, subtract the correction term (Step 5), the sum of squares for between subjects (Step 8), the sum of squares for rotation (Step 11), and the sum of squares for rotation by groups (Step 13).

$$63{,}535 - 55{,}056.4 - 2503.8 - 3940.2 - 462.4 = \mathbf{1572.2} = \mathrm{SS}_{\mathrm{error}_1}$$

Step 19. Computation of the error term for trials and for the groups-by-trials interaction ($\mathrm{SS}_{\mathrm{error}_2}$): This requires that the data be retabled once again. Sum the scores of each subject on the four trials, disregarding the rotation-direction dimension.

Group 1

Subject	Trial 1	Trial 2	Trial 3	Trial 4
S_1	23 (21 + 2)	35 (26 + 9)	55 (35 + 20)	67 (43 + 24)
S_2	23 (18 + 5)	38 (28 + 10)	55 (41 + 14)	61 (46 + 15)
⋮	⋮	⋮	⋮	⋮
S_{10}	27	43	67	77

Group 2

Subject	Trial 1	Trial 2	Trial 3	Trial 4
S_{11}	26	31	43	50
S_{12}	21	23	36	46
⋮	⋮	⋮	⋮	⋮
S_{20}	18	22	31	32

Then, square all the sums in this table, and divide each square by the number of scores added to obtain each sum. (In the present example, 2 scores were added.) Then, add the quotients.

$$\frac{23^2}{2} + \frac{23^2}{2} + \cdots + \frac{32^2}{2} = \frac{126{,}418}{2} = \mathbf{63{,}209}$$

Then, from this value, subtract the correction term (Step 5), the sum of squares for between subjects (Step 8), the sum of squares for trials (Step 12), and the sum of squares for groups-by-trials (Step 14).

$$63{,}209 - 55{,}056.4 - 2503.8 - 4527.6 - 611.1 = \mathbf{510.1} = \mathrm{SS}_{\mathrm{error}_2}$$

Step 20. Computation of the error term for the trials-by-rotation interaction and for the trials-by-rotation-by-groups interaction ($\mathrm{SS}_{\mathrm{error}_3}$): Simply subtract the final values of Steps 18 and 19 from the final value of Step 17.

$$2635.0 - 1572.2 - 510.1 = \mathbf{552.7} = \mathrm{SS}_{\mathrm{error}_3}$$

Step 21. All the computations based directly on the data have been completed. However, to compute the F ratios, the degrees of freedom (df) are required. These are computed as follows.

df for SS_t = the total number of scores entered in Step 1 minus 1.

$$160 - 1 = \mathbf{159}$$

df for SS_b = the total number of subjects used in the experiment minus 1.

$$20 - 1 = \mathbf{19}$$

df for SS_{groups} = the number of groups minus 1.

$$2 - 1 = \mathbf{1}$$

df for SS_{error_b} = the between-subjects df minus the groups df.

$$19 - 1 = \mathbf{18}$$

df for SS_w = the total df minus the between-subjects df.

$$159 - 19 = \mathbf{140}$$

df for SS_{rotation} = the number of different (rotation) conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for SS_{trials} = the number of trials given within each condition minus 1.

$$4 - 1 = \mathbf{3}$$

df for $SS_{\text{groups} \times \text{rotation}}$ = the df for groups times the df for rotation.

$$1 \times 1 = \mathbf{1}$$

df for $SS_{\text{groups} \times \text{trials}}$ = the df for groups times the df for trials.

$$1 \times 3 = \mathbf{3}$$

df for $SS_{\text{rotation} \times \text{trials}}$ = the df for rotation times the df for trials.

$$1 \times 3 = \mathbf{3}$$

df for $SS_{\text{groups} \times \text{rotation} \times \text{trials}}$ = the df for groups times the df for rotation times the df for trials.

$$1 \times 1 \times 3 = \mathbf{3}$$

df for SS_{error_w} = the df for within subjects minus the dfs for rotation, trials, groups-by-rotation, groups-by-trials, rotation-by-trials, and groups-by-rotation-by-trials.

$$140 - 1 - 3 - 1 - 3 - 3 - 3 = \mathbf{126}$$

df for SS_{error_1} (rotation and rotation-by-groups) = the df for rotation, times the number of subjects in each group minus 1, times the number of groups.

$$1 \times 9 \times 2 = \mathbf{18}$$

df for SS_{error_2} (trials and groups-by-trials) = the *df* for trials, times the number of subjects in each group minus 1, times the number of groups.

$$3 \times 9 \times 2 = \mathbf{54}$$

df for SS_{error_3} (rotation-by-trials and groups-by-rotation-by-trials) = the *df* for rotation, times the *df* for trials, times the number of subjects in each group minus 1, times the number of groups.

$$1 \times 3 \times 9 \times 2 = \mathbf{54}$$

Step 22. The mean squares are then computed as SS/*df*.

$$ms_{groups} = \frac{(\text{Step 7})}{1} = \frac{616.2}{1} = \mathbf{616.2}$$

$$ms_{error_b} = \frac{(\text{Step 9})}{18} = \frac{1887.6}{18} = \mathbf{104.9}$$

$$ms_{rotation} = \frac{(\text{Step 11})}{1} = \frac{3940.2}{1} = \mathbf{3940.2}$$

$$ms_{trials} = \frac{(\text{Step 12})}{3} = \frac{4527.6}{3} = \mathbf{1509.2}$$

$$ms_{groups \times rotation} = \frac{(\text{Step 13})}{1} = \frac{462.4}{1} = \mathbf{462.4}$$

$$ms_{groups \times trials} = \frac{(\text{Step 14})}{3} = \frac{611.1}{3} = \mathbf{203.7}$$

$$ms_{rotation \times trials} = \frac{(\text{Step 15})}{3} = \frac{65.4}{3} = \mathbf{21.8}$$

$$ms_{groups \times rotation \times trials} = \frac{(\text{Step 16})}{3} = \frac{8.1}{3} = \mathbf{2.7}$$

$$ms_{error_1} = \frac{(\text{Step 18})}{18} = \frac{1572.2}{18} = \mathbf{87.3}$$

$$ms_{error_2} = \frac{(\text{Step 19})}{54} = \frac{510.1}{54} = \mathbf{9.4}$$

$$ms_{error_3} = \frac{(\text{Step 20})}{54} = \frac{552.7}{54} = \mathbf{10.2}$$

Step 23. The appropriate F ratios are as follows.

Experimental groups: $\dfrac{\text{ms}_{\text{groups}}}{\text{ms}_{\text{error}_b}}$

Rotation direction: $\dfrac{\text{ms}_{\text{rotation}}}{\text{ms}_{\text{error}_1}}$

Trials: $\dfrac{\text{ms}_{\text{trials}}}{\text{ms}_{\text{error}_2}}$

Groups-by-rotation: $\dfrac{\text{ms}_{G \times R}}{\text{ms}_{\text{error}_1}}$

Groups-by-trials: $\dfrac{\text{ms}_{G \times T}}{\text{ms}_{\text{error}_2}}$

Rotation-by-trials: $\dfrac{\text{ms}_{R \times T}}{\text{ms}_{\text{error}_3}}$

Groups-by-rotation-by-trials: $\dfrac{\text{ms}_{G \times R \times T}}{\text{ms}_{\text{error}_3}}$

Step 24. Table the results as follows.

Source	SS	*df*	ms	*F*	*p*
Total	14,753.6	159	—	—	—
Between subjects	2,503.8	19	—	—	—
Groups	616.2	1	616.2	5.88	<.05
Error$_b$	1,887.6	18	104.9	—	—
Within subjects	12,249.8	140	—	—	—
Rotation	3,940.2	1	3940.2	45.11	<.001
Trials	4,527.6	3	1509.2	159.77	<.001
G × R	462.4	1	462.4	5.29	<.05
G × T	611.1	3	203.7	21.56	<.001
R × T	65.4	3	21.8	2.13	n.s.
G × R × T	8.1	3	2.7	<1	n.s.
Error$_1$	1,572.2	18	87.3	—	—
Error$_2$	510.1	54	9.4	—	—
Error$_3$	552.7	54	10.2	—	—

Therefore, it is concluded (see Appendix D) that:

1. The overall performance of the groups was different.
2. The effects of directional change (rotation) were significant.
3. The subjects' performance changed across trials.
4. There was a significant groups-by-direction (rotation) interaction.
5. There was also a significant trials-by-groups interaction.

SECTION 2.10

Latin Square Design: Simple

This Latin square design is basically equivalent to the treatments-by-subjects design of Section 2.5, except that the effects of the order of treatment presentation can also be analyzed. This design is discussed in the following textbooks.[1]

Cochran, W. G., and G. M. Cox. *Experimental Designs*, 2nd ed. New York: John Wiley & Sons, Inc., 1957. Chapter 4, pp. 117ff.

Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 15, pp. 254–280.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 12, pp. 258–265; Chapter 13, 273–281.

McNemar, Q. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 16, pp. 340–345.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 10, pp. 514-543.

Points to Consider When Using the Simple Latin Square Design

1. All subjects in all of the experimental subgroups receive the same experimental treatments. The only difference between the subgroups is in the order of presentation of these treatments, which is systematically counterbalanced. For example, if three drugs were being tested, every subject would receive each of the drugs, but one third of the subjects would be given Drug 1 first, the second third Drug 2 first, and the final third Drug 3 first.
2. There should be an equal number of subjects in each of the experimental subgroups. In most experiments, five to fifteen subjects are assigned to each subgroup.
3. It is rare that more than four or five treatments are administered to each of the subjects. *Note:* The number of treatments is always equal to the number of experimental subgroups in a Latin square design.

EXAMPLE

Assume that an experimenter wishes to determine the effects of three different shock intensities on the galvanic skin responses of a group of subjects. Since it seems reasonable to expect that the order in which the shocks are presented might affect the results, he decides to counterbalance the effects of the shock by using a Latin square design. To do this, one third of the subjects receive low-intensity shock followed by medium and then high. The second third of the subjects receive medium-intensity shock first followed by high and then low. The remaining third receive high, low, and then medium. The measure recorded is the number of responses above a certain criterion during the fifteen trials given under each shock condition.

---

[1] There are many variations of this design. The texts listed below present basically the same approach as this analysis but vary in some of the specifics of the Latin square arrangements.

In this experiment, the presentation of the shock intensities will be as follows. (The subgroups will hereafter be referred to simply as "groups.")

Order of Presentation	Shock Intensity		
	Low	Medium	High
First	Group 1	Group 2	Group 3
Second	Group 3	Group 1	Group 2
Third	Group 2	Group 3	Group 1

Or, rearranging the group and order-of-presentation dimensions, the above becomes:

Group	Shock Intensity		
	Low	Medium	High
Group 1	First	Second	Third
Group 2	Third	First	Second
Group 3	Second	Third	First

(*Note:* This rearrangement permits computations to be made more easily but does not affect the actual computational procedures.)

Step 1. After the experiment has been conducted, table the data as follows.

Group 1 (Order: Low, Medium, High)

Subject	Low Shock (First)	Medium Shock (Second)	High Shock (Third)
S_1	4	6	7
S_2	6	8	9
S_3	5	8	10
S_4	3	5	6
S_5	4	5	9
S_6	5	6	6
S_7	7	19	10

Group 2 (Order: Medium, High, Low)

	(Third)	(First)	(Second)
S_8	8	6	2
S_9	9	4	6
S_{10}	11	8	9
S_{11}	6	6	6
S_{12}	14	8	17
S_{13}	9	5	6
S_{14}	8	5	8

Group 3 (Order: High, Low, Medium)

	(Second)	(Third)	(First)
S_{15}	8	9	9
S_{16}	8	9	12
S_{17}	10	9	10
S_{18}	9	11	11
S_{19}	7	8	9
S_{20}	10	8	13
S_{21}	7	4	8

Step 2. Add the scores in each group for each shock intensity.

Group 1

Subject	Low (First)	Medium (Second)	High (Third)
S_1	4	6	7
S_2	6	8	9
⋮	⋮	⋮	⋮
Sums:	**34**	**57**	**57**

Group 2

	(Third)	(First)	(Second)
S_8	8	6	2
S_9	9	4	6
⋮	⋮	⋮	⋮
Sums:	**65**	**42**	**54**

Group 3

	(Second)	(Third)	(First)
S_{15}	8	9	9
S_{16}	8	9	12
⋮	⋮	⋮	⋮
Sums:	**59**	**58**	**72**

Step 3. Obtain the sum for each group by adding the sums of the shock intensities.

$$34 + 57 + 57 = \mathbf{148} = \text{sum for Group 1}$$
$$65 + 42 + 54 = \mathbf{161} = \text{sum for Group 2}$$
$$59 + 58 + 72 = \mathbf{189} = \text{sum for Group 3}$$

Step 4. Add the scores for *each* subject in each group. (*Note:* If you are using a calculator, Step 5 can be done at the same time as Step 4.)

Group 1

Subject	Low		Medium		High	Sums
S_1	4	+	6	+	7	**17**
S_2	6	+	8	+	9	**23**
⋮	⋮		⋮		⋮	⋮
S_7	7	+	19	+	10	**36**

Group 2

S_8	8	+	6	+	2	**16**
S_9	9	+	4	+	6	**19**
⋮	⋮		⋮		⋮	⋮
S_{14}	8	+	5	+	8	**21**

Group 3

S_{15}	8	+	9	+	9	**26**
S_{16}	8	+	9	+	12	**29**
⋮	⋮		⋮		⋮	⋮
S_{21}	7	+	4	+	8	**19**

Step 5. Square each score in the entire table (Step 1), and add these squared values to get a grand sum of the squares. (Note that in the following example we are considering the scores as they go across—not down—the table.)

$$4^2 + 6^2 + 7^2 + 6^2 + 8^2 + 9^2 + \cdots + 7^2 + 4^2 + 8^2 = \mathbf{4500}$$

Step 6. Add the group sums (Step 3) to get the grand sum of the entire table.

$$148 + 161 + 189 = \mathbf{498}$$

Step 7. Square the grand sum, and divide it by the total number of measures in the entire table—i.e., the number of subjects times the number of measures per subject ($21 \times 3 = 63$ in this example). This yields the correction term.

$$\frac{498^2}{63} = \frac{248{,}004}{63} = \mathbf{3936.571}$$

Step 8. Computation of the total sum of squares ($\mathrm{SS_t}$): Simply subtract the correction term (Step 7) from the sum of the squares (Step 5).

$$4500 - 3936.571 = \mathbf{563.429} = \mathrm{SS_t}$$

Between-Subjects Effects (Steps 9–11)

Step 9. Computation of the between-subjects sum of the squares (SS_b): First, square the sum of each subject's scores (Step 4), and add these squared values.

$$17^2 + 23^2 + \cdots + 19^2 = \mathbf{12{,}756}$$

Then, divide this value by the number of different shock intensities given each subject (i.e., the number of different orders).

$$\frac{12{,}756}{3} = \mathbf{4252}$$

Then, subtract the correction term (Step 7) from the above value.

$$4252 - 3936.571 = \mathbf{315.429} = SS_b$$

Step 10. Computation of the effects of groups—SS_G (i.e., the effects of order of presentation of the shock intensities on the overall group performance): First, square the sum of each of the experimental groups (Step 3), divide each squared value by the number of measures on which the sum was based, and then add these values.

$$\frac{148^2}{21} + \frac{161^2}{21} + \frac{189^2}{21} = \frac{83{,}546}{21} = \mathbf{3978.381}$$

Then, subtract the correction term (Step 7) from the above value.

$$3978.381 - 3936.571 = \mathbf{41.810} = SS_G$$

Step 11. Computation of the between-subjects error term (SS_{error_b}): Simply subtract the SS_G (Step 10) from the SS_b (Step 9).

$$315.429 - 41.810 = \mathbf{273.619} = SS_{\text{error}_b}$$

Within-Subjects Effects (Steps 12–16)

Step 12. Computation of the within-subjects sum of squares (SS_w): Simply subtract the SS_b (Step 9) from the SS_t (Step 8).

$$563.429 - 315.429 = \mathbf{248.0} = SS_w$$

Step 13. Computation of the treatment effects (in this example, the effects of low- versus medium- versus high-intensity shock—SS_S): First, add the sums of all subjects when they received low shock (Step 2). Do the same for medium and high shock.

$$34 + 65 + 59 = \mathbf{158} = \text{sum for low shock}$$
$$57 + 42 + 58 = \mathbf{157} = \text{sum for medium shock}$$
$$57 + 54 + 72 = \mathbf{183} = \text{sum for high shock}$$

Then, square each of these sums, divide by the number of measures on which each sum was based, and add the quotients.

$$\frac{158^2}{21} + \frac{157^2}{21} + \frac{183^2}{21} = \frac{83{,}102}{21} = \mathbf{3957.238}$$

Then, subtract the correction term (Step 7) from the above value.

$$3957.238 - 3936.571 = \mathbf{20.667} = \mathrm{SS_S}$$

Step 14. Computation of the order effects—$\mathrm{SS_O}$ (i.e., the general effects of firstness versus secondness versus thirdness, irrespective of the particular shock intensity): First, add the sums of the shock intensities presented first, second, and third (Step 2).

$34 + 42 + 72 = \mathbf{148}$ = sum of shocks presented first
$57 + 54 + 59 = \mathbf{170}$ = sum of shocks presented second
$57 + 65 + 58 = \mathbf{180}$ = sum of shocks presented third

Then, square these sums, divide by the number of measures on which each sum was based, and then add the quotients.

$$\frac{148^2}{21} + \frac{170^2}{21} + \frac{180^2}{21} = \frac{83{,}204}{21} = \mathbf{3962.095}$$

Then, subtract the correction term (Step 7) from the above value.

$$3962.095 - 3936.571 = \mathbf{25.524} = \mathrm{SS_O}$$

Step 15. Computation of the effects of a particular shock intensity being presented first versus second versus third—$\mathrm{SS_{SO_w}}$ (SS for shock × order within subjects): First, square the sums of each shock intensity in each of the nine experimental groups (Step 2), divide by the number of measures on which the sums were based (7 in this example), and then add the quotients.

$$\frac{34^2}{7} + \frac{57^2}{7} + \frac{57^2}{7} + \frac{65^2}{7} + \frac{42^2}{7} + \frac{54^2}{7} + \frac{59^2}{7}$$

$$+ \frac{58^2}{7} + \frac{72^2}{7} = \frac{28{,}588}{7} = \mathbf{4084.0}$$

Then, from this value, subtract the correction term (Step 7).

$$4084 - 3936.571 = \mathbf{147.429}$$

Then, from this value, subtract $\mathrm{SS_G}$ (Step 10), $\mathrm{SS_S}$ (Step 13), and $\mathrm{SS_O}$ (Step 14).

$$147.429 - 41.810 - 20.667 - 25.524 = \mathbf{59.428} = \mathrm{SS_{SO_w}}$$

Step 16. Computation of the within-subjects error term (SS_{error_w}): Simply subtract SS_S (Step 13), SS_O (Step 14), and SS_{SO_w} (Step 15) from SS_w (Step 12).

$$248 - 20.667 - 25.524 - 59.428 = \mathbf{142.381} = SS_{error_w}$$

Step 17. All computations based on the data have been completed:

$SS_t = 563.429$ (Step 8)
$SS_b = 315.429$ (Step 9)
$SS_G = 41.810$ (Step 10)
$SS_{error_b} = 273.619$ (Step 11)
$SS_w = 248.000$ (Step 12)
$SS_S = 20.667$ (Step 13)
$SS_O = 25.524$ (Step 14)
$SS_{SO_w} = 59.428$ (Step 15)
$SS_{error_w} = 142.381$ (Step 16)

Since the F ratios are the ratios of mean squares (SS/df), the df must be determined, as follows.

df for SS_t = total number of measures recorded minus 1.

$$63 - 1 = \mathbf{62}$$

df for SS_b = total number of subjects minus 1.

$$21 - 1 = \mathbf{20}$$

df for SS_G = number of experimental subject groups minus 1.

$$3 - 1 = \mathbf{2}$$

df for SS_{error_b} = *df* for SS_b minus the *df* for SS_G.

$$20 - 2 = \mathbf{18}$$

df for SS_w = *df* for SS_t minus the *df* for SS_b.

$$62 - 20 = \mathbf{42}$$

df for SS_S = number of treatment conditions minus 1.

$$3 - 1 = \mathbf{2}$$

df for SS_O = number of different orders of presentation minus 1.

$$3 - 1 = \mathbf{2}$$

df for SS_{SO_w} = number of treatment groups minus 2, times the number of orders minus 1.

$$(3 - 2) \times (3 - 1) = \mathbf{2}$$

df for SS_{error_w} = *df* for SS_w minus the *df*s for SS_S, SS_O, and SS_{SO_w}.

$$42 - 2 - 2 - 2 = \mathbf{36}$$

Step 18. Compute the mean squares needed for the analysis (SS/*df*).

ms_t (This value is not needed.)

ms_b (This value is not needed.)

$$\text{ms}_G = \frac{(\text{Step 10})}{2} = \frac{41.810}{2} = \mathbf{20.905}$$

$$\text{ms}_{\text{error}_b} = \frac{(\text{Step 11})}{18} = \frac{273.619}{18} = \mathbf{15.201}$$

ms_w (This value is not needed.)

$$\text{ms}_S = \frac{(\text{Step 13})}{2} = \frac{20.667}{2} = \mathbf{10.333}$$

$$\text{ms}_O = \frac{(\text{Step 14})}{2} = \frac{25.524}{2} = \mathbf{12.762}$$

$$\text{ms}_{SO_w} = \frac{(\text{Step 15})}{2} = \frac{59.428}{2} = \mathbf{29.714}$$

$$\text{ms}_{\text{error}_w} = \frac{(\text{Step 16})}{36} = \frac{142.381}{36} = \mathbf{3.955}$$

Step 19. Compute the *F* ratios and table the data as follows.

Source	SS	*df*	ms	*F*	*p*
Total	563.429	62	—	—	—
Between subjects	315.429	20	—	—	—
Groups (SO_b)	41.810	2	20.905	1.38	n.s.
Error_b	273.619	18	15.201	—	—
Within subjects	248.000	42	—	—	—
Shock (S)	20.667	2	10.333	2.61	n.s.
Order (O)	25.524	2	12.762	3.23	n.s.
Shock × order_w (SO_w)	59.428	2	29.714	7.51	$<.005$*
Error_w	142.381	36	3.955	—	—

Therefore, it is concluded (see Appendix D) that the within-subjects shock and order effects were interactive. All other effects were nonsignificant.

---

* The *p* value listed is taken from an *F* table for the appropriate degrees of freedom. However, according to Lindquist (1953, p. 280), in the instance where the source of variation is mixed (SO_b and SO_w), each component should be tested at the $X/2\%$ level. In that case, it would be concluded that the SO_w interaction is significant at the .025 rather than the .005 level.

SECTION 2.11

Latin Square Design: Complex

The complex Latin square design is essentially an extension of the simple Latin square. The basic difference is that one extra between-subjects dimension is added. The major advantage of this Latin square design is that it allows the use of many fewer separate groups of subjects to cover the treatment combinations of interest. This design is discussed in the following textbooks.[1]

Cochran, W. G., and Cox, G. M. *Experimental Designs*, 2nd ed. New York: John Wiley & Sons, Inc., 1957. Chapter 4, pp. 117ff.

Lindquist, E. F. *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Chapter 13, pp. 285–288.

Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 10, pp. 554–563.

Points to Consider When Using the Complex Latin Square Design

1. Two or more main experimental subject groups are employed and are either *treated* differently or are *selected* on the basis of some criterion so that they differ (for example, high- versus medium- versus low-anxious subjects).
2. Within each of the main experimental subject groups, a number of subgroups are formed, corresponding to the size of the Latin square (i.e., if the square is 3 × 3, then each main group is divided into three subgroups).
3. An equal number of subjects should be assigned to each of the subgroups. In most experiments, five to fifteen subjects are assigned to each subgroup.
4. It is rare that more than four or five treatments are administered to each of the subjects. (*Note:* The number of treatments administered to each subject is always equal to the number of subgroups in a Latin square design.

EXAMPLE

Assume that an experimenter wishes to determine the effects of three different difficulties of verbal material on the learning ability of high- versus low-anxious subjects. However, he hypothesizes that if the three different difficulties of material were given to each subject, the order in which they were presented would affect the rate of learning. Therefore, instead of presenting the lists in the same order (for example, hard first, medium second, and easy last) to all the subjects in both experimental groups (high- versus low-anxious), he decides to counterbalance the difficulty effects by using a Latin square design. Consequently, within *both* the high- and low-anxiety conditions, the order of presentation of the lists is systematically varied so that an equal proportion of the subjects receive the hard, medium, and easy lists first, second, and third in counterbalanced order. The measure recorded by the experimenter is the number of trials needed to reach the criterion of one perfect recitation of each of the lists.

---

[1] Again, there are many variations of this design. The textbooks listed differ somewhat in the specific details and uses of it. Basically, however, they are the same.

For the high- and low-anxious subjects, the presentation of the lists is as follows. (The subgroups will hereafter be referred to simply as "groups.")

Order of Presentation	Low-Anxious Subjects		
	Type of List		
	Hard	Medium	Easy
First	Group 1	Group 2	Group 3
Second	Group 3	Group 1	Group 2
Third	Group 2	Group 3	Group 1
	High-Anxious Subjects		
First	Group 4	Group 5	Group 6
Second	Group 6	Group 4	Group 5
Third	Group 5	Group 6	Group 4

Or, rearranging the group and order-of-presentation dimensions for both the high- and low-anxious subjects, the above becomes:

Group	Low-Anxious Subjects		
	Type of List		
	Hard	Medium	Easy
Group 1	First	Second	Third
Group 2	Third	First	Second
Group 3	Second	Third	First
	High-Anxious Subjects		
Group 4	First	Second	Third
Group 5	Third	First	Second
Group 6	Second	Third	First

(*Note:* This rearrangement permits computations to be made more easily, but it in no way affects the actual computational procedures.)

Step 1. After the experiment has been completed, table the data as follows.

LOW-ANXIOUS SUBJECTS

Group 1 (Order: Hard, Medium, Easy)

Subject	Hard List (First)	Medium List (Second)	Easy List (Third)
S_1	20	16	9
S_2	14	15	7
S_3	18	10	11
S_4	16	12	10
S_5	18	9	12
S_6	19	14	10

Group 2 (Order: Medium, Easy, Hard)

	(Third)	(First)	(Second)
S_7	17	17	9
S_8	16	18	9
S_9	12	15	8
S_{10}	11	15	10
S_{11}	13	13	12
S_{12}	15	16	12

Group 3 (Order: Easy, Hard, Medium)

	(Second)	(Third)	(First)
S_{13}	19	9	11
S_{14}	14	8	13
S_{15}	13	6	14
S_{16}	14	12	10
S_{17}	16	10	15
S_{18}	18	16	13

HIGH-ANXIOUS SUBJECTS

Group 4 (Order: Hard, Medium, Easy)

Subject	Hard List (First)	Medium List (Second)	Easy List (Third)
S_{19}	26	16	7
S_{20}	23	16	6
S_{21}	21	11	8
S_{22}	19	11	6
S_{23}	24	10	8
S_{24}	27	12	10

Group 5 (Order: Medium, Easy, Hard)

	(Third)	(First)	(Second)
S_{25}	14	17	8
S_{26}	15	18	6
S_{27}	11	16	6
S_{28}	13	16	9
S_{29}	15	13	11
S_{30}	13	11	10

Group 6 (Order: Easy, Hard, Medium)

	(Second)	(Third)	(First)
S_{31}	19	10	10
S_{32}	15	9	11
S_{33}	14	7	6
S_{34}	13	12	10
S_{35}	15	10	9
S_{36}	17	11	8

Step 2. Add the scores in each group for each level of list difficulty.

LOW-ANXIOUS SUBJECTS

Group 1

Subject	Hard (First)	Medium (Second)	Easy (Third)
S_1	20	16	9
S_2	14	15	7
⋮	⋮	⋮	⋮
Sums:	**105**	**76**	**59**

Group 2

	(Third)	(First)	(Second)
S_7	17	17	9
S_8	16	18	9
⋮	⋮	⋮	⋮
Sums:	**84**	**94**	**60**

Group 3

	(Second)	(Third)	(First)
S_{13}	19	9	11
S_{14}	14	8	13
⋮	⋮	⋮	⋮
Sums:	**94**	**61**	**76**

HIGH-ANXIOUS SUBJECTS

Group 4

Subject	Hard (First)	Medium (Second)	Easy (Third)
S_{19}	26	16	7
S_{20}	23	16	6
⋮	⋮	⋮	⋮
Sums:	**140**	**76**	**45**

Group 5

	(Third)	(First)	(Second)
S_{25}	14	17	8
S_{26}	15	18	6
⋮	⋮	⋮	⋮
Sums:	**81**	**91**	**50**

Group 6

	(Second)	(Third)	(First)
S_{31}	19	10	10
S_{32}	15	9	11
⋮	⋮	⋮	⋮
Sums:	**93**	**59**	**54**

Step 3. Obtain the sum for each group by adding the sums for each level of list difficulty.

105 + 76 + 59 = **240** = sum for Group 1
84 + 94 + 60 = **238** = sum for Group 2
94 + 61 + 76 = **231** = sum for Group 3
140 + 76 + 45 = **261** = sum for Group 4
81 + 91 + 50 = **222** = sum for Group 5
93 + 59 + 54 = **206** = sum for Group 6

Step 4. Add the scores for *each* subject in each group. (*Note:* If you are using a calculator, Step 5 can be done at the same time as Step 4.)

LOW-ANXIOUS SUBJECTS

Group 1

Subject	Hard		Medium		Easy	Sums
S_1	20	+	16	+	9	**45**
S_2	14	+	15	+	7	**36**
⋮	⋮		⋮		⋮	⋮
S_6	19	+	14	+	10	**43**

Group 2

S_7	17	+	17	+	9	**43**
S_8	16	+	18	+	9	**43**
⋮	⋮		⋮		⋮	⋮
S_{12}	15	+	16	+	12	**43**

Group 3

S_{13}	19	+	9	+	11	**39**
S_{14}	14	+	8	+	13	**35**
⋮	⋮		⋮		⋮	⋮
S_{18}	18	+	16	+	13	**47**

HIGH-ANXIOUS SUBJECTS

Group 4

Subject	Hard		Medium		Easy	Sums
S_{19}	26	+	16	+	7	**49**
S_{20}	23	+	16	+	6	**45**
⋮	⋮		⋮		⋮	⋮
S_{24}	27	+	12	+	10	**49**

Group 5

S_{25}	14	+	17	+	8	**39**
S_{26}	15	+	18	+	6	**39**
⋮	⋮		⋮		⋮	⋮
S_{30}	13	+	11	+	10	**34**

Group 6

S_{31}	19	+	10	+	10	**39**
S_{32}	15	+	9	+	11	**35**
⋮	⋮		⋮		⋮	⋮
S_{36}	17	+	11	+	8	**36**

Step 5. Square each score in the entire table (Step 1), and add these squared values to get a grand sum of the squared numbers.

$$20^2 + 16^2 + 9^2 + 14^2 + 15^2 + 7^2 + \cdots + 17^2 + 11^2 + 8^2 = \mathbf{20{,}106}$$

Step 6. Add the group totals (Step 3) to get the grand sum for the entire table.

$$240 + 238 + 231 + 261 + 222 + 206 = \mathbf{1398}$$

Step 7. Square the grand sum of Step 6, and divide it by the total number of measures recorded in Step 1—i.e., the number of subjects times the number of measures per subject (36 × 3 = 108 in this example). This yields the correction term.

$$\frac{1398^2}{108} = \frac{1{,}954{,}404}{108} = \mathbf{18{,}096.333}$$

Step 8. Computation of the total sum of squares (SS_t): Simply subtract the correction term (Step 7) from the sum of squares obtained in Step 5.

$$20{,}106 - 18{,}096.333 = \mathbf{2009.667} = SS_t$$

Between-Subjects Effects (Steps 9–13)

Step 9. Computation of the between-subjects sum of squares (SS_b): First, square the sum of each subject's scores (Step 4), and add these squared values.

$$45^2 + 36^2 + \cdots + 36^2 = \mathbf{55{,}080}$$

Then, divide this value by the number of scores added to obtain each of the sums obtained in Step 4. (In the present example, each sum represents 3 scores.)

$$\frac{55{,}080}{3} = \mathbf{18{,}360}$$

Then, subtract the correction term from the above value.

$$18{,}360 - 18{,}096.333 = \mathbf{263.667} = SS_b$$

Step 10. Computation of the "pure" between-groups effects (SS_A): (The other "between" components in this design are partially between- and partially within-subjects sources of variance. In the present example, the overall effects of high versus low anxiety are not confounded.) First, add the group sums (Step 3) for the subjects in the low-anxious conditions (Groups 1, 2, and 3) and for subjects in the high-anxious conditions (Groups 4, 5, and 6).

$$240 + 238 + 231 = \mathbf{709} = \text{sum for low-anxious } S\text{s}$$

$$261 + 222 + 206 = \mathbf{689} = \text{sum for high-anxious } S\text{s}$$

Then, square each of these sums, divide by the total number of scores that make up each sum, and add the quotients.

$$\frac{709^2}{54} + \frac{689^2}{54} = \frac{977{,}402}{54} = \mathbf{18{,}100.037}$$

Then, subtract the correction term (Step 7) from the above value.

$$18{,}100.037 - 18{,}096.333 = \mathbf{3.704} = SS_A$$

Step 11. Computation of the between-subjects list-by-order interaction (SS_{LO_b}): First add the sums for the groups that received the same order of presentation of the lists (Step 3), disregarding the effects of high versus low anxiety.

$$\begin{array}{rl} 240 & = \text{sum for Group 1} \\ +261 & = \text{sum for Group 4} \\ \hline \mathbf{501} & = \text{sum for all } S\text{s who received the lists in the order: hard, medium, easy} \end{array}$$

$$\begin{array}{rl} 238 & = \text{sum for Group 2} \\ +222 & = \text{sum for Group 5} \\ \hline \mathbf{460} & = \text{sum for all } S\text{s who received the lists in the order: easy, hard, medium} \end{array}$$

$$\begin{array}{rl} 231 & = \text{sum for Group 3} \\ +206 & = \text{sum for Group 6} \\ \hline \mathbf{437} & = \text{sum for all } S\text{s who received the lists in the order: medium, easy, hard} \end{array}$$

Then, square the above sums, divide each squared value by the number of measures on which the sum was based, and then add the quotients.

$$\frac{501^2}{36} + \frac{460^2}{36} + \frac{437^2}{36} = \frac{653{,}570}{36} = \mathbf{18{,}154.722}$$

Then, subtract the correction term (Step 7) from the above value.

$$18{,}154.722 - 18{,}096.333 = \mathbf{58.389} = SS_{LO_b}$$

Step 12. Computation of the between-subjects list-by-order-by-anxiety interaction (SS_{LOA_b}): First, square the sum of each of the experimental groups (Step 3), divide each squared value by the number of measures on which the sum is based, and then add the quotients.

$$\frac{240^2}{18} + \frac{238^2}{18} + \frac{231^2}{18} + \frac{261^2}{18} + \frac{222^2}{18} + \frac{206^2}{18} = \frac{327{,}446}{18} = \mathbf{18{,}191.444}$$

Then, from the above value, subtract the correction term (Step 7), the SS_A (Step 10), and the SS_{LO_b} (Step 11).

$$18{,}191.444 - 18{,}096.333 - 3.704 - 58.389 = \mathbf{33.018} = SS_{LOA_b}$$

Step 13. Computation of the between-subjects error term (SS_{error_b}): Simply subtract the SS_A (Step 10), SS_{LO_b} (Step 11), and SS_{LOA_b} (Step 12) from the SS_b (Step 9).

$$263.667 - 3.704 - 58.389 - 33.018 = \mathbf{168.556} = SS_{\text{error}_b}$$

Within-Subjects Effects (Steps 14–21)

Step 14. Computation of the within-subjects sum of squares (SS_w): Simply subtract the SS_b (Step 9) from the SS_t (Step 8)

$$2009.667 - 263.667 = \mathbf{1746.0} = SS_w$$

Step 15. Computation of the list effects (SS_L): (In this example, these are the effects of a hard versus medium versus easy list, disregarding the order and anxiety effects.) First, add the scores of all the subjects when they received the hard list (Step 2). Do the same for the medium and easy lists.

$105 + 84 + 94 + 140 + 81 + 93 = \mathbf{597}$ = sum for hard list
$76 + 94 + 61 + 76 + 91 + 59 = \mathbf{457}$ = sum for medium list
$59 + 60 + 76 + 45 + 50 + 54 = \mathbf{344}$ = sum for easy list

Then, square each of these sums, divide by the number of measures on which each sum was based, and add the quotients.

$$\frac{597^2}{36} + \frac{457^2}{36} + \frac{344^2}{36} = \frac{683{,}594}{36} = \mathbf{18{,}988.722}$$

Then, subtract the correction term (Step 7) from the above value.

$$18{,}988.722 - 18{,}096.333 = \mathbf{892.389} = SS_L$$

Step 16. Computation of the order effects (SS_O): (In this example, these are the general effects of firstness versus secondness versus thirdness of list presentation, irrespective of anxiety level or the particular list presented.) First, add the sums of the lists presented first, second, and third (Step 2).

$105 + 94 + 76 + 140 + 91 + 54 = \mathbf{560}$ = sum of lists presented first
$76 + 60 + 94 + 76 + 50 + 93 = \mathbf{449}$ = sum of lists presented second
$59 + 84 + 61 + 45 + 81 + 59 = \mathbf{389}$ = sum of lists presented third

Then, square these sums, divide by the number of measures on which each sum was based, and add the quotients.

$$\frac{560^2}{36} + \frac{449^2}{36} + \frac{389^2}{36} = \frac{666{,}522}{36} = \mathbf{18{,}514.5}$$

Then, subtract the correction term (Step 7) from the above value.

$$18{,}514.5 - 18{,}096.333 = \mathbf{418.167} = SS_O$$

Step 17. Computation of the effects of each type of list being presented first versus second versus third, disregarding the effects of anxiety

(SS_{LO_w}): First, add the sums of the high- and low-anxious groups for each of the treatment and order combinations (Step 2).

105 + 140 = **245** = sum of hard list first
76 + 76 = **152** = sum of medium list second
59 + 45 = **104** = sum of easy list third
94 + 91 = **185** = sum of medium list first
60 + 50 = **110** = sum of easy list second
84 + 81 = **165** = sum of hard list third
76 + 54 = **130** = sum of easy list first
94 + 93 = **187** = sum of hard list second
61 + 59 = **120** = sum of medium list third

Then, square each of the above sums, divide each square by the number of measures on which each sum is based, and then add the quotients.

$$\frac{245^2}{12} + \frac{152^2}{12} + \cdots + \frac{120^2}{12} = \frac{233{,}764}{12} = \mathbf{19{,}480.333}$$

Then, from this value, subtract the correction term (Step 7), the sum of squares for LO_b (Step 11), the sum of squares for lists (Step 15), and the sum of squares for order (Step 16).

$$19{,}480.333 - 18{,}096.333 - 58.389 - 892.389 - 418.167$$
$$= \mathbf{15.055} = SS_{LO_w}$$

Step 18. Computation of the treatments-by-"pure"-between-groups interaction (in this example, the list-by-anxiety interaction—SS_{LA}): First, add the sums of the hard, medium, and easy lists within each of the anxiety conditions (Step 2), disregarding the effects of order of presentation of these lists.

105 + 84 + 94 = **283** = sum of hard lists, low-anxious *S*s
76 + 94 + 61 = **231** = sum of medium lists, low-anxious *S*s
59 + 60 + 76 = **195** = sum of easy lists, low-anxious *S*s
140 + 81 + 93 = **314** = sum of hard lists, high-anxious *S*s
76 + 91 + 59 = **226** = sum of medium lists, high-anxious *S*s
45 + 50 + 54 = **149** = sum of easy lists, high-anxious *S*s

Then, square the above sums, divide by the number of measures on which each sum was based, and add these quotients.

$$\frac{283^2}{18} + \frac{231^2}{18} + \frac{195^2}{18} + \frac{314^2}{18} + \frac{226^2}{18} + \frac{149^2}{18} = \frac{343{,}348}{18} = \mathbf{19{,}074.889}$$

Then, from this value, subtract the correction term (Step 7), the sum of squares for anxiety (Step 10), and the sum of squares for lists (Step 15).

$$19{,}074.889 - 18{,}096.333 - 3.704 - 892.389 = \mathbf{82.463} = SS_{LA}$$

Step 19. Computation of the order-by-"pure"-between-groups interaction (here, the order-by-anxiety interaction—SS_{OA}): First, add the sums of the lists presented first versus second versus third within each of the anxiety conditions (Step 2), disregarding the effects of list difficulty.

$105 + 94 + 76 = \mathbf{275}$ = sum of first lists, low-anxious *S*s
$76 + 60 + 94 = \mathbf{230}$ = sum of second lists, low-anxious *S*s
$59 + 84 + 61 = \mathbf{204}$ = sum of third lists, low-anxious *S*s
$140 + 91 + 54 = \mathbf{285}$ = sum of first lists, high-anxious *S*s
$76 + 50 + 93 = \mathbf{219}$ = sum of second lists, high-anxious *S*s
$45 + 81 + 59 = \mathbf{185}$ = sum of third lists, high-anxious *S*s

Then, square the above sums, divide by the number of measures on which each sum was based, and add these quotients.

$$\frac{275^2}{18} + \frac{230^2}{18} + \frac{204^2}{18} + \frac{285^2}{18} + \frac{219^2}{18} + \frac{185^2}{18} = \frac{333{,}552}{18} = \mathbf{18{,}530.667}$$

Then, from this value, subtract the correction term (Step 7), the sum of squares for anxiety (Step 10), and the sum of squares for order (Step 16).

$$18{,}530.667 - 18{,}096.333 - 3.704 - 418.167 = \mathbf{12.463} = SS_{OA}$$

Step 20. Computation of the within-subjects lists-by-order-by-anxiety interaction (SS_{LOA_w}): First, square the sums of each list within each experimental group (Step 2), divide by the number of measures on which each sum was based, and then add the quotients.

$$\frac{105^2}{6} + \frac{76^2}{6} + \frac{59^2}{6} + \frac{84^2}{6} + \frac{94^2}{6} + \frac{60^2}{6}$$

$$+ \cdots + \frac{93^2}{6} + \frac{59^2}{6} + \frac{54^2}{6} = \frac{117{,}896}{6} = \mathbf{19{,}649.333}$$

From the above value, subtract the correction term (Step 7), the sum of squares for anxiety (Step 10), the sum of squares LO_b (Step 11), the sum of squares LOA_b (Step 12), the sum of squares for lists (Step 15), the sum of squares for order (Step 16), the sum of squares LO_w (Step 17), the sum of squares LA (Step 18), and the sum of squares OA (Step 19).

$$19{,}649.333 - 18{,}096.333 - 3.704 - 58.389 - 33.018 - 892.389$$
$$- 418.167 - 15.055 - 82.463 - 12.463 = \mathbf{37.352} = SS_{LOA_w}$$

Step 21. Computation of the within-subjects error term (SS_{error_w}): From the within-subjects sum of squares (Step 14), subtract the sums of squares for lists (Step 15), order (Step 16), lists × order within (Step 17), lists × anxiety (Step 18), order × anxiety (Step 19), and list × order × anxiety (Step 20).

$$1746.000 - 892.389 - 418.167 - 15.055 - 82.463$$
$$- 12.463 - 37.352 = \mathbf{288.111} = SS_{\text{error}_w}$$

Step 22. All computations based on the data have been completed:

$SS_t = 2009.667$ (Step 8)
$SS_b = 263.667$ (Step 9)
$SS_A = 3.704$ (Step 10)
$SS_{LO_b} = 58.389$ (Step 11)
$SS_{LOA_b} = 33.018$ (Step 12)
$SS_{error_b} = 168.556$ (Step 13)
$SS_w = 1746.000$ (Step 14)
$SS_L = 892.389$ (Step 15)
$SS_O = 418.167$ (Step 16)
$SS_{LO_w} = 15.055$ (Step 17)
$SS_{LA} = 82.463$ (Step 18)
$SS_{OA} = 12.463$ (Step 19)
$SS_{LOA_w} = 37.352$ (Step 20)
$SS_{error_w} = 288.111$ (Step 21)

Since the F ratios are ratios of mean squares (SS/df), the df must be determined, as follows.

df for SS_t = total number of measures recorded minus 1.

$$108 - 1 = \mathbf{107}$$

df for SS_b = total number of subjects minus 1.

$$36 - 1 = \mathbf{35}$$

df for SS_A = number of "pure" between-subjects groups minus 1.

$$2 - 1 = \mathbf{1}$$

df for SS_{LO_b} = number of treatments (lists) minus 1.

$$3 - 1 = \mathbf{2}$$

df for SS_{LOA_b} = number of treatments minus 1, times the number of "pure" between-subjects groups minus 1.

$$(3 - 1) \times (2 - 1) = \mathbf{2}$$

df for SS_{error_b} = df for SS_b minus the dfs for SS_A, SS_{LO_b}, and SS_{LOA_b}.

$$35 - 1 - 2 - 2 = \mathbf{30}$$

df for SS_w = df for SS_t minus the df for SS_b.

$$107 - 35 = \mathbf{72}$$

df for SS_L = number of treatment groups minus 1.

$$3 - 1 = \mathbf{2}$$

df for SO_O = number of orders of presentation minus 1.

$$3 - 1 = \mathbf{2}$$

df for SS_{LO_w} = number of treatments minus 1, times the number of orders minus 2.

$$(3 - 1) \times (3 - 2) = \mathbf{2}$$

df for SS_{LA} = number of treatments minus 1, times the number of "pure" between factors minus 1.

$$(3 - 1) \times (2 - 1) = \mathbf{2}$$

df for SS_{OA} = number of orders minus 1, times the number of "pure" between factors minus 1.

$$(3 - 1) \times (2 - 1) = \mathbf{2}$$

df for SS_{LOA_w} = number of treatments minus 1, times the number of orders minus 2, times the number of "pure" between factors minus 1.

$$(3 - 1) \times (3 - 2) \times (2 - 1) = \mathbf{2}$$

df for SS_{error_w} = *df* for SS_w minus the *df*s for SS_L, SS_O, SS_{LO_w}, SS_{LA}, SS_{OA}, and SS_{LOA_w}.

$$72 - 2 - 2 - 2 - 2 - 2 - 2 = \mathbf{60}$$

Step 23. Computation of the mean squares needed for the analysis ($ms = SS/df$):

ms_t (This value is not needed.)

ms_b (This value is not needed.)

$$ms_A = \frac{(\text{Step } 10)}{1} = \frac{3.704}{1} = \mathbf{3.704}$$

$$ms_{LO_b} = \frac{(\text{Step } 11)}{2} = \frac{58.389}{2} = \mathbf{29.194}$$

$$ms_{LOA_b} = \frac{(\text{Step } 12)}{2} = \frac{33.018}{2} = \mathbf{16.509}$$

$$ms_{error_b} = \frac{(\text{Step } 13)}{30} = \frac{168.556}{30} = \mathbf{5.619}$$

ms_w (This value not needed.)

$$ms_L = \frac{(\text{Step } 15)}{2} = \frac{892.389}{2} = \mathbf{446.194}$$

$$ms_O = \frac{(\text{Step } 16)}{2} = \frac{418.167}{2} = \mathbf{209.083}$$

$$\text{ms}_{\text{LO}_\text{w}} = \frac{(\text{Step 17})}{2} = \frac{15.055}{2} = 7.527$$

$$\text{ms}_{\text{LA}} = \frac{(\text{Step 18})}{2} = \frac{82.463}{2} = 41.231$$

$$\text{ms}_{\text{OA}} = \frac{(\text{Step 19})}{2} = \frac{12.463}{2} = 6.231$$

$$\text{ms}_{\text{LOA}_\text{w}} = \frac{(\text{Step 20})}{2} = \frac{37.352}{2} = 18.676$$

$$\text{ms}_{\text{error}_\text{w}} = \frac{(\text{Step 21})}{60} = \frac{288.111}{60} = 4.802$$

Step 24. Compute the F ratios and table the data as follows.

Source	SS	*df*	ms	*F*	*p*
Total	2009.667	107	—	—	—
Between subjects	263.667	35	—	—	—
Anxiety	3.704	1	3.704	.66	n.s.
List × order$_b$	58.389	2	29.194	5.20	<.025*
List × order$_b$ × anxiety$_b$	33.018	2	16.509	2.94	n.s.
Error$_b$	168.556	30	5.619	—	—
Within subjects	1746.000	72	—	—	—
List	892.389	2	446.194	92.92	<.001
Order	418.167	2	209.083	43.54	<.001
List × order$_w$	15.055	2	7.527	1.57	n.s.
List × anxiety	82.463	2	41.231	8.59	<.001
Order × anxiety	12.463	2	6.231	1.30	n.s.
List × order$_w$ × anxiety$_w$	37.352	2	18.676	3.89	<.05
Error$_w$	288.111	60	4.802	—	—

Therefore, it is concluded (see Appendix D) that:

1. The effects of list difficulty and order of presentation interacted.
2. The overall effects of list difficulty were significant.
3. The overall effects of order of presentation were significant.
4. The effects of anxiety and list difficulty interacted.

---

* The p values listed are those taken from F tables for the appropriate degrees of freedom. However, according to Lindquist (1953, p. 280), in those instances where the sources of variation are mixed (LO$_b$ and LO$_w$, and LOA$_b$ and LOA$_w$) each component should be tested at the $X/2\%$ level. In that case, it would be concluded that only the LO$_b$ of the mixed sources is significant (at the .05 rather than the .025 level).

PART 3

Supplemental Computations for Analysis of Variance

The next seven sections present supplemental computations that are often needed to make a more complete and accurate analysis of data than is possible with the tests in Parts 1 and 2. The seven tests specifically treated in these sections are grouped below into three general types; then, a more detailed description of each test follows. This introductory material should help you to decide which test or tests will satisfy the demands of your particular data analysis.

1. Tests for equality of variances
2. Tests for "simple effects"—usually used when an interaction is found to be significant
3. Trend tests—used to determine whether linear, quadratic, and higher complex relationships are present in the data

Tests for Equality of Variances (Sections 3.1, 3.2, and 3.3)

Most statistical tests have a clearly stated requirement that population variances must be equal. Sections 3.1, 3.2, and 3.3 present simple tests to determine whether this is true. Any one of these tests may also be used to determine whether two experimental treatments produce an effect in terms of different variances.

Section 3.1 presents a test for equality of two independent variances. Section 3.2 is concerned with a test for equality of two related variances. (Related variances are variances computed from two measures of the same people or measures of matched people.) Section 3.3 presents a test for equality of several independent variances.

Tests for Simple Effects (Sections 3.4, 3.5, and 3.6)

When the more complex analyses, such as those presented in Part 2, are completed, a researcher may wish to perform additional analyses on certain portions of the data. Sections 3.4–3.6 present several ways of dealing with such "simple effects." If the experimenter plans from the beginning to compare several specific pairs of means, the best approach is the one presented in Section 3.4. If he waits until he looks at his data and then decides to make certain tests on the basis of the way the data look, the procedure in Section 3.5 should be followed.

When the very complex analyses of Sections 2.6–2.11 are used, there are several complete factorial or completely randomized analyses that may be done *before* the more simple comparisons of two means are done. Section 3.6 presents a series of questions that might be asked and subsequently answered in such complex research situations.

Tests for Trend (Section 3.7)

Occasionally, a researcher is confronted with a problem which requires that he describe the shape of the data points, or the type of equation that best fits the several points. In this case, the analyses presented in Section 3.7 would best serve his needs.

SECTION 3.1

Test for Difference Between Variances of Two Independent Samples (Test for Homogeneity of Independent Variances)

There are two cases in which it is desired to do a significance test of the difference between two independent variances: (1) when an experimental hypothesis is concerned with the variability of the samples and (2) when there is doubt concerning the requirement of equality of variances in a mean-difference test. In either case, the following procedure may be followed in testing the hypothesis of equal variances.

Step 1. Presumably, you already have the two variances (or the two variance estimates, i.e., mean squares). If you do not, go to Section 1.2 and follow the instructions in the first six steps. The value arrived at in Step 6 will be the variance for one of the samples. When you have both variances, go on to Step 2 of this section.

Step 2. Obtain the F ratio by placing the larger variance of Step 1 over the smaller. For example, suppose the two variances are

$$\text{variance}_1 = 126.81$$

$$\text{variance}_2 = 35.27$$

Then,

$$F = \frac{126.81}{35.27} = 3.60$$

Step 3. Before the F value can be evaluated for significance, the degrees of freedom associated with each variance must be determined. In all cases, the degrees of freedom for a variance (actually a population-variance estimate) will be one less than the number of cases on which the variance is based. For example, suppose variance$_1$ of Step 2 had been computed from a sample of 41 cases. Then,

$$df \text{ for variance}_1 = 41 - 1 = 40$$

In the same way, suppose variance$_2$ had been computed from a sample of 31 cases. Then,

$$df \text{ for variance}_2 = 31 - 1 = 30$$

Step 4. The F value of Step 2 (3.60 in this example) has the two degrees of freedom 40 and 30 associated with it. From tabled F values (see Appendix D), we find that when $df = 40/30$,[1] F values larger than 2.47 are significant at the .01 level. Since the ratio of the variances in the present example equals 3.60, it is concluded that the variances are not homogeneous (equal).

However, there is one slight complication due to the fact that we selected *only* the larger variance as the numerator of the F ratio. A correction is needed to take care of the selection of only the larger variance as the numerator of F. In all cases, this correction is accomplished very simply by multiplying the probability value by 2. Thus, an F value of 2.47, with degrees of freedom equal to 24/30, is considered significant at the .02 level rather than at the .01 level.

---

[1] Since the df of 40/30 are not listed, go to the next closest df values. In this instance, it would be 24/30.

SECTION 3.2

Test for Difference Between Variances of Two Related Samples (Test for Homogeneity of Related Variances)

There are two cases in which it is desired to do a significance test of the difference between two related (correlated) variances: (1) when an experimental hypothesis is concerned with the variability of the samples and (2) when there is doubt concerning the requirement of equality of variances in a mean-difference test. In either case, the following procedure may be followed in testing the hypothesis of equal variances.

Step 1. Presumably, you already have the two variances (or the two variance estimates, i.e., mean squares). If you do not, go to Section 1.2 and follow the instructions in the first six steps. The value arrived at in Step 6 will be the variance for one of the samples. When you have both variances, go on to Step 2 of this section.

Step 2. Since these samples are related (correlated), the degree of the relationship must be determined. Compute the correlation between the pairs of scores. This is done by following the directions from Step 1 through Step 18 in Section 4.1. For the present example, suppose the computed correlation is +.64.

Step 3. Subtract the smaller variance of Step 1 from the larger variance. For example, suppose

$$\text{variance}_1 = 65.27$$

$$\text{variance}_2 = 243.61$$

Then,

$$243.61 - 65.27 = \mathbf{178.34}$$

Step 4. Multiply the two variances together.

$$243.61 \times 65.27 = \mathbf{15{,}900.425}$$

Step 5. There were two groups of scores from which the two variances of Step 1 were computed. Subtract 2 from the number of cases in one group. For example, suppose each group is composed of 36 scores.

$$36 - 2 = \mathbf{34}$$

Step 6. Take the square root of the final value of Step 5.

$$\sqrt{34} = \mathbf{5.831}$$

Step 7. Multiply the final value of Step 6 by the final value of Step 3.

$$5.831 \times 178.34 = \mathbf{1039.9}$$

Step 8. Subtract the result of Step 2 from the number 1. (*Note:* The number 1 is always used.)

$$1 - .64 = \mathbf{.36}$$

Step 9. Multiply the final value of Step 4 by the number 4. (*Note:* The number 4 is always used.)

$$4 \times 15{,}900.425 = \mathbf{63{,}601.7}$$

Step 10. Multiply the value obtained in Step 9 by the result of Step 8.

$$63{,}601.7 \times .36 = \mathbf{22{,}896.612}$$

Then, take the square root of the product.

$$\sqrt{22{,}896.612} = \mathbf{151.316}$$

Step 11. Divide the final value of Step 7 by the final value of Step 10. This yields the t statistic.

$$t = \frac{1039.9}{151.316} = \mathbf{6.87}$$

Step 12. The degrees of freedom for the t is the result of Step 5 (34 in this example). A t with 34 degrees of freedom that is larger than 2.042 (see Appendix B) is significant at the .05 level using a two-tailed test. If the computed t value is large enough to be significant, this means that the variances are very likely really different.

SECTION 3.3

Test for Differences Among Several Independent Variances (*F*-Maximum Test for Homogeneity of Variances)[1]

There are two cases in which it is desired to do a significance test of the differences among several independent variances: (1) when an experimental hypothesis is concerned with the variability of the samples and (2) when there is doubt concerning the requirement of equality of variances in a mean-difference test. In either case, the following procedure may be followed in testing the hypothesis of equal variances. There is one restriction on the use of this $F_{\max}$ test: the sizes of all the samples *must be the same.*

---

[1] A Master's thesis completed at Ohio University in 1967 by Mr. Henry Winkler titled "An empirical sampling study testing the robustness of several tests of homogeneity of variance and Type I errors in the simple randomized analysis of variance," dealt with the power of the $F_{\max}$, Bartlett's, Cochran's, and Levine's two tests for homogeneity of variance. His results indicate that the $F_{\max}$ test and Bartlett's test are the most robust and are to be preferred over the others. Because of its simplicity, the $F_{\max}$ is probably the best choice.

Step 1. Presumably, you already have the several variances (or the variance estimates, i.e., mean squares). If you do not, go to Section 1.2 and follow the instructions in the first six steps. The value arrived at in Step 6 will be the variance for one of the samples. When you have all the variances, go on to Step 2 of this section.

Step 2. Obtain the $F_{\max}$ ratio by placing the largest variance of Step 1 over the smallest. For example, suppose we had computed five variances in Step 1:

$$\text{variance}_1 = 18.99$$

$$\text{variance}_2 = 27.62$$

$$\text{variance}_3 = 92.63$$

$$\text{variance}_4 = 77.38$$

$$\text{variance}_5 = 26.40$$

Then,

$$F_{\max} = \frac{92.63}{18.99} = \mathbf{4.88}$$

Step 3. The number of degrees of freedom is always one less than the number of cases in each sample. Suppose that the size of each sample is 18. Then,

$$df = 18 - 1 = \mathbf{17}$$

Step 4. The significance of the $F_{\max}$ ratio computed in Step 2 is obtained from a table of $F_{\max}$ values (see Appendix H). We must know both the number of degrees of freedom (17 in this example) and the number of variances being considered (5 in this example). From the table of $F_{\max}$ values, we see that when $df = 17$ and the number of variances $= 5$, a value larger than 4.37 is significant at the .05 level. Since the value computed in Step 2 above (4.88) is larger than 4.37, we conclude that these variances are not homogeneous (i.e., they are different).

SECTION 3.4

The *t*-Test for Differences Among Several Means

After an F ratio has been found to be significant, the t-test is often used to determine which specific means differ significantly from each other. There is a subtle theoretical difference between when it is appropriate to use the t-tests for multiple comparisons and other tests such as Duncan's (see Section 3.5), Newman–Keuls', Tukey's, etc.

When multiple comparisons are planned before the experiment has been carried out, you can be sure that the tabled probability will be accurate for each of the tests (provided that the assumptions underlying the t-test are not grossly violated). Therefore, if you plan from the beginning to do all possible t-tests (or any selected few), there will be no problem.

However, if you wait until the experiment has been completed, look at the data, and then do tests on only the few most widely different values, you are guilty of a severe violation of a tenet of probability theory. The tabled probability value will hold only when you have planned from the beginning of the data-gathering phase to compare specific mean differences; there is a basic difference between selecting a difference at random and selecting only the largest difference among many. The various multiple-comparison techniques have been developed to keep the probability at the tabled value when the already-collected data are used to aid in stating the hypothesis to be tested.

In summary, if you plan from the beginning of the design phase to analyze specific mean differences, each t-test can be used with the tabled probability of being wrong even though the hypothesis is true. If you wait until the data are in front of you before you choose specific hypotheses based on the data, then you must use a test that will correct the probability for your choosing to state your hypothesis after the fact.

EXAMPLE

The set of data used to demonstrate the computational procedures is that used for the simple randomized design presented in Section 2.1. The data presented in the final analysis were:

Source	SS	*df*	ms	*F*	*p*
Total	1132	47	—	—	—
Between groups	567	3	189.00	14.71	$<.001$
Within groups	565	44	12.84	—	—

The significance of the overall F indicates only that out of the means of the four groups, at least two differ. The problem is to determine *which* pairs of means are significantly different.

Step 1. Obtain the mean score for each group. (In this example, there are 12 scores in each group.)

$$\text{Sum of Group 1} = 111 \qquad \frac{111}{12} = \mathbf{9.25} = \text{mean of Group 1}$$

$$\text{Sum of Group 2} = 88 \qquad \frac{88}{12} = \mathbf{7.33} = \text{mean of Group 2}$$

$$\text{Sum of Group 3} = 156 \qquad \frac{156}{12} = \mathbf{13.00} = \text{mean of Group 3}$$

$$\text{Sum of Group 4} = 195 \qquad \frac{195}{12} = \mathbf{16.25} = \text{mean of Group 4}$$

Step 2. Derive the standard error of the difference among means by

$$\sqrt{\frac{2\ \text{ms}_{\text{within}}}{n\ (\text{per gp.})}}$$

For this example,

$$\sqrt{\frac{2 \times 12.84}{12}} = \sqrt{2.14} = \mathbf{1.463}$$

If there are unequal numbers of cases in some groups, you must account for those unequal ns. If the ns are not too disparate (as a rough rule of thumb, if there are not more than twice as many in the largest group as in the smallest group), you may use $\tilde{n}$, the harmonic mean of the numbers, where

$$\tilde{n} = \frac{\text{number of groups}}{\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \cdots + \frac{1}{n_i}}$$

For example, suppose that $n_1 = 12$, $n_2 = 8$, $n_3 = 10$, and $n_4 = 6$. Then,

$$\tilde{n} = \frac{4}{\frac{1}{12} + \frac{1}{8} + \frac{1}{10} + \frac{1}{6}} = \frac{4}{.475} = \mathbf{8.42}$$

So the standard error of the differences between means would be

$$\sqrt{\frac{2(12.84)}{8.42}} = \sqrt{3.05} = \mathbf{1.746}$$

If the *ns* are widely different, the standard error must be computed separately for each comparison as

$$\sqrt{\text{ms}_{\text{within}}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

For example, if $n_1 = 16$, $n_2 = 6$, and $\text{ms}_{\text{within}} = 12.84$, we would have

$$\sqrt{12.84\left(\tfrac{1}{16}+\tfrac{1}{6}\right)} = \sqrt{\frac{12.84}{16}+\frac{12.84}{6}}$$

$$= \sqrt{.80 + 2.14} = \sqrt{2.94} = \mathbf{1.715}$$

A new standard error must then be computed for each new *t*-test.

Step 3. The needed *t* value is obtained from Appendix B ($p = .05$). The table is entered at the row headed by the *df* for the mean square within groups. In the present example, the *df* for $\text{ms}_{\text{within}}$ is 44. Since 44 is not listed, go to the next closest *df*, which is 40. Reading into the table, for 40 *df*, at the .05 level,

$$t = \mathbf{2.02}$$

Step 4. Now multiply the *t* value of Step 3 by the value obtained in Step 2. This gives the critical difference against which the several mean differences will be compared.

$$2.02 \times 1.463 = \mathbf{2.955} = \text{C. diff.}$$

Step 5. Testing the differences between the various means: In all instances, if the difference between any two means (see Step 1) is larger than the critical difference (in this case, 2.955), then the means are assumed to be significantly different.

Group 1 vs. Group 4 (C. diff. = 2.955):

16.25 − 9.25 = **7.00** (significant)

Group 1 vs. Group 3 (C. diff. = 2.955):

13.00 − 9.25 = **3.75** (significant)

Group 1 vs. Group 2 (C. diff. = 2.955):

7.33 − 9.25 = **1.92** (nonsignificant)

Group 2 vs. Group 4 (C. diff. = 2.955):

16.25 − 7.33 = **8.92** (significant)

Group 2 vs. Group 3 (C. diff. = 2.955):

13.00 − 7.33 = **5.67** (significant)

Group 3 vs. Group 4 (C. diff. = 2.955):

16.25 − 13.00 = **3.25** (significant)

SECTION 3.5

Duncan's Multiple-Range Test for Nearly Equal *ns*[1,2]

While the t test is often used for making multiple comparisons after the F has been found to be significant, a more stringent test procedure is Duncan's multiple-range test (see the discussion at the beginning of Section 3.4).

EXAMPLE

The set of data used to demonstrate the computational procedures is that used for the simple randomized design presented in Section 2.1. Data from the final analysis of that example are:

Source	SS	*df*	ms	*F*	*p*
Total	1132	47	—	—	—
Between groups	567	3	189.00	14.71	$<.001$
Within groups	565	44	12.84	—	—

The significance of the overall F means only that among the means of the four groups, at least two differ. The problem is to determine which specific groups actually differ significantly.

Step 1. Obtain the mean score for each group. (In this example, there are 12 scores per group.)

$$\text{Sum of Group 1} = 111 \qquad \frac{111}{12} = \mathbf{9.25} = \text{mean of Group 1}$$

$$\text{Sum of Group 2} = 88 \qquad \frac{88}{12} = \mathbf{7.33} = \text{mean of Group 2}$$

$$\text{Sum of Group 3} = 156 \qquad \frac{156}{12} = \mathbf{13.00} = \text{mean of Group 3}$$

$$\text{Sum of Group 4} = 195 \qquad \frac{195}{12} = \mathbf{16.25} = \text{mean of Group 4}$$

---

[1] For unequal *ns*, see C. Y. Kramer, Extension of multiple range tests to group means with unequal numbers of replications, *Biometrics*, 1956, **12,** 307–310.

[2] Other multiple-range tests are the Newman-Keuls, Tukey (a) and (b), and Scheffe, discussed by B. J. Winer, *Statistical Principles in Experimental Design*, New York, McGraw-Hill Book Company, 1962, pp. 77–104.

Step 2. Derive the standard error of the means by

$$\sqrt{\frac{\mathrm{ms}_{\mathrm{within}}}{n\ (\text{per gp.})}}$$

For this example,

$$\sqrt{\frac{12.84}{12}} = \sqrt{1.07} = \mathbf{1.03}$$

If there are unequal numbers of cases in some groups, you must account for those unequal *n*s. If the *n*s are not too disparate (as a rough rule of thumb, if there are not more than twice as many in the largest group as in the smallest group), you may use $\tilde{n}$, the harmonic mean of the numbers, where

$$\tilde{n} = \frac{\text{number of groups}}{\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \cdots + \frac{1}{n_i}}$$

For example, suppose that $n_1 = 12$, $n_2 = 8$, $n_3 = 10$, and $n_4 = 6$. Then,

$$\tilde{n} = \frac{4}{\frac{1}{12} + \frac{1}{8} + \frac{1}{10} + \frac{1}{6}} = \frac{4}{.475} = \mathbf{8.42}$$

So the standard error of the means would be

$$\sqrt{\frac{12.84}{8.42}} = \sqrt{1.525} = \mathbf{1.235}$$

Step 3. Go to Appendix I ($p = .05$) to obtain the "significant studentized ranges." The table is entered at the row headed by the *df* for the mean square within groups. In the present example, the *df* for $\mathrm{ms}_{\mathrm{within}}$ is 44. Since 44 *df* is not listed, use the next closest *df*, which is 40. Reading to the right in this row, for the different ranges of means (k) to be compared, the values are

$k = 2$: **2.858**
$k = 3$: **3.006**
$k = 4$: **3.102**

Step 4. Multiply each significant range by the value obtained in Step 2. The values obtained are the minimum mean differences (R) for the given ranges of comparisons.

$k = 2$: $R_2 = 2.858 \times 1.03 = \mathbf{2.944}$
$k = 3$: $R_3 = 3.006 \times 1.03 = \mathbf{3.096}$
$k = 4$: $R_4 = 3.102 \times 1.03 = \mathbf{3.195}$

Step 5. Testing the differences between the various means: The means must first be ranked from smallest to largest in order that the range (k value) for each difference can be determined. In this example, the means are ranked as: 7.33, 9.25, 13.00, 16.25. In each instance, if the difference between the means (see Step 1) is larger than the minimum for that range (Step 4), it is considered to be significant.

Group 2 vs. Group 4 ($R_4 = 3.195$)

$16.25 - 7.33 = \mathbf{8.92}$ (significant)

Group 2 vs. Group 3 ($R_3 = 3.096$)

$13.00 - 7.33 = \mathbf{5.67}$ (significant)

Group 2 vs. Group 1 ($R_2 = 2.944$)

$9.25 - 7.33 = \mathbf{1.92}$ (nonsignificant)

Group 1 vs. Group 4 ($R_3 = 3.096$)

$16.25 - 9.25 = \mathbf{7.00}$ (significant)

Group 1 vs. Group 3 ($R_2 = 2.944$)

$13.00 - 9.25 = \mathbf{3.75}$ (significant)

Group 3 vs. Group 4 ($R_2 = 2.944$)

$16.25 - 13.00 = \mathbf{3.25}$ (significant)

Therefore, it is concluded that Groups 2 and 4, 2 and 3, 1 and 4, 1 and 3, and 3 and 4 differ significantly in terms of their overall performance.

Note: There are important theoretical assumptions which determine how many comparisons may be made. Since the presentation of these assumptions is beyond the scope of this book, the user is referred to: Edwards, A. L. *Experimental Designs in Psychological Research*, rev. ed. New York: Holt, Rinehart & Winston, Inc., 1960. Chapter 10, pp. 136–140. Or, Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Company, 1962. Chapter 3, pp. 85–89.

SECTION 3.6

F-Tests for Simple Effects

The *F*-test for simple effects is most commonly used when one of the factorial or mixed designs (Sections 2.2–2.11) yields a significant interaction.

EXAMPLE

To demonstrate some of the more commonly used tests of simple effects, a complete analysis of the two-factor mixed design of Section 2.7 will be presented. Although the presentation in this section represents a considerably more detailed analysis than is usually carried out, the several operations are presented for the sake of completeness.

First, recall the final analysis of the mixed design of Section 2.7:

Source	SS	*df*	ms	*F*	*p*
Total	467	98	—	—	—
Between subjects	181	32	—	—	—
Conditions	39	2	19.50	4.12	$<.05$
Error_b	142	30	4.73	—	—
Within subjects	286	66	—	—	—
Trials	170	2	85.00	52.46	$<.001$
Trials × conditions	19	4	4.75	2.93	$<.05$
Error_w	97	60	1.62	—	—

The conclusions drawn from this analysis were:

1. Meaningfulness of material significantly affected overall amount learned.
2. The overall tendency of the subjects was to change (learn) as a function of practice (trials).
3. The subjects in the various groups learned at different rates.

The specific questions to be answered by the analyses of simple effects relate to the differences noted above and can be stated as follows.

First Question. Which of the three groups (conditions) differ significantly from each other? Since the overall *F* merely means that of the three groups, at least two differ significantly, the problem is to determine which specific pairs differ. To analyze these differences, the Duncan's range test will be applied.[1]

Step 1. Obtain the overall mean for each of the three groups. The sums obtained in Step 3 of the original analysis (Section 2.7) must be divided by the number of subjects in each of the experimental groups (in this case, 11).

$$\text{Sum of Group 1} = 111 \qquad \frac{111}{11} = \mathbf{10.0909} = \text{mean of Group 1}$$

$$\text{Sum of Group 2} = 124 \qquad \frac{124}{11} = \mathbf{11.2727} = \text{mean of Group 2}$$

$$\text{Sum of Group 3} = 160 \qquad \frac{160}{11} = \mathbf{14.5454} = \text{mean of Group 3}$$

Step 2. Derive the standard error by

$$\sqrt{\frac{\text{ms}_{\text{error}_b}}{n\ (\text{per gp.})}} = \sqrt{\frac{4.73}{11}} = \sqrt{.43} = \mathbf{.656}$$

---

[1] Obviously, *t*-tests could also be used.

Step 3. The significant studentized ranges are obtained from Appendix I of Duncan's multiple-range values ($p = .05$). The table is entered at the row headed by the *df* for $\text{ms}_{\text{error}_b}$, which is 30. Reading to the right in this row, for the different ranges of means (k) to be compared, the values are

$$k = 2:\quad \mathbf{2.888}$$

$$k = 3:\quad \mathbf{3.035}$$

Step 4. Multiply each significant range by the value obtained in Step 2. This gives the minimum mean differences (R) against which the actual differences in the means will be compared.

$$k = 2:\quad R_2 = 2.888 \times .656 = \mathbf{1.895}$$

$$k = 3:\quad R_3 = 3.035 \times .656 = \mathbf{1.991}$$

Step 5. Testing the difference between the various means: In each instance, if the difference between the means (see Step 1) is larger than the minimum for that range (Step 4), it is considered to be significant.

Group 1 vs. Group 3 ($R_3 = 1.991$):

$$14.5454 - 10.0909 = \mathbf{4.4545} \text{ (significant)}$$

Group 1 vs. Group 2 ($R_2 = 1.895$):

$$11.2727 - 10.0909 = \mathbf{1.1818} \text{ (nonsignificant)}$$

Group 2 vs. Group 3 ($R_2 = 1.895$):

$$14.5454 - 11.2727 = \mathbf{3.2727} \text{ (significant)}$$

Therefore, it is concluded that Groups 1 and 3 and Groups 2 and 3 differ significantly in terms of their overall performance.

Second Question. Although the overall tendency of the groups was to change (learn) as a function of trials (practice), did each of the groups actually improve? (*Note:* In actuality, most researchers would not be interested in this question until the third question had been answered. However, since this is merely an example, the various significant effects will be treated in order.)

Step 1. From Step 2 of the original analysis (Section 2.7), obtain the sums of the trials for Group 1.

Sum for Trial 1 = **24**
Sum for Trial 2 = **38**
Sum for Trial 3 = **49**

Step 2. Square each of these values, divide by the number of scores on which each sum was based (in this case, 11), and add the quotients.

$$\frac{24^2}{11} + \frac{38^2}{11} + \frac{49^2}{11} = \frac{4421}{11} = \mathbf{401.909}$$

Step 3. Obtain the grand sum for Group 1.

$$24 + 38 + 49 = \mathbf{111}$$

Then, square this sum and divide by the total number of scores recorded in Group 1.

$$\frac{111^2}{33} = \frac{12{,}321}{33} = \mathbf{373.364}$$

Step 4. Subtract the value of Step 3 from that of Step 2. This yields the Group 1 sum of squares for trials.

$$401.909 - 373.364 = \mathbf{28.545} = \text{SS}_{\text{trials}} \text{ for Group 1}$$

Step 5. The *df* for this component is equal to the number of trials minus 1.

$$3 - 1 = \mathbf{2}$$

Step 6. Computation of the mean square for the simple trials effects: Merely divide the $\text{SS}_{\text{trials}}$ for Group 1 (Step 4) by the *df* (Step 5).

$$\frac{28.545}{2} = \mathbf{14.272} = \text{ms}_{\text{trials}} \text{ for Group 1}$$

Step 7. The test of significance (*F*-test) of the trials effect for Group 1 is $\text{ms}_{\text{trials (Gp. 1)}}/\text{ms}_{\text{error}_w \text{ (total)}}$.

$$F = \frac{14.272}{1.62} = \mathbf{8.81}$$

From Appendix D, it is found that this value, with *df* 2 and 60, is significant ($p < .001$). Therefore, it is concluded that performance of the subjects in Group 1 changed as a function of practice. The same procedures should be repeated for Groups 2 and 3. In each instance, the *F* ratio equals $\text{ms}_{\text{trials(Group 2 or 3)}}/\text{ms}_{\text{error}_w \text{ (total)}}$.

Third Question. The trials-by-conditions interaction indicates that at least two of the groups of subjects learned at different rates: Which groups were they? Comparisons must be made between Groups 1 and 2, Groups 1 and 3, and Groups 2 and 3. In the present example, the first comparison will be made between Group 1 and Group 2.

Step 1. From Step 2 of the original analysis (Section 2.7), obtain the sums of each of the trials for Group 1 and Group 2.

	Trial 1	Trial 2	Trial 3
Sums for Group 1 =	24	38	49
Sums for Group 2 =	24	43	57

Step 2. Obtain the overall sum for each of the groups.

$$24 + 38 + 49 = \mathbf{111} = \text{sum for Group 1}$$
$$24 + 43 + 57 = \mathbf{124} = \text{sum for Group 2}$$

Then, square these sums, divide by the number of scores on which each sum was based, and add the quotients.

$$\frac{111^2}{33} + \frac{124^2}{33} = \frac{27{,}697}{33} = \mathbf{839.303}$$

Step 3. Obtain the grand sum for both groups.

$$111 + 124 = \mathbf{235}$$

Then, square this value, and divide it by the total number of scores recorded in Groups 1 and 2. This yields the correction term for the Group 1 versus Group 2 comparison.

$$\frac{235^2}{66} = \frac{55{,}225}{66} = \mathbf{836.742}$$

Step 4. Computation of the Group 1 versus Group 2 effects: Merely subtract the correction term (Step 3) from the value obtained in Step 2.

$$839.303 - 836.742 = \mathbf{2.561} = SS_{\text{groups}} \text{ for Groups 1 and 2}$$

Step 5. Computation of the effects of trials for Groups 1 and 2: First, obtain the sum of Trials 1, 2, and 3 for both experimental groups (see Step 1).

$$24 + 24 = \mathbf{48} = \text{sum for Trial 1}$$
$$38 + 43 = \mathbf{81} = \text{sum for Trial 2}$$
$$49 + 57 = \mathbf{106} = \text{sum for Trial 3}$$

Then, square each of these sums, divide by the number of scores on which each was based, and add the quotients.

$$\frac{48^2}{22} + \frac{81^2}{22} + \frac{106^2}{22} = \frac{20{,}101}{22} = \mathbf{913.682}$$

Then, subtract the correction term (Step 3) from the above value.

$$913.682 - 836.742 = \mathbf{76.940} = SS_{\text{trials}} \text{ for Groups 1 and 2}$$

Step 6. Computation of interaction of trials by conditions (groups) for Groups 1 and 2: First, square each of the sums for each trial for Groups 1 and 2 (Step 1). Divide each value by the number of scores on which each sum was based, and then add the quotients.

$$\frac{24^2}{11} + \frac{38^2}{11} + \frac{49^2}{11} + \frac{24^2}{11} + \frac{43^2}{11} + \frac{57^2}{11} = \frac{10{,}095}{11} = \mathbf{917.727}$$

Then, subtract the correction term (Step 3), the SS_{groups} (Step 4), and SS_{trials} (Step 5) from the above value.

$$917.727 - 836.742 - 2.561 - 76.940$$

$$= \mathbf{1.484} = SS_{\text{trials} \times \text{groups}} \text{ for Groups 1 and 2}$$

Step 7. The *df* for this component is equal to the number of trials minus 1, times the number of groups minus 1.

$$(3 - 1)(2 - 1) = \mathbf{2}$$

Step 8. Computation of the ms for the trials-by-groups interaction for Groups 1 and 2: Merely divide the $SS_{\text{trials} \times \text{groups}}$ (Step 6) by the *df* (Step 7).

$$\frac{1.484}{2} = \mathbf{.742} = ms_{\text{trials} \times \text{groups}} \text{ for Groups 1 and 2}$$

Step 9. The test of significance (*F*-test) is computed as

$$F = \frac{ms_{\text{trials} \times \text{groups (Gps. 1 and 2)}}}{ms_{\text{error}_w \text{ (total)}}}$$

Thus,

$$F = \frac{.742}{1.62} = \mathbf{.46}$$

From Appendix D, it is found that this *F* value, with $df = 2$ and 60, is nonsignificant. Therefore, it is concluded that the subjects in Groups 1 and 2 learned at the same rate. The remaining tests of simple interactions are carried out in exactly the same way, except that Groups 1 and 3 are compared and then Groups 2 and 3. In all instances, the test of significance is $F = ms_{\text{trials} \times \text{groups}}/ms_{\text{error}_w}$.

One Final Question. Was there a sampling error? Although the problem is not present in this example, sometimes in mixed designs there is some question as to whether the means of the several groups were significantly different initially (sampling error). If this were the case, any differences noted in the overall analysis would merely be reflecting this error rather than indicating any real differences.

The most common technique used to determine if there has been a sampling error is a comparison of the performance of the several groups on Trial 1. Although many analyses are used (*t*-tests of all pairs of groups, Duncan's range test on all pairs, etc.), the *F*-test is employed most often and is a completely randomized design for a particular trial.

This *F*-test of groups effects for Trial 1[2] is computed as follows.

---

[2] If desired, this same analysis can be applied to any trial. For example, it is often necessary to determine if certain groups differed significantly at the *end* of training. In this case, the analysis would be applied to the last trial.

Step 1. From Step 1 of the original analysis (Section 2.7), obtain the scores for each subject in all three groups on Trial 1. Table these scores as follows.

TRIAL 1

Group 1		Group 2		Group 3	
Subject	Score	Subject	Score	Subject	Score
S_1	3	S_{12}	2	S_{23}	2
S_2	3	S_{13}	1	S_{24}	3
S_3	1	S_{14}	3	S_{25}	4
S_4	1	S_{15}	2	S_{26}	1
S_5	2	S_{16}	1	S_{27}	3
S_6	4	S_{17}	4	S_{28}	1
S_7	4	S_{18}	1	S_{29}	1
S_8	1	S_{19}	1	S_{30}	2
S_9	1	S_{20}	2	S_{31}	3
S_{10}	2	S_{21}	3	S_{32}	1
S_{11}	2	S_{22}	4	S_{33}	4

Step 2. Turn to the completely randomized design as presented in Section 2.1 and continue with Step 2 of that analysis. The final F value (Step 12) will indicate whether there are significant differences among the group means on this particular trial.

Note: If, when the above analysis has been completed, a significant conditions (groups) effect is found, multiple t-tests (Section 3.4) or a Duncan's range test (Section 3.5) may be used to determine which specific pairs of groups differ.

SECTION 3.7

Use of Orthogonal Components in Tests for Trend

In general, after an F ratio has been found to be significant, the only additional tests are those for simple effects (Sections 3.4, 3.5, and 3.6). Occasionally, however, it is of interest to determine the shape of the curves that best fit the data points of the various experimental groups. If the treatments form an increasing or decreasing continuum, such as trials or variations in shock intensity or anxiety level, the treatment effects can be further subdivided into trend components by use of orthogonal polynomials. The theoretical rationale underlying these analyses is presented in the following textbooks.

Lewis, D. *Quantitative Methods in Psychology.* New York: McGraw-Hill Book Company, 1960. Chapter 10, pp. 398–419.

Winer, B. J. *Statistical Principles in Experimental Design.* New York: McGraw-Hill Book Company, 1962. Chapter 3, pp. 70–77; Chapter 7, pp. 353–369.

Three examples will be presented: The first is based on the data of Section 2.1 (the completely randomized design), the second on data presented in Section 2.7 (the two-factor mixed design), and the third on data from Section 2.8 (the three-factor mixed design). These examples represent the most frequent instances where this type of trend analysis is used, but they by no means exhaust the possibilities of application since the analysis can be used with any design. It is important to note that this type of trend analysis merely gives information regarding the shape of the performance curves. The nature of the data and the purpose of the experiment determine entirely whether these analyses are worthwhile.

EXAMPLE 1

This example is based on the data and results of the completely randomized analysis presented in Section 2.1. First, recall the final analysis of that section:

Source	SS	*df*	ms	*F*	*p*
Total	1132	47	—	—	—
Between groups	567	3	189.00	14.71	$<.001$
Within groups	565	44	12.84	—	—

Question. What degree polynomial best approximates the form of the curve that connects the data points (group means) of the four groups?

Step 1. Turn to Appendix K and obtain the appropriate coefficients for the re-analysis. Since there are four groups, the coefficients for $k = 4$ are:

Linear:	-3	-1	$+1$	$+3$
Quadratic:	$+1$	-1	-1	$+1$
Cubic:	-1	$+3$	-3	$+1$

Step 2. The linear component will be analyzed first. To do this,

multiply the *sum* of each group (Step 2 of the original analysis) by its appropriate linear coefficient.

$$\begin{array}{llll}
\text{Sum of Group 1} = 111 & 111 \times (-3) & = & \mathbf{-333} \\
\text{Sum of Group 2} = 88 & 88 \times (-1) & = & \mathbf{-88} \\
\text{Sum of Group 3} = 156 & 156 \times (+1) & = & \mathbf{+156} \\
\text{Sum of Group 4} = 195 & 195 \times (+3) & = & \mathbf{+585}
\end{array}$$

Step 3. Get the algebraic sum of the values obtained in Step 2.

$$(-333) + (-88) + 156 + 585 = \mathbf{+320}$$

Then, square that sum.

$$320^2 = \mathbf{102{,}400}$$

Step 4. Square each of the linear coefficients (Step 1), and sum these squared values.

$$(-3)^2 + (-1)^2 + 1^2 + 3^2 = \mathbf{20}$$

Then, multiply the sum by the number of subjects in each group (12 in the present example).

$$20 \times 12 = \mathbf{240}$$

Step 5. Divide the value of Step 3 by that of Step 4.

$$\frac{102{,}400}{240} = \mathbf{426.67} = \mathrm{SS}_{\text{linear}}$$

Step 6. The test for linear trend is an F ratio that is equal to the value of Step 5 divided by the mean square within groups of the original analysis. (In the present example, $\mathrm{ms}_{\text{within}} = 12.84$.) Compute this F, and record the value for future use.

$$F_{\text{linear}} = \frac{426.67}{12.84} = \mathbf{33.22}$$

The degrees of freedom (*df*) for this F ratio are always equal to 1 and the *df* for $\mathrm{ms}_{\text{within}}$ of the original analysis (44 in the present example).

After completing the linear computations, the same basic steps must be repeated to analyze the quadratic component.

Step 1. From Step 1 of the linear analysis (or Appendix K), obtain the coefficients for the quadratic component. In the present example, where $k = 4$, these are:

$$\text{Quadratic:} \quad +1 \quad -1 \quad -1 \quad +1$$

Step 2. This time multiply the sum of each group by its appropriate quadratic coefficient.

$$\begin{array}{llll}
\text{Sum of Group 1} = 111 & 111 \times (+1) & = & \mathbf{+111} \\
\text{Sum of Group 2} = 88 & 88 \times (-1) & = & \mathbf{-88} \\
\text{Sum of Group 3} = 156 & 156 \times (-1) & = & \mathbf{-156} \\
\text{Sum of Group 4} = 195 & 195 \times (+1) & = & \mathbf{+195}
\end{array}$$

Step 3. Get the algebraic sum of the values obtained in Step 2.

$$111 + (-88) + (-156) + 195 = \mathbf{+62}$$

Then, square this sum.

$$+62^2 = \mathbf{3844}$$

Step 4. Square the quadratic coefficients, and sum these squared values.

$$1^2 + (-1)^2 + (-1)^2 + 1^2 = \mathbf{4}$$

Then, multiply this sum by the number of subjects in each group. (In the present example, the number of subjects per group is 12.)

$$4 \times 12 = \mathbf{48}$$

Step 5. Divide the value of Step 3 by that of Step 4.

$$\frac{3844}{48} = \mathbf{80.08} = \text{SS}_{\text{quadratic}}$$

Step 6. The test for quadratic trend is an F ratio that is equal to the value of Step 5 divided by the mean square within groups of the original analysis. Compute this F, and record the value for future use.

$$F_{\text{quadratic}} = \frac{80.08}{12.84} = \mathbf{6.23}$$

The *df*s for this F ratio are always equal to 1 and the *df* for $\text{ms}_{\text{within}}$ of the original analysis (44 in this example).

Once again the computations must be repeated; this time, analyze the cubic component.

Step 1. From Step 1 of the linear analysis (or Appendix K), obtain the cubic coefficients. In the present example, where $k = 4$, these are:

Cubic: $-1 \quad +3 \quad -3 \quad +1$

Step 2. Multiply the sum of each group by the appropriate cubic coefficient.

Sum of Group 1 = 111 $\quad 111 \times (-1) = \mathbf{-111}$
Sum of Group 2 = 88 $\quad 88 \times (+3) = \mathbf{+264}$
Sum of Group 3 = 156 $\quad 156 \times (-3) = \mathbf{-468}$
Sum of Group 4 = 195 $\quad 195 \times (+1) = \mathbf{+195}$

Step 3. Get the algebraic sum of the values obtained in Step 2.

$$(-111) + 264 + (-468) + 195 = \mathbf{-120}$$

Then, square this sum.

$$-120^2 = \mathbf{14{,}400}$$

Step 4. Square each of the cubic coefficients, and sum these squared values.

$$(-1)^2 + 3^2 + (-3)^2 + 1^2 = \mathbf{20}$$

Then, multiply that sum by the number of subjects in each group (12 in this example).

$$20 \times 12 = \mathbf{240}$$

Step 5. Divide the value of Step 3 by that of Step 4.

$$\frac{14{,}400}{240} = \mathbf{60.00} = \mathrm{SS}_{\text{cubic}}$$

Step 6. The test of significance for cubic trend is again an F ratio that is equal to the value of Step 5 divided by the mean square within groups of the original analysis. Compute this F, and record the value for future use.

$$F_{\text{cubic}} = \frac{60.00}{12.84} = \mathbf{4.67}$$

The *df*s are equal to 1 and the *df* for $\mathrm{ms}_{\text{within}}$ (44 in this example).

Step 7. Final check for accuracy: Since the sum of squares between groups was broken into its several trend components, the sums of the linear, quadratic, and cubic components must equal (within rounding error) the sum of squares between groups. (In the present example, $\mathrm{SS}_b = 567$.)

$$426.67 + 80.08 + 60.00 = \mathbf{566.75}$$

Note that if the sum of the trend components does not equal the sum of squares between groups, an error in computation has been made.

Step 8. Final tabling of the data: Although there are several ways to do this, the following is most often used.

Source	SS	*df*	ms	*F*	*p*
Total	1132	47	—	—	—
Between groups	567	3	189.00	14.71	<.001
Linear	426.67	1	426.67	33.22	<.001
Quadratic	80.08	1	80.08	6.23	<.025
Cubic	60.00	1	60.00	4.67	<.05
Within groups	565	44	12.84	—	—

The results are interpreted as showing that the data can be best described mathematically by a complex equation. The significant linear component shows that there is some tendency for the data to lie along a straight line. The significant quadratic component shows that there is a tendency for the data to deviate from a straight line in an arcing manner. A quadratic function is represented by a curving line, such as a parabolic arc. The significant cubic component shows that there is a tendency for the data to crisscross a straight line in a regular fashion. A cubic function is similar to two connected parabolic arcs, each peaking in opposite directions.

EXAMPLE 2

First, recall the two-factor mixed analysis as presented in Section 2.7. The final table of the results was as follows:

Source	SS	*df*	ms	*F*	*p*
Total	467	98	—	—	—
Between subjects	181	32	—	—	—
Conditions	39	2	19.50	4.12	$<.05$
Error_b	142	30	4.73	—	—
Within subjects	286	66	—	—	—
Trials	170	2	85.00	52.46	$<.001$
Trials by conditions	19	4	4.75	2.93	$<.05$
Error_w	97	60	1.62	—	—

Question. What level of polynomial functions best describes the form of the data points of the various within-subjects effects?

Step 1. Turn to Appendix K, and obtain the appropriate coefficients for the re-analysis. In the present example, each subject was measured three times (three trials). Therefore, the coefficients for $k = 3$ are:

Linear:	-1	0	$+1$
Quadratic:	$+1$	-2	$+1$

Step 2. The linear component will be analyzed first. Go back to the data recorded in Step 1 of the original analysis (Section 2.7) and multiply each score of each subject by the appropriate linear coefficient. In the present example, the retabled data would appear as follows.

GROUP 1 (LOW-MEANINGFUL MATERIAL)

	Trial Block 1	Trial Block 2	Trial Block 3
Subject	Score × −1	Score × 0	Score × +1
S_1	−3	0	4
S_2	−3	0	4
S_3	−1	0	4
S_4	−1	0	3
S_5	−2	0	5
S_6	−4	0	6
S_7	−4	0	6
S_8	−1	0	5
S_9	−1	0	5
S_{10}	−2	0	3
S_{11}	−2	0	4

GROUP 2 (MEDIUM-MEANINGFUL MATERIAL)

S_{12}	−2	0	6
S_{13}	−1	0	5
S_{14}	−3	0	7
S_{15}	−2	0	6
S_{16}	−1	0	3
S_{17}	−4	0	5
S_{18}	−1	0	4
S_{19}	−1	0	4
S_{20}	−2	0	4
S_{21}	−3	0	6
S_{22}	−4	0	7

GROUP 3 (HIGH-MEANINGFUL MATERIAL)

S_{23}	−2	0	8
S_{24}	−3	0	3
S_{25}	−4	0	7
S_{26}	−1	0	4
S_{27}	−3	0	12
S_{28}	−1	0	2
S_{29}	−1	0	3
S_{30}	−2	0	6
S_{31}	−3	0	6
S_{32}	−1	0	12
S_{33}	−4	0	7

Step 3. Obtain the algebraic sum of the "weighted scores" for each subject in all groups. (*Note:* All subsequent linear computations will be based on these algebraic sums.)

Group 1

Subject	Trial Block 1		Trial Block 2		Trial Block 3	Sum
S_1	−3	+	0	+	4	+1
S_2	−3	+	0	+	4	+1
⋮	⋮		⋮		⋮	⋮
S_{11}	−2	+	0	+	4	+2

Group 2

S_{12}	−2	+	0	+	6	+4
S_{13}	−1	+	0	+	5	+4
⋮	⋮		⋮		⋮	⋮
S_{22}	−4	+	0	+	7	+3

Group 3

S_{23}	−2	+	0	+	8	+6
S_{24}	−3	+	0	+	3	0
⋮	⋮		⋮		⋮	⋮
S_{33}	−4	+	0	+	7	+3

Step 4. Add the subject sums obtained in Step 3 to get the algebraic sum for each experimental group. (*Note:* These group sums may be negative.)

$$1 + 1 + \cdots + 2 = \mathbf{+25} = \text{sum of Group 1}$$
$$4 + 4 + \cdots + 3 = \mathbf{+33} = \text{sum of Group 2}$$
$$6 + 0 + \cdots + 3 = \mathbf{+45} = \text{sum of Group 3}$$

Step 5. Square each of the subject sums obtained in Step 3, and add these squared values to get the overall sum of squares.

$$1^2 + 1^2 + \cdots + 4^2 + 4^2 + \cdots + 6^2 + 0^2 + \cdots + 3^2 = \mathbf{473}$$

Step 6. Add the group sums obtained in Step 4 to get the overall algebraic sum. (*Note:* Again, this may be a negative value.)

$$25 + 33 + 45 = \mathbf{+103}$$

Step 7. Square each of the linear coefficients (Step 1), and add the squared values.

$$(-1)^2 + 0^2 + 1^2 = \mathbf{2}$$

Step 8. Computation of the within-subjects linear component: Divide the sum of the squared values obtained in Step 5 by the value of Step 7.

$$\frac{473}{2} = \mathbf{236.5}$$

Step 9. Computation of the repeated-measure effect (trials, in the present example) for the linear component: Square the overall algebraic sum obtained in Step 6, and then divide by the value of Step 7 times the total number of subjects in the experiment (33, in the present example).

$$\frac{103^2}{2 \times 33} = \frac{10{,}609}{66} = \mathbf{160.742}$$

Step 10. Computation of the trials-by-groups effect for the linear component: First, square each of the group sums obtained in Step 4, divide each by the value of Step 7 times the number of subjects in each group (11 in the present example), and add the quotients.

$$\frac{25^2}{2 \times 11} + \frac{33^2}{2 \times 11} + \frac{45^2}{2 \times 11} = \frac{3739}{22} = \mathbf{169.955}$$

Then, from this sum, subtract the value obtained in Step 9.

$$169.955 - 160.742 = \mathbf{9.213}$$

Step 11. Computation of the error term for the linear component:

Simply subtract the values obtained in Steps 9 and 10 from the value of Step 8.

$$236.5 - 160.742 - 9.213 = \mathbf{66.545}$$

Step 12. Computation of the *df* for each of the above components:

df for within subjects (see Step 8) = the total number of subjects used in the experiment.

$$\text{No. of subjects} = \mathbf{33}$$

df for trials (see Step 9) = **1** in all instances.
df for the trials-by-groups interaction (see Step 10) = the number of experimental groups minus 1.

$$3 - 1 = \mathbf{2}$$

df for the linear-component error term (see Step 11) = *df* for within subjects minus the *df* for trials and the *df* for trials by groups.

$$33 - 1 - 2 = \mathbf{30}$$

Step 13. Computation of the required linear mean squares: Simply divide each sum of squares value as computed in Steps 8, 9, 10, and 11 by its appropriate degrees of freedom (Step 12). The test of significance in each instance is an *F* ratio of the appropriate linear component divided by the linear error term. Table the mean-square values and the *F* ratios temporarily as follows.

Source	SS	*df*	ms	*F*	*p*
$\text{Within}_{\text{linear}}$	236.50	33	(not needed)	—	—
Trials	160.742	1	160.742	72.47	<.001
Trials by groups	9.213	2	4.606	2.08	n.s.
$\text{Error}_{\text{linear}}$	66.545	30	2.218	—	—

After completing the linear-component computations, the same basic steps must be repeated to analyze the quadratic component.

Step 1. From Step 1 of the linear analysis (or Appendix K), obtain the coefficients for the quadratic component. In the present example, where $k = 3$, the coefficients are:

$$\text{Quadratic:} \quad +1 \quad -2 \quad +1$$

Step 2. The data must again be retabled. The same basic procedure of Step 2 of the linear analysis is followed except that each score is multiplied by its appropriate quadratic coefficient.

GROUP 1

	Trial Block 1	Trial Block 2	Trial Block 3
Subject	Score × +1	Score × −2	Score × +1
S_1	3	−6	4
S_2	3	−8	4
S_3	1	−6	4
S_4	1	−4	3
S_5	2	−6	5
S_6	4	−10	6
S_7	4	−10	6
S_8	1	−8	5
S_9	1	−8	5
S_{10}	2	−6	3
S_{11}	2	−4	4

GROUP 2

S_{12}	2	−6	6
S_{13}	1	−8	5
S_{14}	3	−12	7
S_{15}	2	−8	6
S_{16}	1	−4	3
S_{17}	4	−10	5
S_{18}	1	−6	4
S_{19}	1	−4	4
S_{20}	2	−6	4
S_{21}	3	−10	6
S_{22}	4	−12	7

GROUP 3

S_{23}	2	−8	8
S_{24}	3	−12	3
S_{25}	4	−14	7
S_{26}	1	−14	4
S_{27}	3	−14	12
S_{28}	1	−8	2
S_{29}	1	−10	3
S_{30}	2	−10	6
S_{31}	3	−12	6
S_{32}	1	−10	12
S_{33}	4	−18	7

Step 3. Obtain the algebraic sum of the "weighted scores" for each subject in all groups. (*Note:* All remaining quadratic computations will be based on these sums.)

Group 1

Subject	Trial Block 1		Trial Block 2		Trial Block 3	Sum
S_1	3	+	(−6)	+	4	+1
S_2	3	+	(−8)	+	4	−1
⋮	⋮		⋮		⋮	⋮
S_{11}	2	+	(−4)	+	4	+2

Group 2

S_{12}	2	+	(−6)	+	6	+2
S_{13}	1	+	(−8)	+	5	−2
⋮	⋮		⋮		⋮	⋮
S_{22}	4	+	(−12)	+	7	−1

Group 3

S_{23}	2	+	(−8)	+	8	+2
S_{24}	3	+	(−12)	+	3	−6
⋮	⋮		⋮		⋮	⋮
S_{33}	4	+	(−18)	+	7	−7

Step 4. Add the subject sums obtained in Step 3 to get the algebraic sum for each group.

$$\begin{aligned} 1 + (-1) + \cdots + 2 &= \mathbf{-3} = \text{sum for Group 1} \\ 2 + (-2) + \cdots + (-1) &= \mathbf{-5} = \text{sum for Group 2} \\ 2 + (-6) + \cdots + (-7) &= \mathbf{-35} = \text{sum for Group 3} \end{aligned}$$

Step 5. Square each of the subject sums obtained in Step 3, and add these squared values.

$$1^2 + (-1)^2 + \cdots + 2^2 + (-2)^2 + \cdots + 2^2 + (-6)^2 + \cdots + (-7)^2 = \mathbf{297}$$

Step 6. Add the group sums obtained in Step 4 to get the overall algebraic sum.

$$(-3) + (-5) + (-35) = \mathbf{-43}$$

Step 7. Square each of the quadratic coefficients (Step 1), and add the squared values.

$$1^2 + (-2)^2 + 1^2 = \mathbf{6}$$

Step 8. Computation of the within-subjects quadratic component: Divide the value of Step 5 by that of Step 7.

$$\frac{297}{6} = \mathbf{49.50}$$

Step 9. Computation of the trials effect for the quadratic component:

Square the value of Step 6, and divide it by the value of Step 7 times the total number of subjects used in the experiment (33 in this example).

$$\frac{(-43)^2}{6 \times 33} = \frac{1849}{198} = \mathbf{9.34}$$

Step 10. Computation of the trials-by-groups effect for the quadratic component: First, square each sum obtained in Step 4, divide each by the value of Step 7 times the number of subjects in each group (11 in this example), and add the quotients.

$$\frac{(-3)^2}{6 \times 11} + \frac{(-5)^2}{6 \times 11} + \frac{(-35)^2}{6 \times 11} = \frac{1259}{66} = \mathbf{19.08}$$

Then, from this sum subtract the value obtained in Step 9.

$$19.08 - 9.34 = \mathbf{9.74}$$

Step 11. Computation of the error term for the quadratic component: Simply subtract the values of Step 9 and 10 from the value of Step 8.

$$49.50 - 9.34 - 9.74 = \mathbf{30.42}$$

Step 12. Computation of the *df* for each quadratic component:

df for within subjects = total number of subjects used in the experiment.

No. of subjects = **33**

df for trials = **1** in all instances.

df for trials-by-group interaction = the number of experimental groups minus 1.

$$3 - 1 = \mathbf{2}$$

df for the quadratic component error term = *df* for within subjects minus the *df* for trials and the *df* for trials by groups.

$$33 - 1 - 2 = \mathbf{30}$$

Step 13. Computation of the quadratic mean squares: Simply divide each of the sums of squares of Steps 9, 10, and 11 by the appropriate *df* (Step 12). The test of significance in each case is an *F* ratio of the appropriate quadratic component divided by the quadratic error term. Table the data temporarily as follows.

Source	SS	*df*	ms	*F*	*p*
$\text{Within}_{\text{quadratic}}$	49.50	33	(not needed)	—	—
Trials	9.34	1	9.34	9.21	<.001
Trials by groups	9.74	2	4.87	4.80	<.05
$\text{Error}_{\text{quadratic}}$	30.42	30	1.01	—	—

Step 14. Final check for accuracy: Since each of the within-subjects sums of squares of the original analysis was broken into trend components,

the sums of the components must equal (within rounding error) the original values.

Source	Linear		Quadratic		Sum	Original
Within subjects	236.500	+	49.500	=	286.000	286.00
Trials	160.742	+	9.340	=	170.082	170.00
Trials × groups	9.213	+	9.740	=	18.953	19.00
Error$_{\text{within}}$	66.545	+	30.420	=	96.965	97.00

Step 15. Final tabling of the analysis: Although there are several techniques, the following is used most often.

Source	SS	*df*	ms	*F*	*p*
Within subjects	286	66	—	—	—
Trials	170	2	85.000	52.46	$<.001$
Linear	160.742	1	160.742	72.47	$<.001$
Quadratic	9.340	1	9.340	9.21	$<.001$
Trials × groups	19	4	4.750	2.93	$<.05$
Linear	9.213	2	4.606	2.08	n.s.
Quadratic	9.740	2	4.870	4.80	$<.05$
Error	97	60	1.620	—	—
Linear	66.545	30	2.218	—	—
Quadratic	30.420	30	1.014	—	—

A significant linear component for the trials effect means that there is a tendency for the data to lie along a straight line. The significance of the quadratic component means that there is also a tendency for the data to follow a quadratic (arcing) function.

It is more difficult to interpret the quadratic component of the trials-by-groups interaction, since at least two sets of data points must be represented.

EXAMPLE 3

This example is based on the three-factor mixed design presented in Section 2.8. First, recall the final analysis in that section:

Source	SS	*df*	ms	*F*	*p*
Total	1657	111	—	—	—
Between subjects	576	27	—	—	—
High/low esteem	4	1	4	<1	—
Failure/nonfailure	290	1	290	34.27	$<.001$
Esteem × failure	79	1	79	9.34	$<.001$
Error$_b$	203	24	8.46	—	—
Within subjects	1081	84	—	—	—
Trials	784	3	261.33	235.43	$<.001$
Trials × esteem	1	3	.33	<1	—
Trials × failure	167	3	55.67	50.15	$<.001$
Trials × esteem × failure	49	3	16.33	14.71	$<.001$
Error$_w$	80	72	1.11	—	—

Question: What level of polynomial functions best describes the form of the data points of the several within-subjects effects in this analysis?

Step 1. Obtain from Appendix K the appropriate coefficients for the re-analysis. In the present example, four measures were recorded for each subject (four trials). Therefore, the coefficients for $k = 4$ are:

Linear:	-3	-1	$+1$	$+3$
Quadratic:	$+1$	-1	-1	$+1$
Cubic:	-1	$+3$	-3	$+1$

Step 2. The linear component will be analyzed first. Multiply each score used in the original analysis by the appropriate linear coefficient, and retable the data as follows.

GROUP 1

(HIGH SELF-ESTEEM/FAILURE INSTRUCTIONS)

Subject	Trial 1 Score × −3	Trial 2 Score × −1	Trial 3 Score × +1	Trial 4 Score × +3
S_1	−12	−6	7	27
S_2	−18	−7	9	33
S_3	−15	−8	10	39
S_4	−9	−4	6	27
S_5	−12	−6	9	30
S_6	−15	−5	6	24
S_7	−21	−8	9	36

GROUP 2

(HIGH SELF-ESTEEM/NONFAILURE INSTRUCTIONS)

S_8	−15	−8	9	36
S_9	−9	−5	10	42
S_{10}	−21	−10	12	39
S_{11}	−15	−8	11	45
S_{12}	−12	−9	13	45
S_{13}	−9	−5	9	36
S_{14}	−18	−7	8	33

GROUP 3

(LOW SELF-ESTEEM/FAILURE INSTRUCTIONS)

S_{15}	−9	−4	4	15
S_{16}	−15	−6	8	27
S_{17}	−15	−6	6	24
S_{18}	−15	−6	7	24
S_{19}	−9	−4	4	15
S_{20}	−21	−7	8	21
S_{21}	−18	−8	8	24

GROUP 4

(LOW SELF-ESTEEM/NONFAILURE INSTRUCTIONS)

S_{22}	−12	−8	10	42
S_{23}	−15	−10	14	54
S_{24}	−9	−7	12	54
S_{25}	−15	−8	11	51
S_{26}	−18	−11	15	57
S_{27}	−18	−8	10	45
S_{28}	−15	−11	17	60

Step 3. Obtain and record the algebraic sum of the "weighted scores" for each subject in all groups. (*Note:* All linear computations will be based on these sums.)

Group 1

Subject	Trial 1		Trial 2		Trial 3		Trial 4	Sum
S_1	(−12)	+	(−6)	+	7	+	27	+16
S_2	(−18)	+	(−7)	+	9	+	33	+17
⋮	⋮		⋮		⋮		⋮	⋮
S_7	(−21)	+	(−8)	+	9	+	36	+16

Group 2

S_8	(−15)	+	(−8)	+	9	+	36	+22
S_9	(−9)	+	(−5)	+	10	+	42	+38
⋮	⋮		⋮		⋮		⋮	⋮
S_{14}	(−18)	+	(−7)	+	8	+	33	+16

Group 3

S_{15}	(−9)	+	(−4)	+	4	+	15	+6
S_{16}	(−15)	+	(−6)	+	8	+	27	+14
⋮	⋮		⋮		⋮		⋮	⋮
S_{21}	(−18)	+	(−8)	+	8	+	24	+6

Group 4

S_{22}	(−12)	+	(−8)	+	10	+	42	+32
S_{23}	(−15)	+	(−10)	+	14	+	54	+43
⋮	⋮		⋮		⋮		⋮	⋮
S_{28}	(−15)	+	(−11)	+	17	+	60	+51

Step 4. Add the subject sums obtained in Step 3 to get the algebraic

sum for each group. Record these sums and whether they are positive or negative; they will be used later.

$$\begin{aligned} 16 + 17 + \cdots + 16 &= +\mathbf{126} = \text{sum of Group 1} \\ 22 + 38 + \cdots + 16 &= +\mathbf{197} = \text{sum of Group 2} \\ 6 + 14 + \cdots + 6 &= +\ \mathbf{52} = \text{sum of Group 3} \\ 32 + 43 + \cdots + 51 &= +\mathbf{287} = \text{sum of Group 4} \end{aligned}$$

Step 5. Square each of the subject sums obtained in Step 3. Add these squared values, and record the sum for later use.

$$16^2 + 17^2 + \cdots + 51^2 = \mathbf{21{,}092}$$

Step 6. Add the group sums obtained in Step 4 to get the overall algebraic sum. Record this sum for later use.

$$126 + 197 + 52 + 287 = +\mathbf{662}$$

Step 7. Square each of the linear coefficients (Step 1), and add the squared values.

$$(-3)^2 + (-1)^2 + 1^2 + 3^2 = \mathbf{20}$$

Step 8. Computation of the within-subjects linear component: Simply divide the overall sum of the squared values obtained in Step 5 by the value of Step 7.

$$\frac{21{,}092}{20} = \mathbf{1054.60}$$

Step 9. Computation of the repeated-measures (trials) effect for the linear component: Square the algebraic sum obtained in Step 6, and divide by the value of Step 7 times the total number of subjects used in the experiment (28 in the present example).

$$\frac{662^2}{20 \times 28} = \frac{438{,}244}{560} = \mathbf{782.58}$$

Step 10. Computation of the trials-by-first-factor (self-esteem) linear component: First, obtain the algebraic sums for all subjects who were the same in terms of the first factor (self-esteem), disregarding the second factor (instructions). (See Step 4 for group sums to be added.)

$$\begin{aligned} 126 + 197 &= \mathbf{323} = \text{sum of all high-self-esteem subjects} \\ 52 + 287 &= \mathbf{339} = \text{sum of all low-self-esteem subjects} \end{aligned}$$

Then, square these sums, divide each by the value of Step 7 times the number of scores on which each sum was based (14 in this example), and add the quotients.

$$\frac{323^2}{20 \times 14} + \frac{339^2}{20 \times 14} = \frac{219{,}250}{280} = \mathbf{783.03}$$

Then, from this sum, subtract the value obtained in Step 9.

$$783.03 - 782.58 = \mathbf{0.45}$$

Step 11. Computation of the trials-by-second-factor (instructions) linear component: Repeat the computations of Step 10, except using the algebraic sums of subjects who were the same in terms of the second factor, disregarding the first factor (self-esteem). (See Step 4 for the group sums to be added.) First,

$126 + 52 = \mathbf{178} =$ sum of all failure subjects
$197 + 287 = \mathbf{484} =$ sum of all nonfailure subjects

Then,

$$\frac{178^2}{20 \times 14} + \frac{484^2}{20 \times 14} = \frac{265{,}940}{280} = \mathbf{949.78}$$

Then,

$$949.78 - 782.58 = \mathbf{167.20}$$

Step 12. Computation of the repeated-measures-by-first-by-second-factor linear component (trials × self-esteem × instructions, in the present example): Square the algebraic sum of each group (Step 4), divide each square by the value of Step 7 times the number of subjects in each group, and add the quotients.

$$\frac{126^2}{20 \times 7} + \frac{197^2}{20 \times 7} + \frac{52^2}{20 \times 7} + \frac{287^2}{20 \times 7} = \frac{139{,}758}{140} = \mathbf{998.27}$$

Then, from this sum, subtract the values obtained in Steps 9, 10, and 11.

$$998.27 - 782.58 - .45 - 167.20 = \mathbf{48.04}$$

Step 13. Computation of the linear-component error term: Simply subtract the value obtained in the first computation of Step 12 (998.27, in the present example) from the value of Step 8 (or subtract the values obtained in Steps 9, 10, 11, and 12 from Step 8).

$$1054.60 - 998.27 = \mathbf{56.33}$$

Step 14. Computation of the *df* for the linear components:

df for $\text{within}_{\text{linear}}$ = the total number of subjects used in the experiment.

No. of subjects = **28**

df for repeated measures (trials) = **1** (always).
df for trials by first factor (self-esteem) = the number of experimental conditions within the first factor minus 1.

$$2 - 1 = \mathbf{1}$$

df for trials by second factor (instructions) = the number of experimental conditions within the second factor minus 1.

$$2 - 1 = \mathbf{1}$$

df for trials by first by second factor = the *df* for the first factor times the *df* for the second factor.

$$1 \times 1 = \mathbf{1}$$

df for error = *df* for within subjects minus *df* for trials, trials by first factor, trials by second factor, and trials by first by second factor.

$$28 - 1 - 1 - 1 - 1 = \mathbf{24}$$

Step 15. Computation of the mean squares for the linear components: Simply divide each sum-of-squares value as computed in Steps 8, 9, 10, 11, 12, and 13 by its appropriate degrees of freedom. The test of significance in each instance is an *F* ratio of the appropriate linear component divided by the linear-component error term. The mean squares and *F* ratios should be tabled as follows for future use.

Source	SS	*df*	ms	*F*
$\text{Within}_{\text{linear}}$	1054.60	28	(not needed)	—
Trials	782.58	1	782.58	333.01
T × SE	.45	1	.45	<1
T × Instr.	167.20	1	167.20	71.15
T × SE × Instr.	48.04	1	48.04	20.44
$\text{Error}_{\text{linear}}$	56.33	24	2.35	—

The computations for the linear components have been completed. The computations for the quadratic components must now be undertaken.

Step 1. Obtain the coefficients for the quadratic component from Step 1 of the linear analysis (or Appendix K). In the present example, where $k = 4$:

Quadratic: +1 −1 −1 +1

Step 2. The data must again be retabled. The same basic procedure of Step 2 of the linear analysis is followed, except that each score is multiplied by its appropriate quadratic coefficient.

GROUP 1

(HIGH SELF-ESTEEM/FAILURE INSTRUCTIONS)

Subject	Trial 1 Score × +1	Trial 2 Score × −1	Trial 3 Score × −1	Trial 4 Score × +1
S_1	4	−6	−7	9
S_2	6	−7	−9	11
S_3	5	−8	−10	13
S_4	3	−4	−6	9
S_5	4	−6	−9	10
S_6	5	−5	−6	8
S_7	7	−8	−9	12

GROUP 2

(HIGH SELF-ESTEEM/NONFAILURE INSTRUCTIONS)

S_8	5	−8	−9	12
S_9	3	−5	−10	14
S_{10}	7	−10	−12	13
S_{11}	5	−8	−11	15
S_{12}	4	−9	−13	15
S_{13}	3	−5	−9	12
S_{14}	6	−7	−8	11

GROUP 3

(LOW SELF-ESTEEM/FAILURE INSTRUCTIONS)

S_{15}	3	−4	−4	5
S_{16}	5	−6	−8	9
S_{17}	5	−6	−6	8
S_{18}	5	−6	−7	8
S_{19}	3	−4	−4	5
S_{20}	7	−7	−8	7
S_{21}	6	−8	−8	8

GROUP 4

(LOW SELF-ESTEEM/NONFAILURE INSTRUCTIONS)

S_{22}	4	−8	−10	14
S_{23}	5	−10	−14	18
S_{24}	3	−7	−12	18
S_{25}	5	−8	−11	17
S_{26}	6	−11	−15	19
S_{27}	6	−8	−10	15
S_{28}	5	−11	−17	20

Step 3. Using the data of Step 2, obtain and record the algebraic sum of the "weighted scores" of each subject in all groups. (*Note:* All quadratic computations will be based on these scores.)

Group 1

Subject	Trial 1		Trial 2		Trial 3		Trial 4	Sum
S_1	4	+	(−6)	+	(−7)	+	9	**0**
S_2	6	+	(−7)	+	(−9)	+	11	**+1**
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
S_7	7	+	(−8)	+	(−9)	+	12	**+2**

Group 2

S_8	5	+	(−8)	+	(−9)	+	12	**0**
S_9	3	+	(−5)	+	(−10)	+	14	**+2**
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
S_{14}	6	+	(−7)	+	(−8)	+	11	**+2**

Group 3

S_{15}	3	+	(−4)	+	(−4)	+	5	**0**
S_{16}	5	+	(−6)	+	(−8)	+	9	**0**
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
S_{21}	6	+	(−8)	+	(−8)	+	8	**−2**

Group 4

S_{22}	4	+	(−8)	+	(−10)	+	14	**0**
S_{23}	5	+	(−10)	+	(−14)	+	18	**−1**
$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
S_{28}	5	+	(−11)	+	(−17)	+	(20)	**−3**

Step 4. Add the subject sums of Step 3 to get the algebraic sums for each group. Record these sums and whether they are positive or negative.

$$\begin{aligned} 0 + 1 + \cdots + 2 &= \mathbf{+6} = \text{sum of Group 1} \\ 0 + 2 + \cdots + 2 &= \mathbf{+1} = \text{sum of Group 2} \\ 0 + 0 + \cdots + (-2) &= \mathbf{-2} = \text{sum of Group 3} \\ 0 + (-1) + \cdots + (-3) &= \mathbf{+3} = \text{sum of Group 4} \end{aligned}$$

Step 5. Square each of the subject sums of Step 3. Add these squared values, and record the sum for later use.

$$0^2 + 1^2 + \cdots + 0^2 + 2^2 + \cdots + 0^2 + 0^2 + \cdots + 0^2 + (-1)^2 + \cdots + (-3)^2 = \mathbf{76}$$

Step 6. Add the subject sums of Step 4 to get the overall algebraic sum.

$$6 + 1 + (-2) + 3 = \mathbf{+8}$$

Step 7. Square and sum the quadratic coefficients (Step 1).

$$1^2 + (-1)^2 + (-1)^2 + 1^2 = \mathbf{4}$$

Step 8. Computation of the within-subjects quadratic component: Simply divide the value of Step 5 by that of Step 7.

$$\frac{76}{4} = \mathbf{19.00}$$

Step 9. Computation of the trials effect for the quadratic component: Square the value of Step 6, and then divide by the value of Step 7 times the total number of subjects used in the experiment (28 in this example).

$$\frac{8^2}{4 \times 28} = \frac{64}{112} = \mathbf{.57}$$

Step 10. Computation of the trials-by-first-factor (self-esteem) quadratic component: First, obtain the algebraic sum of the subjects who were the same in terms of the first factor (self-esteem in the present example), disregarding the second factor. (See Step 4 for the group sums to be added.)

$$6 + 1 = \mathbf{+7} = \text{sum for all high-self-esteem subjects}$$
$$(-2) + 3 = \mathbf{+1} = \text{sum for all low-self-esteem subjects}$$

Then, square these sums, divide each by the value of Step 7 times the number of scores on which each sum was based (14 in this example), and add the quotients.

$$\frac{7^2}{4 \times 14} + \frac{1^2}{4 \times 14} = \frac{50}{56} = \mathbf{.89}$$

Then, from this value, subtract the value of Step 9.

$$.89 - .57 = \mathbf{.32}$$

Step 11. Computation of the trials-by-second-factor (instructions) quadratic component: Repeat the computations of Step 10, except using the algebraic sums of subjects who were the same in terms of the second factor, disregarding the first factor. (See Step 4 for the group sums to be added.) First,

$$6 + (-2) = \mathbf{+4} = \text{sum of all failure subjects}$$
$$1 + 3 = \mathbf{+4} = \text{sum of all nonfailure subjects}$$

Then,

$$\frac{4^2}{4 \times 14} + \frac{4^2}{4 \times 14} = \frac{32}{56} = \mathbf{.57}$$

Then,

$$.57 - .57 = \mathbf{.00}$$

Step 12. Computation of the trials-by-first-by-second-factor quadratic component: Square the sums of Step 4, divide each square by the value of Step 7 times the number of subjects in each group, and add the quotients.

$$\frac{6^2}{4 \times 7} + \frac{1^2}{4 \times 7} + \frac{(-2)^2}{4 \times 7} + \frac{3^2}{4 \times 7} = \frac{50}{28} = \mathbf{1.79}$$

Then, from this sum, subtract the values obtained in Steps 9, 10, and 11.

$$1.79 - .57 - .32 - .00 = \mathbf{.90}$$

Step 13. Computation of the quadratic-component error term: Simply subtract the value obtained in the first computation of Step 12 from the value of Step 8.

$$19.00 - 1.79 = \mathbf{17.21}$$

Step 14. The *df* computations for the quadratic component are the same as for the linear component.

df for within$_{\text{quadratic}}$ = the total number of subjects used in the experiment.

No. of subjects = **28**

df for trials = **1** (always).

df for trials by first factor = number of first-factor conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for the trials by second factor = number of second-factor conditions minus 1.

$$2 - 1 = \mathbf{1}$$

df for trials by first by second factor = the *df* for the first factor times the *df* for the second factor.

$$1 \times 1 = \mathbf{1}$$

df for error$_{\text{quadratic}}$ = *df* for within subjects minus the *df* for trials, trials by first factor, trials by second factor, and trials by first by second factor.

$$28 - 1 - 1 - 1 - 1 = \mathbf{24}$$

Step 15. Computation of the quadratic mean squares: Again, divide the values of Steps 8, 9, 10, 11, 12, and 13 by the appropriate degrees of freedom. Compute each F ratio by dividing the appropriate quadratic component by the quadratic error term. Table the data temporarily as follows.

Source	SS	*df*	ms	*F*
$\text{Within}_{\text{quadratic}}$	19.00	28	(not needed)	—
Trials	.57	1	.57	<1
T × SE	.32	1	.32	<1
T × Instr.	.00	1	.00	<1
T × SE × Instr.	.90	1	.90	1.25
$\text{Error}_{\text{quadratic}}$	17.21	24	.72	—

When the computations of the quadratic components have been completed, the computation of the cubic components must be undertaken.

Step 1. Obtain the coefficients for the cubic component from Step 1 of the linear analysis (or Appendix K). In the present example, where $k = 4$:

$$\text{Cubic:} \quad -1 \quad +3 \quad -3 \quad +1$$

Step 2. Once again, the data must be retabled. This time each score is multiplied by its appropriate cubic coefficient.

GROUP 1

	Trial 1	Trial 2	Trial 3	Trial 4
Subject	Score × −1	Score × +3	Score × −3	Score × +1
S_1	−4	18	−21	9
S_2	−6	21	−27	11
S_3	−5	24	−30	13
S_4	−3	12	−18	9
S_5	−4	18	−27	10
S_6	−5	15	−18	8
S_7	−7	24	−27	12

GROUP 2

S_8	−5	24	−27	12
S_9	−3	15	−30	14
S_{10}	−7	30	−36	13
S_{11}	−5	24	−33	15
S_{12}	−4	27	−39	15
S_{13}	−3	15	−27	12
S_{14}	−6	21	−24	11

GROUP 3

S_{15}	−3	12	−12	5
S_{16}	−5	18	−24	9
S_{17}	−5	18	−18	8
S_{18}	−5	18	−21	8
S_{19}	−3	12	−12	5
S_{20}	−7	21	−24	7
S_{21}	−6	24	−24	8

GROUP 4

S_{22}	−4	24	−30	14
S_{23}	−5	30	−42	18
S_{24}	−3	21	−36	18
S_{25}	−5	24	−33	17
S_{26}	−6	33	−45	19
S_{27}	−6	24	−30	15
S_{28}	−5	33	−51	20

Step 3. Using the data of Step 2, obtain the algebraic sum of each subject's "weighted scores." These sums will be used for all cubic computations.

Group 1

Subject	Trial 1		Trial 2		Trial 3		Trial 4	Sum
S_1	(−4)	+	18	+	(−21)	+	9	+2
S_2	(−6)	+	21	+	(−27)	+	11	−1
⋮	⋮		⋮		⋮		⋮	⋮
S_7	(−7)	+	24	+	(−27)	+	12	+2

Group 2

S_8	(−5)	+	24	+	(−27)	+	12	+4
S_9	(−3)	+	15	+	(−30)	+	14	−4
⋮	⋮		⋮		⋮		⋮	⋮
S_{14}	(−6)	+	21	+	(−24)	+	11	+2

Group 3

S_{15}	(−3)	+	12	+	(−12)	+	5	+2
S_{16}	(−5)	+	18	+	(−24)	+	9	−2
⋮	⋮		⋮		⋮		⋮	⋮
S_{21}	(−6)	+	24	+	(−24)	+	8	+2

Group 4

S_{22}	(−4)	+	24	+	(−30)	+	14	+4
S_{23}	(−5)	+	30	+	(−42)	+	18	+1
⋮	⋮		⋮		⋮		⋮	⋮
S_{28}	(−5)	+	33	+	(−51)	+	20	−3

Step 4. Add the sums of Step 3 to get the algebraic sum for each group.

$$2 + (-1) + \cdots + 2 = \mathbf{+2} = \text{sum of Group 1}$$
$$4 + (-4) + \cdots + 2 = \mathbf{-1} = \text{sum of Group 2}$$
$$2 + (-2) + \cdots + 2 = \mathbf{+4} = \text{sum of Group 3}$$
$$4 + 1 + \cdots + (-3) = \mathbf{+9} = \text{sum of Group 4}$$

Step 5. Square the sums of Step 3. Add these squared values, and record them for later use.

$$2^2 + (-1)^2 + \cdots + 4^2 + (-4)^2 + \cdots + 2^2 + (-2)^2 + \cdots + 4^2 + 1^2 + \cdots + (-3)^2 = \mathbf{148}$$

Step 6. Add the group sums of Step 4 to obtain the overall algebraic sum.

$$2 + (-1) + 4 + 9 = \mathbf{+14}$$

Step 7. Square and sum the quadratic coefficients (Step 1).

$$(-1)^2 + 3^2 + (-3)^2 + 1^2 = \mathbf{20}$$

Step 8. Computation of the within-subjects cubic component: Simply divide Step 5 by Step 7.

$$\frac{148}{20} = \mathbf{7.40}$$

Step 9. Computation of the cubic-component trials effect: Square the value of Step 6. Then, divide by the value of Step 7 times the total number of subjects used in the experiment.

$$\frac{14^2}{20 \times 28} = \frac{196}{560} = \mathbf{.35}$$

Step 10. Computation of the trials-by-first-factor (self-esteem) cubic component: First, obtain the algebraic sum of the subjects who were the same in terms of the first factor, disregarding the second factor. (See Step 4 for the group sums to be added.)

$$2 + (-1) = \mathbf{+\ 1} = \text{sum of high-self-esteem subjects}$$

$$4 + 9 = \mathbf{+13} = \text{sum of low-self-esteem subjects}$$

Then, square these sums, divide each by the value of Step 7 times the number of scores on which each sum was based, and add the quotients.

$$\frac{1^2}{20 \times 14} + \frac{13^2}{20 \times 14} = \frac{170}{280} = \mathbf{.61}$$

Then, from this value, subtract the final value of Step 9.

$$.61 - .35 = \mathbf{.26}$$

Step 11. Computation of the trials-by-second-factor (instructions) cubic component: Repeat the computations of Step 10, except that the algebraic sums of the second factor are used, disregarding the first factor. (See Step 4 for the group sums to be added.) First,

$$2 + 4 = \mathbf{6} = \text{sum of failure subjects}$$

$$(-1) + 9 = \mathbf{8} = \text{sum of nonfailure subjects}$$

Then,

$$\frac{6^2}{20 \times 14} + \frac{8^2}{20 \times 14} = \frac{100}{280} = \mathbf{.36}$$

Then,

$$.36 - .35 = \mathbf{.01}$$

Step 12. Computation of the trials-by-first-by-second-factor cubic component: Square the sums of Step 4, divide each by the value of Step 7 times the number of subjects in each group, and add the quotients.

$$\frac{2^2}{20 \times 7} + \frac{(-1)^2}{20 \times 7} + \frac{4^2}{20 \times 7} + \frac{9^2}{20 \times 7} = \frac{102}{140} = \mathbf{.73}$$

Then, from this sum, subtract the values of Steps 9, 10, and 11.

$$.73 - .35 - .26 - .01 = \mathbf{.11}$$

Step 13. Computation of the cubic-component error term: Simply subtract the value obtained in the initial computation of Step 12 from that of Step 8.

$$7.40 - .73 = \mathbf{6.67}$$

Step 14. The *df* computations for the cubic components are the same as for the linear and quadratic components. In the present example, these are:

df for within$_{\text{cubic}}$ = **28**
df for trials = **1**
df for trials by first factor = **1**
df for trials by second factor = **1**
df for trials by first by second factor = **1**
df for error$_{\text{cubic}}$ = **24**

Step 15. Computation of the mean squares and *F* ratios for the cubic component: These are computed in the same manner as for the linear and quadratic components. Again, table the results temporarily as follows.

Source	SS	*df*	ms	*F*
Within$_{\text{cubic}}$	7.40	28	(not needed)	—
Trials	.35	1	.35	1.25
T × SE	.26	1	.26	<1
T × Instr.	.01	1	.01	<1
T × SE × Instr.	.11	1	.11	<1
Error$_{\text{cubic}}$	6.67	24	.28	—

Step 16. Final check for accuracy: Since each of the within-subjects sums of squares of the original analysis was broken into trend components, the sums of these components must equal (within rounding error) the

original values. In the present example:

Source	Linear		Quadratic		Cubic		Sum	Original
Within subjects	1054.60	+	19.00	+	7.40	=	**1081.00**	1081
Trials	782.58	+	.57	+	.35	=	**783.50**	784
T × SE	.45	+	.32	+	.26	=	**1.03**	1
T × Instr.	167.20	+	.00	+	.01	=	**167.21**	167
T × SE × Instr.	48.04	+	.90	+	.11	=	**49.05**	49
$\text{Error}_{\text{within}}$	56.33	+	17.21	+	6.67	=	**80.21**	80

Step 17. Final tabling of the results: Although several procedures can be used, the following is probably used most often.

Source	SS	*df*	ms	*F*	*p*
Within subjects	1081	84	—	—	—
Trials	784	3	261.33	235.43	<.001
Linear	782.58	1	782.58	333.01	<.001
Quadratic	.57	1	.57	<1	—
Cubic	.35	1	.35	1.25	n.s.
Trials × SE	1	3	.33	<1	—
Linear	.45	1	.45	<1	—
Quadratic	.32	1	.32	<1	—
Cubic	.26	1	.26	<1	—
Trials × Instr.	167	3	55.67	50.15	<.001
Linear	167.20	1	167.20	71.15	<.001
Quadratic	.00	1	.00	<1	—
Cubic	.01	1	.01	<1	—
Trials × SE × Instr.	49	3	16.33	14.71	<.001
Linear	48.04	1	48.04	20.44	<.001
Quadratic	.90	1	.90	1.25	n.s.
Cubic	.11	1	.11	<1	—
$\text{Error}_{\text{within}}$	80	72	1.11	—	—
Linear	56.33	24	2.35	—	—
Quadratic	17.21	24	.72	—	—
Cubic	6.67	24	.28	—	—

The significant linear components mean that the data points tend to lie along a straight line.

If there had been significant quadratic components, it would have meant that there was a tendency for the data to follow quadratic functions (such as a parabola). Significant cubic components would have meant that there was a tendency for the data to follow an even more complex path with two inflections in the functions. The interpretation of linear, quadratic, and cubic components of interactions is very complex due to the fact that at least two sets of data points are involved in each. Only inspection of the data or additional analyses of each individual set of data points (using the same procedures just outlined) can give an indication as to which of the sets is contributing to the significant findings.

PART 4

Correlation and Related Topics

The fourteen sections in this part present six rather different kinds of analyses. These six types are listed below, followed by a more detailed description of the different kinds of problems that can be handled with each analysis. This material should enable you to choose quickly the section that most clearly suits your individual needs.

1. Pearson product-moment correlation (the common or usual correlation)
2. Rank-order correlation, point-biserial correlation, and the correlation ratio (*eta*)
3. Partial and multiple correlation
4. Analyses of covariance
5. Tests for reliability of measurement
6. Significance tests for deciding whether two correlation coefficients are different in magnitude

Pearson Product-Moment Correlation (*Section 4.1*)

Use the correlation coefficient described in Section 4.1 if your numbers represent amounts of some measurable quantity—such as height, age, IQ, grade points, test scores, etc. This analysis assumes that the two variables for which you have measures are linearly related. If you have ranked data or are reasonably sure that the variables are related in some complex curved way, use one of the correlation coefficients below.

Rank-Order Correlation, Point-Biserial Correlation, and eta *(Sections 4.2, 4.3, 4.4, and 4.5)*

If you have ranked data or if you wish to convert measures to ranks, use one of the rank-order correlations described in Sections 4.2 and 4.3. These analyses make no assumptions about the shape of the relationship between variables.

If you have measures along one variable, such as grade points, and have another variable that is dichotomized, such as sex, use the point-biserial correlation described in Section 4.4.

If you are fairly certain that the variables are related in some complex curved manner or if you have already computed an F ratio, you can use the correlation ratio (*eta*), described in Section 4.5, to express the relationship between the two variables.

Partial and Multiple Correlation (Sections 4.6, 4.7, and 4.8)

If you have three variables and wish to know how highly two of them are related when the mutual relationships with the third variable are taken out, use the partial correlation described in either Section 4.6 or Section 4.7.

If you have three variables and wish to know how highly two of them, taken together, are related to the third, use the multiple correlation described in Section 4.8.

Covariance (Sections 4.9 and 4.10)

When you wish to make the kinds of tests of significance described in Sections 2.1 and 2.2 (the completely randomized design and the two-factor factorial design) but cannot select random groups that are essentially equal, you must make statistical corrections for such differences. For example, if one group is composed of grade-school children and another of high-school students and you wish to measure the amount of verbal material learned in a certain interval of time, you would reasonably expect the groups to differ initially by a large amount because of their different levels of education. The analyses of covariance, described in Sections 4.9 and 4.10, provide a technique whereby you can equate the two groups before training begins.

Reliability of Measurement (Sections 4.11 and 4.12)

Whenever measures are gathered by some sort of testing, the validity of the measures depends upon the quality of the test being used. The tests for the reliability of measurement involve particular interpretations of a correlation coefficient. Sections 4.11 and 4.12 consider the topic of reliability of measurement.

Significance Tests of Differences Between Correlations (Sections 4.13 and 4.14)

Each correlation coefficient is regarded as being either (1) indicative of a real relationship or (2) due to chance variation. Each of Sections 4.1 through 4.8 presents a test to help you decide which of the two cases is most likely for a particular correlation value. However, it is often desirable to know whether *two* correlation coefficients differ by more than would be expected by chance. Sections 4.13 and 4.14 present tests to help evaluate differences between correlation values. If the two correlation coefficients are computed using completely *different* groups of people (that is, if the correlations are independent), use the test presented in Section 4.13. If the correlation values have been computed using the *same* people (that is, if the correlations are dependent), use the test presented in Section 4.14.

In this part, computational formulas will again be presented in those instances where a complete analysis can be shown by a single formula or where a standard set of terms and symbols are used in nearly all textbooks.

SECTION 4.1

Pearson Product-Moment Correlation

The Pearson product-moment correlation (r) is used to determine if there is a relationship between two sets of paired numbers. Generally, the paired numbers are either (1) two different measures on each of several objects or persons or (2) one measure on each of several pairs of objects or people where the pairing is based on some natural relationship, such as father to son, or on some initial matching according to one specific variable, such as IQ score.

EXAMPLE

Assume that an experimenter wishes to determine whether there is a relationship between the grade point averages (GPAs) and the scores on a reading-comprehension test of fifteen college freshmen.

The basic computational formula for the Pearson product-moment correlation is

$$r = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{[N\sum X^2 - (\sum X)^2][N\sum Y^2 - (\sum Y)^2]}}$$

where N = number of pairs of scores
$\sum XY$ = sum of the products of the paired scores
$\sum X$ = sum of scores on one variable
$\sum Y$ = sum of scores on the other variable
$\sum X^2$ = sum of the squared scores on the X variable
$\sum Y^2$ = sum of the squared scores on the Y variable

Step 1. The scores must be paired in some meaningful way in order to use the Pearson r. In the present example, the two different scores—reading comprehension and grade point average—are paired and recorded for each of the fifteen students.

Student	Reading Score (X)	Freshman GPA (Y)
S_1	38	2.1
S_2	54	2.9
S_3	43	3.0
S_4	45	2.3
S_5	50	2.6
S_6	61	3.7
S_7	57	3.2
S_8	25	1.3
S_9	36	1.8
S_{10}	39	2.5
S_{11}	48	3.4
S_{12}	46	2.6
S_{13}	44	2.4
S_{14}	39	2.5
S_{15}	48	3.3

Step 2. Multiply the two numbers in each pair; then add the products.

$$(38 \times 2.1) + (54 \times 2.9) + \cdots + (48 \times 3.3) = \mathbf{1846.1}$$

Step 3. Multiply the number obtained in Step 2 by N, the number of *paired* scores (15 in this example).

$$1846.1 \times 15 = \mathbf{27{,}691.5}$$

Step 4. Square each number in the first column, and add the squared values. (*Note:* As these squares are accumulated—that is, added—by the calculating machine, the value of Step 6 will be accumulating in the upper dial.)

$$38^2 + 54^2 + \cdots + 48^2 = \mathbf{31{,}327}$$

Step 5. Multiply the sum of Step 4 by the number of paired scores ($N = 15$ in this example).

$$31{,}327 \times 15 = \mathbf{469{,}905}$$

Step 6. Add all the scores in the first column (in this example, the reading-comprehension scores).

$$38 + 54 + \cdots + 48 = \mathbf{673}$$

Step 7. Square the value obtained in Step 6.

$$673^2 = \mathbf{452{,}929}$$

Step 8. Square each number in the second column, and add the squared values. (*Note:* The value of Step 10 can be obtained at the same time, as explained in Step 4.)

$$2.1^2 + 2.9^2 + \cdots + 3.3^2 = \mathbf{110.2}$$

Step 9. Multiply the result of Step 8 by the number of paired scores ($N = 15$ in this example).

$$110.2 \times 15 = \mathbf{1653}$$

Step 10. Add all the scores in the second column (in this example, the freshman GPAs).

$$2.1 + 2.9 + \cdots + 3.3 = \mathbf{39.6}$$

Step 11. Square the value obtained in Step 10.

$$39.6^2 = \mathbf{1568.16}$$

Step 12. Multiply the final value of Step 6 by that of Step 10.

$$673 \times 39.6 = \mathbf{26{,}650.8}$$

Step 13. The numerator of r is now computed by subtracting the value obtained in Step 12 from that obtained in Step 3. Be careful to note whether the answer is positive or negative.

$$27{,}691.5 - 26{,}650.8 = \mathbf{+1040.7}$$

Step 14. Subtract the value of Step 7 from that of Step 5.

$$469{,}905 - 452{,}929 = \mathbf{16{,}976}$$

Note: If the variance of these first-column scores is desired, divide the result of Step 14 by N^2, i.e., $16{,}976/15^2 = 16{,}976/225 = 75.45$. Or if an estimate of the variance of the population from which these scores were drawn is desired, divide by $N(N - 1)$, i.e., $16{,}976/[15(15 - 1)] = 16{,}976/210 = 80.84$.

Step 15. Subtract the final value of Step 11 from that of Step 9.

$$1653 - 1568.16 = \mathbf{84.84}$$

Note: The variance of these second-column scores can be obtained by dividing this result by N^2, i.e., $84.84/15^2 = 84.84/225 = .38$. For the estimate of the population variance, divide by $N(N - 1)$, i.e., $84.84/210 = .40$.

Step 16. Multiply the result of Step 14 by the result of Step 15.

$$16{,}976 \times 84.84 = \mathbf{1{,}440{,}243.84}$$

Step 17. Take the square root of the result of Step 16.

$$\sqrt{1{,}440{,}243.84} = \mathbf{1200.1}$$

Step 18. Divide the value of Step 13 by that of Step 17. This yields the value of the Pearson product-moment correlation.

$$r = \frac{+1040.7}{1200.1} = \mathbf{+.87}$$

SUPPLEMENT

Testing the significance of r: Two different procedures are used to test the hypothesis that $r = 0$. If N (the number of pairs) is 30 or larger, a critical-ratio z-test can easily be done. If N is smaller than 30, a slightly more complicated t-test should be done.

Step 19. If N is 30 or larger, compute $z = r\sqrt{N - 1}$. For example, suppose $r = -.56$ and $N = 37$. Then,

$$z = (-.56)\sqrt{37 - 1} = (-.56)\sqrt{36} = -.56 \times 6 = \mathbf{-3.36}$$

If z is greater than ± 1.96, then r is significant at the .05 level using a two-tailed test (see Appendix A).

Step 20. If N is smaller than 30, compute $t = r\sqrt{(N - 2)/(1 - r^2)}$. Using the value of r from Step 18 and the value of N used to compute that r, we have $r = +.87$ and $N = 15$. Thus,

$$t = (+.87)\sqrt{\frac{15 - 2}{1 - .87^2}} = .87\sqrt{\frac{13}{1 - .76}} = .87\sqrt{54.16}$$

$$= .87 \times 7.36 = \mathbf{+6.40}$$

The degrees of freedom for this t are $N - 2$.

$$df = 15 - 2 = \mathbf{13}$$

With 13 degrees of freedom, a t value larger than ± 2.16 (see Appendix B) is significant at the .05 level when a two-tailed test is used.

Note: Several critical values of r are given in Appendix F for various degrees of freedom.

SECTION 4.2

Spearman Rank-Order Correlation (*rho*)

Spearman's *rho* is used when an experimenter wishes to determine whether two sets of rank-ordered data are related.

EXAMPLE

The experimenter first asked an experienced teacher to rank twenty children in her class according to what she believed their intelligence to be. Then the children were tested and their actual Wechsler IQ scores obtained. The data were as follows.

Child	Teacher's Ranking	IQ Score
A	1	116
B	2	111
C	3	97
D	4	122
E	5	116
F	6	105
G	7	108
H	8	95
I	9	124
J	10	98
K	11	116
L	12	109
M	13	103
N	14	103
O	15	96
P	16	90
Q	17	134
R	18	87
S	19	96
T	20	91

The following formula for *rho* (ρ) describes the computational procedures.

$$rho = 1 - \frac{6\sum D^2}{N(N^2 - 1)}$$

where D = difference score between each X and Y pair
N = number of pairs of scores

Step 1. Rank the IQ scores so that both variables are ranked.

Note: When two children have the same IQ score, the same rank is given to each. But notice that the rank given is the mean value of the two ranks for the two tied scores. For example, Child M and Child N both have IQ scores of 103. These scores should fall into ranks 11 and 12. But since both children have the same IQ score, both are ranked as 11.5 and the next score (Child J, with an IQ score of 98) is ranked as 13.

Child	Teacher's Ranking	IQ Rank on the Basis of Test Score
A	1	5
B	2	7
C	3	14
D	4	3
E	5	5
F	6	10
G	7	9
H	8	17
I	9	2
J	10	13
K	11	5
L	12	8
M	13	11.5
N	14	11.5
O	15	15.5
P	16	19
Q	17	1
R	18	20
S	19	15.5
T	20	18

Step 2. Compute the difference between the two ranks for each child. The resulting value is called the *D value.* List these values in a column, making sure to note whether they are positive or negative.

Child	*D* Value	
A	−4	(1–5)
B	−5	(2–7)
C	−11	(3–14)
D	+1	(4–3)
E	0	(5–5)
F	−4	(6–10)
G	−2	(7–9)
H	−9	(8–17)
I	+7	(9–2)
J	−3	(10–13)
K	+6	(11–5)
L	+4	(12–8)

Child	*D* Value	
M	+1.5	(13–11.5)
N	+2.5	(14–11.5)
O	−0.5	(15–15.5)
P	−3	(16–19)
Q	+16	(17–1)
R	−2	(18–20)
S	+3.5	(19–15.5)
T	+2	(20–18)

Step 3. Square all the *D* values from Step 2, and add all the squared values.

$$(-4^2) + (-5^2) + \cdots + 2^2 = \mathbf{668}$$

Step 4. Multiply the result of Step 3 by the number 6. (*Note:* The number 6 is *always* used, regardless of the number of ranks, etc., involved.)

$$668 \times 6 = \mathbf{4008}$$

Step 5. Compute $N(N^2 - 1)$. (In our example $N = 20$.)

$$20(20^2 - 1) = 20(400 - 1) = 20(399) = \mathbf{7980}$$

Step 6. Divide the result of Step 4 by the result of Step 5.

$$\frac{4008}{7980} = \mathbf{.50}$$

Step 7. Subtract the final value of Step 6 from the number 1. (*Note:* The number 1 is also always used.) This yields the value of Spearman's *rho*. Be careful to record whether it is positive or negative.

$$rho = 1 - .50 = \mathbf{+.50}$$

SUPPLEMENT

Testing the significance of *rho*: Two different procedures are needed to test the hypothesis that $rho = 0$. If N (the number of ranks) is 30 or larger, a critical-ratio *z*-test can easily be done. If N is between 10 and 30, a slightly more complicated *t*-test should be done. No test is presented here for the significance of *rho* where N is less than 10 since it is rare that so few ranks would be considered.

Step 8. When N is 30 or greater, compute $z = rho\sqrt{N - 1}$. For example, suppose $rho = -.21$ and $N = 50$. Then,

$$z = (-.21)\sqrt{50 - 1} = (-.21)\sqrt{49} = -.21 \times 7 = \mathbf{-1.47}$$

If z is greater than ± 1.96, then *rho* is significant at the .05 level using a two-tailed test (see Appendix A).

Step 9. When N is between 10 and 30, compute

$$t = rho\sqrt{\frac{N-2}{1-rho^2}}$$

Using the value of *rho* from Step 7 above and the value of N used to compute that *rho*, we have $rho = +.50$ and $N = 20$. Thus,

$$t = (+.50)\sqrt{\frac{20-2}{1-rho^2}} = (+.50)\sqrt{\frac{18}{1-.25}} = (+.50)\sqrt{\frac{18}{.75}}$$

$$= (+.50)\sqrt{24} = +.50 \times 4.9 = \mathbf{+2.45}$$

The degrees of freedom for this t are $N - 2$. Thus,

$$df = 20 - 2 = \mathbf{18}$$

With 18 degrees of freedom, a t value larger than ± 2.10 (see Appendix B) is significant at the .05 level when a two-tailed test is used.

SECTION 4.3

Kendall Rank-Order Correlation (*tau*)

Kendall's *tau* is sometimes used in place of Spearman's *rho*, which was presented in Section 4.2. If you have a pair of ranks for each of several individuals, the *tau* statistic can be computed to express the degree of relationship between the ranks. Its significance can be tested as follows.

EXAMPLE

Assume that an experimenter wishes to determine the degree of relationship between the ratings of two judges on ten contestants in a beauty contest. The measures recorded represent the ranks given to the contestants by each of the judges.

The usual formula for *tau* when there are no tied ranks is

$$tau = \frac{P-Q}{\frac{1}{2}N(N-1)}$$

where P = total number of higher ranks
Q = total number of lower ranks
N = number of pairs of ranks

(*Note:* See the following for the explanation of "higher" and "lower.")

A more general computation, covering a case where some ranks are tied, is presented here. Steps 7 through 12 show how to correct for the tied ranks.

Step 1. Table the data as follows. In computing *tau*, always arrange the first column of ranks so that they range from the lowest to the highest as you go down the column.

Contestant	Ranking by Judge A	Ranking by Judge B
A	1	3
B	2	5
C	3.5	1.5
D	3.5	4
E	5	1.5
F	6	6
G	8	8
H	8	7
I	8	10
J	10	9

Step 2. The ranks in the first column run from the lowest to the highest. The ranks in the second column are "mixed up." We must determine the extent of the "mix-up" in Column 2. In order to do so, count the number of ranks in Column 2 that are higher than that for Contestant A—i.e., the number of ranks that are higher in numerical value than the number 3. In this example, only the two 1.5's are lower than 3; therefore there are seven numbers higher than 3.

$$\text{No. of higher ranks} = \mathbf{7}$$

Then, count the number of ranks that are lower than 3.

$$\text{No. of lower ranks} = \mathbf{2}$$

Step 3. Next, count the number of ranks in Column 2 that are higher than the rank for Contestant B—i.e., the number of ranks higher than 5. In this example, 6, 8, 7, 10, and 9 are higher than 5. Thus,

$$\text{No. of higher ranks} = \mathbf{5}$$

Then count the number of ranks lower than 5. Do *not* include ranks that have already been considered (Contestant A). With the ranking for A excluded,

$$\text{No. of lower ranks} = \mathbf{3}$$

Step 4. Now count the number of ranks that are higher than the rank for Contestant C—i.e., the number of ranks higher than 1.5. All ranks except the other 1.5 are higher, but ties are not counted and we have already taken care of numbers 3 and 5 for Contestants A and B. Thus,

$$\text{No. of higher ranks} = \mathbf{6}$$

Now count the number of ranks lower than 1.5.

$$\text{No. of lower ranks} = \mathbf{0}$$

Step 5. Continue counting both the number of ranks higher and the number of ranks lower than the one under consideration for all the rest of the ranks in Column 2. Table the results as follows. (*Note:* The values for the bottom contestant are both zero since no ranks appear below them.)

Contestant	No. of Higher Ranks	No. of Lower Ranks
A	7	2
B	5	3
C	6	0
D	5	1
E	5	0
F	4	0
G	2	1
H	2	0
I	0	1
J	0	0

Step 6. Add the numbers in each of the columns in Step 5.

$$7 + 5 + \cdots + 0 = \mathbf{36} = \text{sum of Column 1 (higher ranks)}$$
$$2 + 3 + \cdots + 0 = \mathbf{8} = \text{sum of Column 2 (lower ranks)}$$

Then, subtract the sum of Column 2 from the sum of Column 1. Be careful to record whether the result is positive or negative.

$$36 - 8 = \mathbf{+28}$$

Step 7. Go to the table in Step 1 and count the number of ranks that are tied in the first column (the ranks assigned by Judge A). Categorize each tie as being a set of two, three, four, etc., according to the number of contestants in that tie. Then, compute $x(x - 1)$ for each tie, where x is the number of contestants tied for that place—e.g., $2(2 - 1) = 2(1) = 2$; $5(5 - 1) = 5(4) = 20$; etc. Add these products, and divide by 2. (*Note:* The number 2 is always used.) In this example, Contestants C and D tied for the third and fourth positions (both were assigned to the rank of 3.5) and Contestants G, H, and I tied for the seventh, eighth, and ninth places (all were assigned to the rank of 8). In this example, then, there is one set of two and one set of three. Thus,

$$\frac{2(2-1)+3(3-1)}{2}=\frac{2(1)+3(2)}{2}=\frac{2+6}{2}=\frac{8}{2}=\mathbf{4}$$

Step 8. Repeat all the computations of Step 7 using the ranks in the second column of Step 1 (the ranks assigned by Judge B). Notice that Contestants C and E are tied at a rank of 1.5. Thus, we have one set of two, so

$$\frac{2(2-1)}{2}=\frac{2(1)}{2}=\frac{2}{2}=\mathbf{1}$$

Step 9. Compute $[N(N-1)]/2$, where N refers to the total number of ranks in each column (10 in this example).

$$\frac{N(N-1)}{2}=\frac{10(10-1)}{2}=\frac{10(9)}{2}=\frac{90}{2}=\mathbf{45}$$

Step 10. Subtract the result of Step 7 from the result of Step 9.

$$45-4=\mathbf{41}$$

Step 11. Subtract the result of Step 8 from the result of Step 9.

$$45-1=\mathbf{44}$$

Step 12. Multiply the result of Step 10 by the result of Step 11.

$$41\times 44=\mathbf{1804}$$

Then, take the square root of the product.

$$\sqrt{1804}=\mathbf{42.474}$$

Step 13. Divide the result of Step 6 by the result of Step 12. This yields the value of Kendall's *tau*.

$$tau=+\frac{28}{42.474}=\mathbf{+.66}$$

SUPPLEMENT

Testing the significance of *tau* (i.e., the hypothesis that *tau* = 0): When there are 10 or more ranks and when the number of tied ranks is not too great, the following significance test will yield approximate values.

Step 14. Compute $[N(N-1)(2N+5)]/18$ where N is the total number of ranks in each column of Step 1 (10 in this example).

$$\frac{10(10-1)[2(10)+5]}{18}=\frac{10(9)(20+5)}{18}=\frac{90(25)}{18}=\frac{2250}{18}=\mathbf{125}$$

Step 15. Take the square root of the result of Step 14.

$$\sqrt{125} = \mathbf{11.18}$$

Step 16. Divide the result of Step 6 by the answer of Step 15. This yields a z statistic.

$$z = +\frac{28}{11.18} = \mathbf{+2.50}$$

If the result of Step 16 is larger than ±1.96 (see Appendix A), then *tau* is significant at the .05 level using a two-tailed test.

SECTION 4.4

Point-Biserial Correlation

The point-biserial correlation is used when a coefficient of relationship is desired between one measure that is continuous and another that is dichotomous.

EXAMPLE

Assume that an experimenter wishes to determine the relationship between scores on a certain college entrance examination and whether the student obtained a grade point average of C or better during the freshman year. Twelve students are picked at random and their entrance-examination scores and their course grades are recorded.

The continuous variable in this example is the array of scores on the college entrance examination. The dichotomous variable is the grade point category, which is determined by whether the student got (1) a C or better or (2) lower than a C. The number 1 represents the grade of each person who obtained a C or better. A cipher, 0, represents the grade of those who got lower than a C.

The formula for the point-biserial correlation is

$$r_{pb} = \frac{\bar{Y}_1 - \bar{Y}_0}{s_y}\sqrt{\frac{N_1N_0}{N(N-1)}}$$

where $\bar{Y}_1$ = the mean of the values of the continuous variable for persons in dichotomous category 1
$\bar{Y}_0$ = the mean of the values of the continuous variable for persons in dichotomous category 0
N_1 = the number of persons in dichotomous category 1
N_0 = the number of persons in dichotomous category 0
s_y = the estimated standard deviation of the population of continuous-variable values from which the sample was taken
N = the total number of persons in the sample ($N_1 + N_0$)

The formula for the estimated standard deviation of the population is

$$s_y = \sqrt{\frac{\sum Y^2}{N-1} - \frac{(\sum Y)^2}{N(N-1)}}$$

Step 1. Table the data as follows.

Student	College-Entrance-Examination Score (Y)	GPA Category
A	104	1
B	82	0
C	92	1
D	76	0
E	101	1
F	111	1
G	98	1
H	85	0
I	88	1
J	67	0
K	106	1
L	95	0

Step 2. Make a list of all those examination scores paired with the GPA category 1 and a list of all those scores paired with the GPA category 0.

Y_1 Scores (Paired with GPA 1)	Y_0 Scores (Paired with GPA 0)
104	82
92	76
101	85
111	67
98	95
88	
106	

Step 3. Compute the means of the two groups of scores.

$$\bar{Y}_1 = \frac{104 + 92 + \cdots + 106}{7} = \frac{700}{7} = \mathbf{100}$$

$$\bar{Y}_0 = \frac{82 + 76 + \cdots + 95}{5} = \frac{405}{5} = \mathbf{81}$$

Step 4. Perform the subtraction $\bar{Y}_1 - \bar{Y}_0$, noting and recording the sign of the result.

$$100 - 81 = \mathbf{+19}$$

Step 5. Obtain $\sqrt{(N_1N_0)/[N(N-1)]}$. (In our example, $N_1 = 7$, $N_0 = 5$, and $N = N_1 + N_0 = 12$.)

$$\sqrt{\frac{(7)(5)}{12(11)}} = \sqrt{\frac{35}{132}} = \sqrt{.265} = \mathbf{.515}$$

Step 6. Square all the examination scores of Step 1, and add the squares. (*Note:* Steps 6 and 8 can be done at the same time if you are using a calculating machine.)

$$104^2 + 82^2 + \cdots + 95^2 = \mathbf{103{,}625}$$

Step 7. Multiply the value obtained in Step 6 by N (12 in our example).

$$103{,}625 \times 12 = \mathbf{1{,}243{,}500}$$

Step 8. Add all the examination scores.

$$104 + 82 + \cdots + 95 = \mathbf{1105}$$

Step 9. Square the value obtained in Step 8.

$$1105^2 = \mathbf{1{,}221{,}025}$$

Step 10. Subtract the result of Step 9 from the result of Step 7.

$$1{,}243{,}500 - 1{,}221{,}025 = \mathbf{22{,}475}$$

Step 11. Divide the result of Step 10 by $N(N-1)$.

$$\frac{22{,}475}{N(N-1)} = \frac{22{,}475}{12(11)} = \frac{22{,}475}{132} = \mathbf{170.265}$$

Step 12. Take the square root of the result of Step 11.

$$\sqrt{170.265} = \mathbf{13.05}$$

Step 13. Divide the result of Step 4 by the result of Step 12.

$$+\frac{19}{13.05} = \mathbf{+1.456}$$

Step 14. Multiply the result of Step 13 by the result of Step 5. This yields the value of r_{pb}. (We see from Step 4 that the value will be positive when $\bar{Y}_1$ is larger than $\bar{Y}_0$.)

$$r_{pb} = +1.456 \times .515 = \mathbf{+.75}$$

SUPPLEMENT

Testing the significance of r_{pb}: The hypothesis that $r_{pb} = 0$ can be tested by the t-test

$$t = r_{pb}\sqrt{\frac{N-2}{1-r_{pb}^2}}$$

which has $N - 2$ degrees of freedom. In our example, $r_{pb} = +.75$ and $N = 12$. Thus,

$$t = (+.75)\sqrt{\frac{12-2}{1-(+.75)^2}} = (+.75)\sqrt{\frac{10}{1-.562}} = (+.75)\sqrt{\frac{10}{.438}}$$

$$= (+.75)\sqrt{22.831} = (+.75)(4.78) = \mathbf{+3.58}$$

With $N - 2 = 12 - 2 = 10$ degrees of freedom, a t value larger than ± 2.23 is significant at the .05 level using a two-tailed test (see Appendix B).

SECTION 4.5

The Correlation Ratio (*eta*)

As a measure of relationship between two variables, *eta* can be useful in two different situations: (1) when the relationship is sufficiently nonlinear to make you hesitant about using the Pearson correlation or (2) when the experiment involves independent and dependent measures and you wish to know the degree of their relationship.

Step 1. *Eta* is most clearly defined as

$$eta = \sqrt{\frac{\text{SS}_{\text{between groups}}}{\text{SS}_{\text{total}}}}$$

where SS_{between} and SS_{total} are sums of squares that are computed in a typical analysis of variance. For the purposes of illustration, we will use the SS_b and SS_t from the last step of Section 2.1.

$$SS_b = \mathbf{567} \qquad SS_t = \mathbf{1132}$$

Step 2. From these values we compute

$$eta = \sqrt{\frac{567}{1132}} = \sqrt{.50} = \mathbf{.71}$$

This value of *eta* gives a rough indication of the correlation between the independent and dependent variables in the experiment discussed in Section 2.1.

Step 3. In order to compute *eta* from raw data, turn to Section 2.1 and follow all the steps for the completely randomized design. To test the *eta* for significance, complete the F-test for significance of the group means as given in Section 2.1. The results of that test are also the results of a significance test of *eta*. For the example above, we see that *eta* is judged to be different from zero.

$$F = \frac{ms_{\text{between}}}{ms_{\text{within}}} = \frac{189}{12.84} = \mathbf{14.71} \qquad df = 3/44$$

Regardless of which use of *eta* (as presented at the beginning of this section) you wish to make, compute the sums of squares according to the steps presented in Section 2.1; then follow the directions in the present section.

SUPPLEMENT

It is sometimes important that one know the *eta* for research reported in a publication. The relationship between a particular independent variable and the dependent measure used as a criterion can be found if the F ratio and the degrees of freedom are reported. The procedure for computing *eta* from a given F value is as follows. We will use the data presented in Section 2.7 for the purposes of illustration.

Step 1. Obtain the F of interest (here, the F for trials) and the appropriate degrees of freedom.

$$F = \mathbf{52.46} \qquad df = 2/60$$

Step 2. Multiply the F value of Step 1 by the numerator df.

$$52.46 \times 2 = \mathbf{104.92}$$

Step 3. Add the result of Step 2 to the denominator df from Step 1.

$$104.92 + 60 = \mathbf{164.92}$$

Step 4. Divide the result of Step 2 by the result of Step 3. This yields the square of *eta*.

$$\frac{104.92}{164.92} = .64$$

Step 5. Take the square root of the final value of Step 4.

$$eta = \sqrt{.64} = .80$$

This value of *eta* indicates the degree of relationship between the trials variable and the dependent variable, which is number of correct responses per trial in the verbal-learning situation.

SECTION 4.6

Partial Correlation: Three Variables

Partial correlation is used when you have three sets of measures that are related and you wish to find the relationship between any two when the relationship effect of the third has been taken out of both variables.

EXAMPLE

Suppose you have measures of (a) incomes of a group of fathers, (b) incomes of their eldest sons, and (c) number of years of formal education of the sons. A correlation of $+.60$ between incomes of fathers and sons might be interpreted as being indicative that money-making characteristics run in families. However, an observer might argue that the most probable contributor to the similarity in incomes of fathers and sons is level of education. A partial correlation between incomes of fathers and sons with the effect of education "partialed" out would help solve the controversy.

The notation for partial correlation is $r_{ab.c}$, which is read as "the partial correlation between variables a and b with the relational effects of c taken out." The formula for partial correlation is

$$r_{ab.c} = \frac{r_{ab} - r_{ac}r_{bc}}{\sqrt{1 - r_{ac}^2}\sqrt{1 - r_{bc}^2}}$$

Step 1. Compute the simple correlation between each of the three possible pairs of measures—r_{ab}, r_{ac}, and r_{bc}. To do this, follow the steps provided in Section 4.1 for each of the three correlation coefficients needed. Let us assume that the three correlations for this example turned out to be

$$r_{ab} = +.60 \qquad r_{ac} = +.70 \qquad r_{bc} = +.80$$

Step 2. Compute the partial correlation between a and b.

$$r_{ab.c} = \frac{.60 - (.70)(.80)}{\sqrt{1 - .70^2}\sqrt{1 - .80^2}} = \frac{.60 - .56}{\sqrt{1 - .49}\sqrt{1 - .64}}$$

$$= \frac{+.04}{\sqrt{.51}\sqrt{.36}} = \frac{+.04}{(.714)(.60)} = \frac{+.04}{.428} = +.09$$

Thus, the correlation $r_{ab} = +.60$ is reduced to $r_{ab.c} = +.09$ when the mutual relationships of the variables with c are partialed out. These data suggest that the hypothesis that money-making ability is hereditary is false and that the reason fathers and sons tend to have similar incomes is that they tend to have similar levels of education.

Step 3. The other two partial correlations could be computed in a similar manner if desired.

$$r_{ac.b} = \frac{r_{ac} - r_{ab}r_{cb}}{\sqrt{1 - r_{ab}^2}\sqrt{1 - r_{cb}^2}} = \frac{.70 - (.60)(.80)}{\sqrt{1 - .60^2}\sqrt{1 - .80^2}}$$

$$= \frac{+.22}{.48} = +.46$$

$$r_{bc.a} = \frac{r_{bc} - r_{ba}r_{ca}}{\sqrt{1 - r_{ba}^2}\sqrt{1 - r_{ca}^2}} = \frac{.80 - (.70)(.60)}{\sqrt{1 - .70^2}\sqrt{1 - .60^2}}$$

$$= \frac{+.38}{.571} = +.67$$

SECTION 4.7

Partial Rank-Order Correlation (Using Kendall's *tau*)

The *tau* statistics of Section 4.3 may be extended to the case of partial correlation in either of two cases: (1) when there are no tied ranks or (2) when there are tied ranks but the rank-order correlations are expected to yield only rough indications of true values. No significance test for partial *tau* can be presented since the sampling distribution is unknown.

EXAMPLE

Suppose that a judge has rank ordered fifteen individuals on the basis of IQ, personality, and poise. In this case, the experimenter wishes to determine the correlation between each possible pair of rankings when the effects of the third have been taken out. The data might appear as follows.

Subject	Ranking 1 (IQ)	Ranking 2 (Personality)	Ranking 3 (Poise)
S_1	1	3	1
S_2	2	5	4
⋮	⋮	⋮	⋮
S_{15}	15	13	12

Step 1. First, go to Section 4.3 and compute the *tau* value for each of the three possible pairs of rankings—1 with 2 (tau_{12}), 1 with 3 (tau_{13}), and 2 with 3 (tau_{23}). For this example, suppose

$$tau_{12} = +\mathbf{.50} \qquad tau_{13} = +\mathbf{.60} \qquad tau_{23} = +\mathbf{.80}$$

Step 2. Compute the partial correlation of 1 with 2, partialing out the relationships with 3.

$$tau_{12.3} = \frac{tau_{12} - tau_{13}\,tau_{23}}{\sqrt{1 - tau_{12}^2}\,\sqrt{1 - tau_{23}^2}} = \frac{.50 - (.60)(.80)}{\sqrt{1 - .60^2}\,\sqrt{1 - .80^2}}$$

$$= \frac{.50 - .48}{\sqrt{1 - .36}\,\sqrt{1 - .64}} = \frac{+.02}{\sqrt{.64}\,\sqrt{.36}} = \frac{+.02}{(.80)(.60)}$$

$$= \frac{+.02}{.48} = +\mathbf{.04}$$

Step 3. In a similar manner, compute the partial correlation of 2 with 3, partialing out the relationships with 1.

$$tau_{23.1} = \frac{tau_{23} - tau_{21}\, tau_{31}}{\sqrt{1 - tau_{21}^2}\,\sqrt{1 - tau_{31}^2}} = \frac{.80 - (.50)(.60)}{\sqrt{1 - .50^2}\,\sqrt{1 - .60^2}}$$

$$= \frac{.80 - .30}{\sqrt{1 - .25}\,\sqrt{1 - .36}} = \frac{+.50}{\sqrt{.75}\,\sqrt{.64}} = \frac{+.50}{(.866)(.80)}$$

$$= \frac{+.50}{.693} = +.72$$

Step 4. Finally, compute the partial correlation of 1 with 3, partialing out the relationships with 2.

$$tau_{13.2} = \frac{tau_{13} - tau_{12}\, tau_{23}}{\sqrt{1 - tau_{12}^2}\,\sqrt{1 - tau_{23}^2}} = \frac{.60 - (.50)(.80)}{\sqrt{1 - .50^2}\,\sqrt{1 - .80^2}}$$

$$= \frac{.60 - .40}{\sqrt{1 - .25}\,\sqrt{1 - .64}} = \frac{+.20}{\sqrt{.75}\,\sqrt{.36}} = \frac{+.20}{(.866)(.60)}$$

$$= \frac{+.20}{.520} = +.38$$

SECTION 4.8

Multiple Correlation: Three Variables

Multiple correlation is used when you wish to determine the relationship of one set of numbers (measures on a variable) with two other sets of numbers.

EXAMPLE

In a typical multiple-correlation situation, the first set of numbers represents measures on a criterion variable—in this example, freshman grade point average—and the other two sets of numbers are measures on predictors—here, high-school grades and college-entrance-examination scores. The multiple-correlation coefficient between the criterion variable and the two predictor variables will give an indication of the degree to which the predictors, taken together, actually predict.

Step 1. If you have measures on two predictor variables and one criterion variable, go to Section 4.1 and compute the three simple correlation coefficients according to the instructions there. Let's abbreviate the three variables as:

1 = the criterion variable: freshman grade point average (GPA)
2 = the first predictor: high-school grades
3 = the second predictor: entrance-examination scores

The first correlation coefficient, r_{12}, is between GPA and high-school grades; r_{13} is between GPA and examination scores; and r_{23} is between high-school grades and examination scores. For the purposes of illustration, assume that the simple correlations in this case turned out to be

$$r_{12} = +.60 \qquad r_{13} = +.50 \qquad r_{23} = +.70$$

Step 2. The multiple correlation of 1 with 2 and 3 is given by

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}}$$

Step 3. Simply plug the values of the simple correlations in at the right spots, and carry out the operations.

$$R_{1.23} = \sqrt{\frac{.60^2 + .50^2 - 2(.60)(.50)(.70)}{1 - .70^2}}$$

$$= \sqrt{\frac{.36 + .25 - 1.20(.50)(.70)}{1 - .49}} = \sqrt{\frac{.61 - 1.20(.35)}{.51}}$$

$$= \sqrt{\frac{.19}{.51}} = \sqrt{.373} = \mathbf{.61}$$

This is the value of the multiple correlation between the criterion variable and the two predictor variables. The correlation between high-school grades and freshman GPA was .60, and the correlation between the entrance-examination scores and freshman GPA was .50. By combining the high-school grades with the entrance-examination scores, the multiple correlation with freshman GPA was raised to .61. Thus, the combination of predictors produced a higher correlation with the criterion variable than either predictor taken separately.

Concerning the size of the correlation between the predictor variables and their joint correlation with the criterion: If the simple correlations between each of the predictors and the criterion remain unchanged, the value of R, the multiple correlation, will *increase* as the correlation between the predictors becomes smaller than .65 and the value of R will *decrease* as the correlation between predictors increases above .65. In other words, the more independent of each other the predictors are, the greater their value in jointly predicting the criterion, provided the simple correlations between each predictor and the criterion remain constant.

Computation of the multiple-correlation coefficient becomes much more complicated if there are more than two predictor variables. A procedure such as the Doolittle solution, as explained by McNemar,[1] can be used or, probably better, a competent computer programer can find a ready-made program that will do the job quickly and accurately.

SECTION 4.9

Simple Analysis of Covariance: One Treatment Variable

When experimental control of an important variable—such as age or IQ—is impossible or impractical, statistical control is possible by use of a covariance analysis. This section presents the procedures for such an analysis when there is only one treatment variable.

EXAMPLE

In order to make a decision concerning future equipment needs, an elementary-school principal designed the following experiment to evaluate three different methods of teaching spelling. Eighteen first-grade children were given a vocabulary test (the control measure) to determine their verbal ability before the program was begun. The X values below represent these test scores. The three different teaching methods were used for three months, and then a standard spelling test was given. The Y values represent these criterion spelling scores.

---

[1] Q. McNemar. *Psychological Statistics*, 3rd ed. New York: John Wiley & Sons, Inc., 1962. Chapter 11, pp. 180–184.

Step 1. Table the data as follows.

Teaching Method 1			Teaching Method 2			Teaching Method 3		
Student	X	Y	Student	X	Y	Student	X	Y
A	32	62	G	21	72	M	38	95
B	46	66	H	24	85	N	45	88
C	27	64	I	18	61	O	52	104
D	35	48	J	32	87	P	48	86
E	31	51	K	35	69	Q	41	72
F	40	74	L	26	74	R	37	63

Step 2. Add the scores in each column.

	Method 1		Method 2		Method 3	
	X	Y	X	Y	X	Y
	32	62	21	72	38	95
	46	66	24	85	45	88
	⋮	⋮	⋮	⋮	⋮	⋮
Sums:	**211**	**365**	**156**	**448**	**261**	**508**

Step 3. Square all the numbers in the X columns (Step 1), and add the squared values.

$$32^2 + 46^2 + \cdots + 40^2 + 21^2 + 24^2 + \cdots + 26^2 + 38^2 + 45^2 + \cdots + 37^2 = \mathbf{23{,}448}$$

Step 4. Add the sums of the X columns (Step 2).

$$211 + 156 + 261 = \mathbf{628}$$

Step 5. Square the result of Step 4, and divide by the number of X values (18 in this example).

$$\frac{628^2}{18} = \frac{394{,}384}{18} = \mathbf{21{,}910.222}$$

Step 6. Subtract the result of Step 5 from the result of Step 3.

$$23{,}448 - 21{,}910.222 = \mathbf{1537.778}$$

Step 7. Square the three sums of the X columns (Step 2) and add the squares; then divide by the number of scores composing each sum (6 in this example).

$$\frac{211^2 + 156^2 + 261^2}{6} = \frac{136{,}978}{6} = \mathbf{22{,}829.667}$$

Step 8. Subtract the result of Step 5 from the result of Step 7.

$$22{,}829.667 - 21{,}910.222 = \mathbf{919.445}$$

Step 9. Subtract the value of Step 8 from that of Step 6.

$$1537.778 - 919.445 = \mathbf{618.333}$$

Step 10. Square all the numbers in the Y columns (Step 1), and add the squares.

$$62^2 + 66^2 + \cdots + 74^2 + 72^2 + 85^2 + \cdots + 74^2 + 95^2 + 88^2 + \cdots + 63^2 = \mathbf{100{,}747}$$

Step 11. Add the sums of the Y columns (Step 2).

$$365 + 448 + 508 = \mathbf{1321}$$

Step 12. Square the result of Step 11, and divide by the number of Y values (18 in this example).

$$\frac{1321^2}{18} = \frac{1{,}745{,}041}{18} = \mathbf{96{,}946.722}$$

Step 13. Subtract the result of Step 12 from the result of Step 10.

$$100{,}747 - 96{,}946.722 = \mathbf{3800.278}$$

Step 14. Square the three sums of the Y columns (Step 2), and add the squares; then divide by the number of scores composing each sum (6 in this example).

$$\frac{365^2 + 448^2 + 508^2}{6} = \mathbf{98{,}665.500}$$

Step 15. Subtract the result of Step 12 from the result of Step 14.

$$98{,}665.500 - 96{,}946.722 = \mathbf{1718.778}$$

Step 16. Subtract the final value of Step 15 from that of Step 13.

$$3800.278 - 1718.778 = \mathbf{2081.500}$$

Step 17. Multiply each X score by its paired Y score (see Step 1), and add all the products.

$$(32 \times 62) + (46 \times 66) + \cdots + (37 \times 63) = \mathbf{47{,}131}$$

Step 18. Multiply the result of Step 4 by the result of Step 11, and divide by the number of XY pairs (18 in this example).

$$\frac{628 \times 1321}{18} = \frac{829{,}588}{18} = \mathbf{46{,}088.222}$$

Step 19. Subtract the final value of Step 18 from that of Step 17.

$$47{,}131 - 46{,}088.222 = \mathbf{1042.778}$$

Step 20. Multiply each X sum in Step 2 by its paired Y sum, and add the three products; then divide by the number of pairs under each method (6 in this example).

$$\frac{(211 \times 365) + (156 \times 448) + (261 \times 508)}{6} = \frac{279{,}491}{6} = \mathbf{46{,}581.833}$$

Step 21. Subtract the result of Step 18 from the result of Step 20.

$$46{,}581.833 - 46{,}088.222 = \mathbf{493.611}$$

Step 22. Subtract the value of Step 21 from that of Step 19.

$$1042.778 - 493.611 = \mathbf{549.167}$$

Step 23. Square the result of Step 19, and divide by the result of Step 6.

$$\frac{1042.778^2}{1537.778} = \frac{1{,}087{,}385.957}{1537.778} = \mathbf{707.115}$$

Step 24. Subtract the result of Step 23 from the result of Step 13.

$$3800.278 - 707.115 = \mathbf{3093.163}$$

Step 25. Square the result of Step 22, and divide by the result of Step 9.

$$\frac{549.167^2}{618.333} = \frac{301{,}584.394}{618.333} = \mathbf{487.738}$$

Step 26. Subtract the result of Step 25 from the result of Step 16.

$$2081.500 - 487.738 = \mathbf{1593.762}$$

Step 27. Divide the value of Step 26 by the number of degrees of freedom for the adjusted within-groups measures. The degrees of freedom will always be $N - a - 1$, where N is the total number of XY pairs (18 in our example) and a is the number of different experimental groups (3 in our example, corresponding to the three methods of teaching).

$$\frac{1593.762}{N - a - 1} = \frac{1593.762}{18 - 3 - 1} = \frac{1593.762}{14} = \mathbf{113.840}$$

Step 28. Subtract the value of Step 26 from that of Step 24.

$$3093.163 - 1593.762 = \mathbf{1499.401}$$

Step 29. Divide the result of Step 28 by the number of degrees of freedom for the adjusted between-groups measures. This degrees-of-freedom value will always be $a - 1$, where a is the number of different experimental groups (3 in our example).

$$\frac{1499.401}{a-1} = \frac{1499.401}{3-1} = \frac{1499.401}{2} = 749.700$$

Step 30. Compute the F ratio by dividing the result of Step 29 by the result of Step 27.

$$F = \frac{749.700}{113.840} = 6.59$$

This F has $df = (a - 1)/(N - a - 1)$. (This equals 2/14 in our example.) From Appendix D it is seen that with $df = 2/14$, an F ratio larger than 6.51 would be expected by chance alone less than one time in a hundred. Therefore, the F value is said to be significant at the .01 level. We conclude, then, that when the covariance analysis was used to equate statistically the groups according to vocabulary-test score (this equating was necessary since the groups differed at the beginning of training), the methods of teaching produced significantly different results on the spelling test.

SECTION 4.10

Factorial Analysis of Covariance: Two Treatment Variables

The following example presents the computational steps to be used when there are two treatment variables and it is necessary to have statistical control of some other variable to draw meaningful conclusions.

EXAMPLE

A certain university was engaged in teaching Peace Corps volunteers the foreign languages they would need during their tour of duty. The following experiment was carried out to determine the effectiveness of the different teaching methods that were used. All students were first given a language-aptitude test, which provided the control measures (the X values presented in Step 1 below). It is obvious from these X values that the students' language-aptitude scores differed widely. Since all the students were not able to participate in the experiment at the same time, they were placed in experimental groups on the basis of when they participated rather than on the basis of their aptitude scores. The average aptitude scores of the resulting experimental groups differed considerably. It is for this reason that a covariance analysis was needed. Using statistical procedures, the covariance analysis equated the groups on aptitude scores, so that any differences found after the experiment could be interpreted as results of the experimental manipulations rather than of the original differences in aptitudes.

The experiment that was carried out was to evaluate two methods of teaching the foreign languages and to determine the value of language laboratory sessions. The two teaching methods were (1) formal classroom meetings with lectures and (2) no formal classroom meetings—only conversation periods held in a congenial atmosphere. In addition, half of the students being taught by each teaching method spent three hours a day in the language laboratory using the tape-recording equipment. The other half of the students in each method group never entered the language lab.

Two years later, when the volunteers returned from overseas, they were asked to evaluate the degree to which their language training prepared them for their work. The ratings were on a 10-point scale ranging from 1 to 10. These ratings served as the criterion measures (the Y scores below).

Step 1. Table the data as follows.

Classroom Group 1 (3 Hrs./Day in Lab)			Conversation Group 1 (3 Hrs./Day in Lab)		
Subject	X	Y	Subject	X	Y
S_1	62	5	S_{11}	46	5
S_2	75	7	S_{12}	53	4
S_3	41	3	S_{13}	57	3
S_4	88	8	S_{14}	49	7
S_5	72	7	S_{15}	62	6

Classroom Group 2 (No Time in Lab)			Conversation Group 2 (No Time in Lab)		
Subject	X	Y	Subject	X	Y
S_6	84	2	S_{16}	58	9
S_7	91	3	S_{17}	72	10
S_8	68	1	S_{18}	61	8
S_9	77	1	S_{19}	65	8
S_{10}	85	3	S_{20}	59	10

Step 2. Add the scores in each column, and record the sums.

	Class 1/Lab		Converse 1/Lab	
	X	Y	X	Y
	62	5	46	5
	75	7	53	4
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sums:	**338**	**30**	**267**	**25**

	Class 2/No Lab		Converse 2/No Lab	
	X	Y	X	Y
	84	2	58	9
	91	3	72	10
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Sums:	**405**	**10**	**315**	**45**

The Control Variable (Steps 3–15)

Step 3. Square all the numbers in the X columns (Step 1), and add the squares.

$$62^2 + \cdots + 72^2 + 84^2 + \cdots + 85^2 + 46^2 + \cdots + 62^2 + 58^2 + \cdots + 59^2 = \mathbf{91{,}587}$$

Step 4. Add the sums for all four X columns (Step 2).

$$338 + 405 + 267 + 315 = \mathbf{1325}$$

Step 5. Square the result of Step 4, and divide by the total number of X values (20 in this example).

$$\frac{1325^2}{20} = \frac{1{,}755{,}625}{20} = \mathbf{87{,}781.250}$$

Step 6. Subtract the result of Step 5 from the result of Step 3.

$$91{,}587 - 87{,}781.25 = \mathbf{3805.750}$$

Step 7. Add the sums of the X columns in the two classroom groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(338 + 405)^2}{10} = \frac{743^2}{10} = \frac{552{,}049}{10} = \mathbf{55{,}204.900}$$

Step 8. Add the sums of the X columns in the two conversation groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(267 + 315)^2}{10} = \frac{582^2}{10} = \frac{338{,}724}{10} = \mathbf{33{,}872.400}$$

Step 9. Add the result of Step 7 to the result of Step 8, and subtract the value of Step 5.

$$55{,}204.9 + 33{,}872.4 - 87{,}781.250 = \mathbf{1296.050}$$

Step 10. Add the sums of the X columns in the two lab groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(338 + 267)^2}{10} = \frac{605^2}{10} = \frac{366{,}025}{10} = \mathbf{36{,}602.500}$$

Step 11. Add the sums of the X columns in the two no-lab groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(405 + 315)^2}{10} = \frac{720^2}{10} = \frac{518{,}400}{10} = \mathbf{51{,}840}$$

Step 12. Add the result of Step 10 to the result of Step 11; then subtract the value of Step 5.

$$36{,}602.500 + 51{,}840 - 87{,}781.250 = \mathbf{661.250}$$

Step 13. Square each of the sums under the four X columns (Step 2), divide each square by the number of cases on which each sum is based (5 in our example), and then add the four quotients.

$$\frac{338^2}{5} + \frac{405^2}{5} + \frac{267^2}{5} + \frac{315^2}{5} = \frac{448{,}783}{5} = \mathbf{89{,}756.6}$$

Step 14. Subtract the results of Steps 5, 9, and 12 from the value of Step 13.

$$89{,}756.6 - 87{,}781.250 - 1296.050 - 661.250 = \mathbf{18.05}$$

Step 15. Subtract the result of Step 13 from that of Step 3.

$$91{,}587 - 89{,}756.6 = \mathbf{1830.4}$$

The Criterion Variable (Steps 16–28)

Step 16. Square all the numbers in the Y columns (Step 1), and add the squares.

$$5^2 + \cdots + 7^2 + 2^2 + \cdots + 3^2 + 5^2 + \cdots + 6^2 + 9^2 + \cdots + 10^2 = \mathbf{764}$$

Step 17. Add the sums for all the Y columns (Step 2).

$$30 + 10 + 25 + 45 = \mathbf{110}$$

Step 18. Square the result of Step 17, and divide by the total number of Y values (20 in this example).

$$\frac{110^2}{20} = \frac{12{,}100}{20} = \mathbf{605}$$

Step 19. Subtract the result of Step 18 from the result of Step 16.

$$764 - 605 = \mathbf{159}$$

Step 20. Add the sums of the Y columns in the two classroom groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(30 + 10)^2}{10} = \frac{40^2}{10} = \frac{1600}{10} = \mathbf{160}$$

Step 21. Add the sums under the Y columns in the two conversation groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(25 + 45)^2}{10} = \frac{70^2}{10} = \frac{4900}{10} = \mathbf{490}$$

Step 22. Add the result of Step 20 to the result of Step 21, and subtract the result of Step 18.

$$160 + 490 - 605 = \mathbf{45}$$

Step 23. Add the sums under the Y columns in the two lab groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(30 + 25)^2}{10} = \frac{55^2}{10} = \frac{3025}{10} = \mathbf{302.5}$$

Step 24. Add the sums under the Y columns in the two no-lab groups (Step 2), square that number, and divide it by the number of cases on which the sum is based (10 in our example).

$$\frac{(10 + 45)^2}{10} = \frac{55^2}{10} = \frac{3025}{10} = \mathbf{302.5}$$

Step 25. Add the result of Step 23 to the result of Step 24, and subtract the result of Step 18.

$$302.5 + 302.5 - 605 = \mathbf{0}$$

Step 26. Square each of the sums under the four Y columns (Step 2), divide each square by the number of cases on which each sum is based (5 in our example), and then add the quotients.

$$\frac{30^2}{5} + \frac{10^2}{5} + \frac{25^2}{5} + \frac{45^2}{5} = \frac{3650}{5} = \mathbf{730}$$

Step 27. Subtract the results of Steps 18, 22, and 25 from the result of Step 26.

$$730 - 605 - 45 - 0 = \mathbf{80}$$

Step 28. Subtract the value of Step 26 from that of Step 16.

$$764 - 730 = \mathbf{34}$$

Products of the Two Variables (Steps 29–53)

Step 29. Multiply each of the X scores in Step 1 by its paired Y score, and add the products.

$$(62 \times 5) + \cdots + (72 \times 7) + (84 \times 2) + \cdots + (85 \times 3)$$
$$+ (46 \times 5) + \cdots + (62 \times 6) + (58 \times 9) + \cdots + (59 \times 10)$$
$$= \mathbf{7175}$$

Step 30. Multiply the result of Step 4 by the value of Step 17; then divide by the number of XY pairs (20 in our example).

$$\frac{1325 \times 110}{20} = \frac{145{,}750}{20} = \mathbf{7287.500}$$

Step 31. Subtract the result of Step 30 from the result of Step 29.

$$7175 - 7287.5 = \mathbf{-112.5}$$

Step 32. From Step 2, obtain the X sum and the Y sum for the two classroom groups.

$338 + 405 = \mathbf{743} = X$ sum for classroom groups
$30 + 10 = \mathbf{40} = Y$ sum for classroom groups

Multiply the X sum by the Y sum, and divide by the number of scores in each sum (10 in our example).

$$\frac{743 \times 40}{10} = \frac{29{,}720}{10} = \mathbf{2972}$$

Step 33. From Step 2, obtain the X sum and the Y sum for the two conversation groups.

$267 + 315 = \mathbf{582} = X$ sum for conversation groups
$25 + 45 = \mathbf{70} = Y$ sum for conversation groups

Multiply the X sum by the Y sum, and divide by the number of scores in each sum (10 in our example).

$$\frac{582 \times 70}{10} = \frac{40{,}740}{10} = \mathbf{4074}$$

Step 34. Add the result of Step 32 to the result of Step 33, and subtract the value of Step 30.

$$2972 + 4074 - 7287.5 = \mathbf{-241.5}$$

Step 35. From Step 2, obtain the X sum and the Y sum for the two lab groups.

$$338 + 267 = \mathbf{605} = X \text{ sum for lab groups}$$
$$30 + 25 = \mathbf{55} = Y \text{ sum for lab groups}$$

Multiply the X sum by the Y sum, and divide by the number of scores in each sum (10 in our example).

$$\frac{605 \times 55}{10} = \frac{33{,}275}{10} = \mathbf{3327.5}$$

Step 36. From Step 2, obtain the X sum and the Y sum for the two no-lab groups.

$$405 + 315 = \mathbf{720} = X \text{ sum for no-lab groups}$$
$$10 + 45 = \mathbf{55} = Y \text{ sum for no-lab groups}$$

Multiply the X sum by the Y sum, and divide by the number of scores in each sum (10 in our example).

$$\frac{720 \times 55}{10} = \frac{39{,}600}{10} = \mathbf{3960}$$

Step 37. Add the result of Step 35 to the result of Step 36, and subtract the result of Step 30.

$$3327.5 + 3960 - 7287.5 = \mathbf{0}$$

Step 38. Multiply each X sum in Step 2 by its paired Y sum, divide each product by the number of scores on which each sum is based (5 in our example), and then add the quotients.

$$\frac{338 \times 30}{5} + \frac{405 \times 10}{5} + \frac{267 \times 25}{5} + \frac{315 \times 45}{5} = \frac{35{,}040}{5} = \mathbf{7008}$$

Step 39. Subtract the results of Steps 30, 34, and 37 from the result of Step 38.

$$7008 - 7287.5 - (-241.5) - 0 = \mathbf{-38}$$

Step 40. Subtract the value of Step 38 from the value of Step 29.

$$7175 - 7008 = \mathbf{167}$$

Step 41. Square the result of Step 40, and divide by the value of Step 15.

$$\frac{167^2}{1830.4} = \frac{27{,}889}{1830.4} = \mathbf{15.237}$$

Then, subtract that quotient from the result of Step 28.

$$34 - 15.237 = \mathbf{18.763}$$

Step 42. Add the result of Step 9 to the result of Step 15.

$$1296.050 + 1830.4 = \mathbf{3126.45}$$

Step 43. Add the value of Step 22 to that of Step 28.

$$45 + 34 = \mathbf{79}$$

Step 44. Add the result of Step 34 to the result of Step 40.

$$(-241.5) + 167 = \mathbf{-74.5}$$

Step 45. Square the result of Step 44, and divide by the result of Step 42.

$$\frac{(-74.5)^2}{3126.5} = \frac{5550.25}{3126.5} = \mathbf{1.775}$$

Then, subtract that quotient and the result of Step 41 from the result of Step 43.

$$79 - 1.775 - 18.763 = \mathbf{58.462}$$

Step 46. Add the final value of Step 12 to that of Step 15.

$$661.250 + 1830.4 = \mathbf{2491.65}$$

Step 47. Add the result of Step 25 to that of Step 28.

$$0 + 34 = \mathbf{34}$$

Step 48. Add the result of Step 37 to that of Step 40.

$$0 + 167 = \mathbf{167}$$

Step 49. Square the result of Step 48, and divide by the result of Step 46.

$$\frac{167^2}{2491.65} = \frac{27{,}889}{2491.65} = \mathbf{11.193}$$

Then subtract that quotient and the result of Step 41 from the result of Step 47.

$$34 - 11.193 - 18.763 = \mathbf{4.044}$$

Step 50. Add the value of Step 14 to that of Step 15.

$$18.05 + 1830.4 = \mathbf{1848.450}$$

Step 51. Add the value of Step 27 to that of Step 28.

$$80 + 34 = \mathbf{114}$$

Step 52. Add the value of Step 39 to that of Step 40.

$$(-38) + 167 = \mathbf{129}$$

Step 53. Square the result of Step 52, and divide by the result of Step 50.

$$\frac{129^2}{1848.45} = \frac{16{,}641}{1848.45} = \mathbf{9.003}$$

Then, subtract that quotient and the result of Step 41 from the result of Step 51.

$$114 - 9.003 - 18.763 = \mathbf{86.234}$$

Mean Squares and F *Ratios* (*Steps 54–60*)

Step 54. Divide the result of Step 41, which is the adjusted-error sum of squares, by the degrees of freedom for adjusted error. The adjusted-error *df* are always $N - CL - 1$, where N is the total number of XY pairs (20 in this example), C is the number of experimental conditions on the first variable (2 in this example, for classroom versus conversation), and L is the number of experimental conditions on the other variable (2 in this example, for lab versus no lab). Thus, $N - CL - 1 = 20 - (2 \times 2) - 1 = 20 - 5 = 15$.

$$\frac{18.763}{N - CL - 1} = \frac{18.763}{15} = \mathbf{1.251}$$

This final value is the error mean square, which will make up the denominator for the three F-tests to follow.

Step 55. Divide the result of Step 45, which is the adjusted sum of squares for teaching methods, by the appropriate degrees of freedom. The *df* are $C - 1$, where C is the number of experimental conditions in this variable (2 in this example, for classroom versus conversation).

$$\frac{58.462}{C - 1} = \frac{58.462}{1} = \mathbf{58.462}$$

Step 56. Divide the value of Step 55 by that of Step 54. This will yield the F for teaching methods. The *df* are $(C - 1)/(N - CL - 1) = 1/15$.

$$F = \frac{58.462}{1.251} = \mathbf{46.73}$$

From Appendix D it is seen that with $df = 1/15$, an F ratio larger than 16.59 would be expected by chance alone less than one time in a thousand. Therefore, the F is said to be significant at the .001 level. This means that the different teaching methods were rated as being significantly different in their effectiveness.

Step 57. Divide the result of Step 49, which is the adjusted sum of squares for language-lab usage, by the appropriate degrees of freedom. The *df* are $L - 1$, where L is the number of experimental conditions in this variable (2 in this example, for lab versus no lab).

$$\frac{4.044}{L-1} = \frac{4.044}{1} = \mathbf{4.044}$$

Step 58. Divide the value of Step 57 by that of Step 54. This will yield the F for language-lab usage. The *df* are $(L - 1)/(N - CL - 1) = (2 - 1)/(20 - 5) = 1/15$.

$$F = \frac{4.044}{1.251} = \mathbf{3.23}$$

Appendix D shows that the probability of getting a value larger than this F is between five and ten in one thousand. Therefore, we conclude that there is no difference in language effectiveness depending upon whether one spends three hours in the lab or no time in the lab.

Step 59. Divide the result of Step 53, which is the adjusted sum of squares for the methods-by-lab interaction, by the appropriate degrees of freedom. The *df* are $(C - 1)(L - 1)$, or the product of the *df* from Step 55 and the *df* from Step 57.

$$\frac{86.234}{(C-1)(L-1)} = \frac{86.234}{1 \times 1} = \mathbf{86.234}$$

Step 60. Divide the value of Step 59 by that of Step 54. This will yield the F for the methods-by-lab interaction.

$$F = \frac{86.234}{1.251} = \mathbf{68.93}$$

The *df* for this F are

$$\frac{(C-1)(L-1)}{N-CL-1} = \frac{1}{15}$$

Appendix D shows that this F is significant at the .001 level. Since we have found that the teaching methods affect (interact with) the lab experience, we would wish to seek out the source of the interaction. The statistical procedures to use for such an undertaking are presented in Section 3.6. If you find a significant interaction in your data, go to Section 3.6 and proceed.

SECTION 4.11

Reliability of Measurement: The Test As a Whole (Test-Retest, Parallel Forms, and Split Halves)

The three coefficients of reliability presented here all start with the computation of a Pearson product-moment correlation coefficient.

1. The *test-retest* reliability measure is computed when you have pairs of scores obtained from two different administrations of the same test to the same people. If you have such pairs of scores, label the scores obtained on the first administration as X and those obtained on the second administration as Y. Then, go to Section 4.1 and follow the instructions. The computed value will be the test-retest reliability coefficient (sometimes called the *coefficient of stability*).
2. The *parallel-forms* reliability measure is computed when you have pairs of test scores from two tests composed of "equated" items. If you have such pairs of scores, label them X and Y, go to Section 4.1, and follow the instructions. The computed value will be the parallel-forms reliability coefficient (sometimes called the *measure of equivalence and stability*).
3. The *split-halves* reliability measure is computed when you have given a test to several individuals and then formed pairs of scores by dividing the test items into two equal groups. (The usual method is to put even-numbered items in one group and odd-numbered items in the other group). The pairs of scores are then used to compute the correlation. Carry out the following steps in order to compute the split-halves reliability coefficient.

Step 1. Go to Section 4.1 and follow the instructions. When you have the correlation coefficient, return to this section and go on to the next step.

Step 2. In order to compute the correlation, you divided the test in half. This means that the real or whole test is twice as long as either half used to compute the reliability coefficient. When a test is lengthened, the reliability increases. You must therefore "correct" the value obtained in Step 1 so that it effectively pertains to the whole test rather than to just half of it.

In order to correct the value of Step 1, first multiply the answer of Step 1 by the number 2. For example, suppose the correlation value from Step 1 is .83.

$$.83 \times 2 = \mathbf{1.66}$$

Step 3. Now add the number 1 to the result of Step 1.

$$1 + .83 = \mathbf{1.83}$$

Step 4. Divide the result of Step 2 by the result of Step 3.

$$\frac{1.66}{1.83} = \mathbf{.91}$$

This value is the split-halves reliability coefficient corrected so that it applies to the whole test. A high reliability value (.70 or higher) shows that the test is reliably (accurately) measuring the characteristic it was designed to measure.

SECTION 4.12

Reliability of Measurement: The Individual Items (Kuder-Richardson and Hoyt)

When your main concern is whether the items in a test are fairly homogeneous in terms of how the individuals responded to the items, the measure of reliability to be used is the Kuder-Richardson coefficient. The computational method presented here was actually developed by Hoyt. It uses the analysis of variance and is somewhat simpler than the original formulation. Hoyt's basic formula for reliability is

$$r_{tt} = \frac{V_e - V_r}{V_e}$$

where V_r = variance for remainder sum of squares
V_e = variance for examinees

EXAMPLE

Suppose that an experimenter wishes to test the reliability of a certain ten-item test. He administers the test to eight subjects and grades their answers. Then, for the purposes of determining reliability, he records for each subject on each test item whether the question was answered correctly (indicated by the number 1) or incorrectly (indicated by the cipher, 0).

Step 1. Table the data as follows.

Subject	Test Items									
	1	2	3	4	5	6	7	8	9	10
S_1	1	1	1	1	1	1	1	1	1	1
S_2	0	0	0	1	1	0	1	1	1	1
S_3	0	0	1	1	1	0	0	0	0	0
S_4	0	0	1	1	1	1	1	0	1	1
S_5	0	1	1	1	1	0	1	0	1	1
S_6	1	1	0	1	0	0	1	1	1	0
S_7	0	0	1	1	0	0	1	0	0	1
S_8	1	1	0	1	1	1	1	0	1	1

Step 2. Count the number of items that each subject answered correctly (in this example, 10 for the first subject, 6 for the second subject, etc.). List the totals for each subject.

Subject	No. of Correct Answers
S_1	10
S_2	6
S_3	3
S_4	7
S_5	7
S_6	6
S_7	4
S_8	8

Step 3. Add the number of correct answers (Step 2), and record the sum.

$$10 + 6 + \cdots + 8 = \mathbf{51}$$

Step 4. Square each number of correct answers in Step 2; then add the squares and divide that sum by the number of items in the test (10 in this example).

$$\frac{10^2 + 6^2 + \cdots + 8^2}{10} = \frac{359}{10} = \mathbf{35.9}$$

Step 5. Square the result of Step 3, and divide by the product of the number of people by the number of items ($8 \times 10 = 80$ in this example).

$$\frac{51^2}{80} = \frac{2601}{80} = \mathbf{32.512}$$

Step 6. Subtract the result of Step 5 from the result of Step 3.

$$51 - 32.512 = \mathbf{18.488}$$

Step 7. Subtract the result of Step 5 from the result of Step 4.

$$35.9 - 32.512 = \mathbf{3.388}$$

Step 8. Count the number of subjects who correctly answered each item. List the totals for each item.

Item	No. of Persons Correct
1	3
2	4
3	5
4	8
5	6
6	3
7	7
8	3
9	6
10	6

Step 9. Square each number of persons correct in Step 8; then add the squares and divide that sum by the number of people who took the test (8 in this example).

$$\frac{3^2 + 4^2 + \cdots + 6^2}{8} = \frac{289}{8} = \mathbf{36.125}$$

Step 10. Subtract the result of Step 5 from the result of Step 9.

$$36.125 - 32.512 = \mathbf{3.613}$$

Step 11. Subtract the result of Step 7 and the result of Step 10 from the result of Step 6.

$$18.488 - 3.388 - 3.613 = \mathbf{11.487}$$

Step 12. Divide the result of Step 7 by $N - 1$, where N is the number of subjects who took the test (8 in this example).

$$\frac{3.388}{N-1} = \frac{3.388}{8-1} = \frac{3.388}{7} = \mathbf{.484}$$

Step 13. Divide the result of Step 11 by $(N - 1)(I - 1)$, where N is the number of subjects who took the test (8 in our example) and I is the number of items in the test (10 in our example).

$$\frac{11.487}{(N-1)(I-1)} = \frac{11.487}{7 \times 9} = \frac{11.487}{63} = \mathbf{.182}$$

Step 14. Subtract the result of Step 13 from the result of Step 12.

$$.484 - .182 = \mathbf{.302}$$

Step 15. Divide the result of Step 14 by the result of Step 12. This yields the value of the Kuder-Richardson (or Hoyt) reliability coefficient.

$$\frac{.302}{.484} = \mathbf{.62}$$

A high reliability coefficient (.70 or higher) would mean that the test was accurately measuring some characteristic of the people taking it. Further, it would mean that the individual items on the test were producing similar patterns of responding in different people. Therefore, a high value would mean that the test items were homogeneous and, therefore, valid.

SECTION 4.13

Test for Difference Between Independent Correlations

If you have two correlations computed from data that were gathered from two different groups of individuals, the correlation coefficients will be experimentally independent. In such a case, you may use the following procedure to test for significance of the difference between the correlations.

EXAMPLE

Suppose we have a correlation coefficient of +.68 that was computed between grades in an English class and IQ scores for thirty-eight people. Suppose further, that we have a correlation coefficient of +.36 between grades in a similar English class and IQ scores for a different group of seventy-three people. We wish to know whether these coefficients are different.

Step 1. First, change the two correlations into Fisher z scores. This can be done by means of any table of such transformations (see Appendix E):

Correlation of .68 = z of **.829**
Correlation of .36 = z of **.377**

Step 2. Subtract either z score of Step 1 from the other.

$$.829 - .377 = \mathbf{.452}$$

Step 3. Subtract 3 from the number of people in the group for which the first correlation was computed (38 in this example). (*Note:* The number 3 is always used.)

$$38 - 3 = \mathbf{35}$$

Step 4. Divide the result of Step 3 into the number 1 (i.e., take the reciprocal of 35). Carry the answer to four decimal places.

$$\frac{1}{35} = \mathbf{.0286}$$

Step 5. Subtract 3 from the number of people in the group for which the second correlation was computed (73 in this example).

$$73 - 3 = \mathbf{70}$$

Step 6. Divide the result of Step 5 into the number 1 (i.e., take the reciprocal of 70). Carry the answer to four decimal places.

$$\frac{1}{70} = \mathbf{.0143}$$

Step 7. Add the result of Step 4 to the result of Step 6.

$$.0286 + .0143 = \mathbf{.0429}$$

Then take the square root of the sum.

$$\sqrt{.0429} = \mathbf{.207}$$

Step 8. Divide the result of Step 2 by the result of Step 7. This yields a z statistic.

$$z = \frac{.452}{.207} = \mathbf{2.18}$$

A z larger than 1.96 is significant at the .05 level using a two-tailed test (see Appendix A). A significant z tells us that the two correlation values are very likely really different.

SECTION 4.14

Test for Difference Between Dependent Correlations

The following procedure is used to determine the significance of the difference between experimentally dependent correlations—i.e., correlations based on data taken from the same group of people.

EXAMPLE

Suppose it is known that the correlation between grades in a statistics course and overall grade point average (GPA) for sixty-three students is +.70. Suppose it is also known that the correlation between grades in an introductory psychology course and overall GPA for those same sixty-three students is +.40. If you wish to test for the significance of the difference between these two correlations, you must first be aware of the fact that they are related or dependent. Then, you must find the remaining correlation between statistics grades and introductory-psychology grades for the sixty-three students. Suppose that correlation is +.30.

Step 1. You have the following three correlations:

Statistics grade with GPA = **+.70**
Introductory psychology grade with GPA = **+.40**
Statistics grade with psychology grade = **+.30**

Compute the difference between the two correlations of interest (in this example, the first two).

$$.70 - .40 = \mathbf{.30}$$

Step 2. Subtract 3 from the number of individuals involved in the correlations (63 in this example). (*Note:* The number 3 is always used.)

$$63 - 3 = \mathbf{60}$$

Step 3. Add 1 to the third correlation in Step 1—i.e., the correlation that you are not presently interested in (+.30 in this example). (*Note:* The number 1 is always used.)

$$.30 + 1 = \mathbf{1.30}$$

Step 4. Multiply the result of Step 2 by the result of Step 3.

$$60 \times 1.30 = \mathbf{78}$$

Then, take the square root of the product.

$$\sqrt{78} = \mathbf{8.832}$$

Step 5. Multiply the result of Step 1 by the result of Step 4.

$$.30 \times 8.832 = \mathbf{2.65}$$

Step 6. Square each of the three correlation values from Step 1, and add the squares.

$$.70^2 + .40^2 + .30^2 = .49 + .16 + .09 = \mathbf{.74}$$

Step 7. Multiply the three correlation values from Step 1.

$$.70 \times .40 \times .30 = \mathbf{.084}$$

Step 8. Multiply the result of Step 7 by 2, and then add 1 to the product. (*Note:* The numbers 2 and 1 are always used.)

$$(2 \times .084) + 1 = .168 + 1 = \mathbf{1.168}$$

Step 9. Subtract the result of Step 6 from the result of Step 8.

$$1.168 - .74 = \mathbf{.428}$$

Step 10. Multiply the result of Step 9 by 2.

$$2 \times .428 = \mathbf{.856}$$

Then, take the square root of the product.

$$\sqrt{.856} = \mathbf{.925}$$

Step 11. Divide the result of Step 5 by the result of Step 10. This yields a t statistic.

$$t = \frac{2.65}{.925} = \mathbf{2.86}$$

The appropriate degrees of freedom are given as the result of Step 2, i.e., 60. A t larger than 2.00, with 60 *df*, is significant at the .05 level using a two-tailed test (see Appendix B). A significant t tells us that the two correlation values are very likely really different.

PART 5

Nonparametric Tests, Miscellaneous Tests of Significance, and Indices of Relationship

In many cases where it is desired to make comparisons of two proportions, or of two medians, or within a table of frequency counts, there are some very simple tests that can be used. Some of these simple tests are listed in the following six sections. Various names—such as *nonparametric tests* or *distribution-free tests*—have been applied to many such tests of significance. We will not attempt to argue the merits and deficiencies of these tests, but will merely point out that they are relatively easy to use and that they are often applicable when the various tests presented elsewhere (e.g., in sections 1.4 through 2.11) are not.

The six sections in this part are discussed below in three sets of two each. These discussions should help you choose the correct analysis for your particular problem.

1. Significance tests for proportions
2. Significance tests for differences between two groups
3. Tests for (1) significance of differences or (2) relationships, when data are gathered into contingency tables

Tests for Proportions (Sections 5.1 and 5.2)

If you have a proportion that has been computed from experimental data (such as the proportion of all American men over forty who have been married at least two times) and you wish to find whether it differs from some specific value (such as a guess that one third, or .33, of all

American men over forty have been married at least two times), the material presented in Section 5.1 will help you solve the problem. If you wish to find whether two independently computed proportions are different, use the analysis presented in Section 5.2.

Tests for Differences Between Two Groups (Sections 5.3 and 5.4)

Section 1.5 (t-test for independent means) shows how to find whether two *mean* values are different. If you wish to find whether two independent *median* values are different, use the procedure in Section 5.3.

Section 1.6 (t-test for related measures) presents the method for finding whether two *means* are different when the samples are paired, matched, or otherwise related. If you wish to test whether two related groups differ on the basis of a simple sign test, use the analysis in Section 5.4.

Tests for Data in Contingency Tables (Sections 5.5 and 5.6)

When you have frequency data (counts of the people exhibiting the characteristic of interest to you) that can be put into the form of a contingency table, you probably wish to ask one side or the other of a two-sided question: (1) Do these variables (treatments) produce different results? or (2) Are these variables related, and if so, how highly related? Both questions can be answered by the tests presented in Sections 5.5 and 5.6. The first question is a chi-square question, and the first part of either Section 5.5 (if you have a 2 × 2 table of data) or Section 5.6 (if you have a larger number of cells in the table) will help you answer it. The second question is a *phi*-coefficient question or a contingency-coefficient question depending on the number of cells in your table, and the last part of either Section 5.5 or 5.6 will help you answer it.

In the following sections, computational formulas are again presented where they contribute to the understanding of the analysis. In those instances where the computation involves only a count or ranking, the formula is omitted.

SECTION 5.1

Test for Significance of a Proportion

In a situation where a logically dichotomous variable, such as passing or failing a course, is considered, the use of proportions can sometimes facilitate the statistical analysis of significance. Before the test of significance can be executed, it is necessary to know the proportion to be expected. If one is tossing a coin, for example, he expects to get heads one half of the times (in the long run); thus, the expected proportion is .50. If one is tossing a die, he expects to get a one-spot one time in six; thus, the expected proportion is 1/6. When the dichotomy is on a variable such as passing or failing a course, you must either (1) know the expected proportion from some previous experience, (2) insist on some a priori proportion, such as "10 per cent will fail," or (3) in some other way determine or state the expected proportion. Then, when the actual proportion has been determined and the expected proportion has been decided upon, the test for significance can be carried out.

EXAMPLE

Assume that a teacher wishes to determine whether a particular class of fifteen students is under par in its performance on an arithmetic achievement test. She decides to do this simply by determining if the proportions who passed and failed correspond to those proportions in classes she has taught in previous years.

Step 1. In order to determine the actual or observed proportion, the first step is to record and table the data on which the proportion will be based (in this case, the arithmetic scores).

Student	Score
A	64
B	35
C	81
D	77
E	65
F	48
G	56
H	31
I	64
J	72
K	43
L	44
M	63
N	59
O	38

Step 2. In order to compute the proportion of students who passed, the teacher must set a pass-fail cutoff point. Suppose she decides that a score of 50 or more is passing and a score of 49 or less is failing. We can quickly see that Students B, F, H, K, L, and O failed. Six out of fifteen failed; thus, nine out of fifteen passed. The proportion (p) of students who passed is the number who passed divided by the total number of students.

$$p = \frac{9}{15} = .60$$

Step 3. Suppose the teacher's information from previous classes indicates that, on the average, 80 per cent (or .80) of the students got a passing grade on this examination. The .80 expected to pass is the P value. The significance test (z-test) for the obtained proportion is computed by use of the following formula.

$$z = \frac{p - P}{\sqrt{\frac{P(1 - P)}{N}}}$$

Thus, for this case,

$$z = \frac{.60 - .80}{\sqrt{\frac{.80(1 - .80)}{15}}} = \frac{-.20}{\sqrt{\frac{.80(.20)}{15}}} = \frac{-.20}{\sqrt{\frac{.16}{15}}} = \frac{-.20}{\sqrt{.0107}} = \frac{-.20}{.1034}$$

$$= -1.93$$

In order to have a significant difference, at the .05 level using a two-tailed test, the z would need to be at least ±1.96 (see Appendix A). A significant z would mean that the p value is significantly different from the P value. In this case, it is concluded that the scores of the fifteen students are not under par for the arithmetic achievement test.

SECTION 5.2

Test for Significance of Difference Between Two Proportions

When dealing with data on a logically dichotomous variable, it is easy to test the significance of a difference between the proportions.

EXAMPLE

In an experiment on short-term memory for a single item, the following conditions were used. The item was presented to all subjects for 2 seconds. One group of twenty subjects was tested to determine if they recalled the item after 5 seconds; the other group of twenty subjects was tested after 10 seconds. If the item was correctly recalled, a "+" was recorded for that subject; if an incorrect response was made, a "0" was recorded.

The formula for the significance of the difference between two proportions is

$$z = \frac{P_1 - P_2}{\sqrt{\frac{p(1-p)}{N_1} + \frac{p(1-p)}{N_2}}}$$

where the value of p under the radical is computed as:

$$p = \frac{N_1P_1 + N_2P_2}{N_1 + N_2}$$

Step 1. In order to compute the two proportions P_1 and P_2, from Groups 1 and 2 respectively, first table the data as follows.

Group 1		Group 2	
Subject	Recall (5 sec)	Subject	Recall (10 sec)
S_1	+	S_{21}	0
S_2	+	S_{22}	0
S_3	+	S_{23}	+
S_4	+	S_{24}	0
S_5	+	S_{25}	+
S_6	+	S_{26}	+
S_7	+	S_{27}	0
S_8	+	S_{28}	+
S_9	+	S_{29}	0
S_{10}	0	S_{30}	0
S_{11}	+	S_{31}	+

Subject	Recall (5 sec)	Subject	Recall (10 sec)
S_{12}	0	S_{32}	0
S_{13}	0	S_{33}	0
S_{14}	+	S_{34}	0
S_{15}	+	S_{35}	0
S_{16}	0	S_{36}	+
S_{17}	+	S_{37}	0
S_{18}	+	S_{38}	0
S_{19}	+	S_{39}	+
S_{20}	+	S_{40}	+

Step 2. Subjects 10, 12, 13, and 16 in Group 1 missed the item; thus, a total of 16 subjects recalled correctly. The number correctly recalling, divided by the number of subjects in the group, gives the proportion who correctly recalled the item.

$$P_1 = \frac{16}{20} = .80$$

Step 3. Subjects 21, 22, 24, 27, 29, 30, 32, 33, 34, 35, 37, and 38 in Group 2 missed the item. Thus, 12 subjects missed, and only 8 correctly recalled the item. Again, the number who correctly recalled, divided by the number of subjects in the group, gives the proportion who correctly recalled the item.

$$P_2 = \frac{8}{20} = .40$$

Step 4. Multiply the result of Step 2 by the value of N_1 (the number of cases in the first sample).

$$.80 \times 20 = 16$$

Then multiply the result of Step 3 by the value of N_2 (the number of cases in the second sample).

$$.40 \times 20 = 8$$

Step 5. Add the two numbers computed in Step 4.

$$16 + 8 = 24$$

Then divide that answer by the total number of cases in both samples ($N_1 + N_2 = 40$).

$24/40 = .60$ *This is the value of* p.

Step 6. Subtract the final result of Step 5 from the number 1.

$$1 - .60 = .40$$

Then multiply that number by the final result of Step 5.

$$.60 \times .40 = .24$$

Step 7. Divide the result of Step 6 by N_1 (the size of the first sample).

$$.24/20 = .012$$

Step 8. Divide the result of Step 6 by N_2 (the size of the second sample).

$$.24/20 = .012$$

Step 9. Add the result of Step 7 to the result of Step 8.

$$.012 + .012 = .024$$

Then take the square root of that sum.

$$\sqrt{.024} = .155$$

Step 10. Subtract the result of Step 3 from the result of Step 2.

$$.80 - .40 = .40$$

Step 11. Divide the result of Step 10 by the result of Step 9.

$$.40/.155 = 2.48$$

A z having value greater than or equal to 1.96 or less than or equal to -1.96 is considered significant at the .05 level using a two-tailed test (see Appendix A). Now, a significant z tells us that the two proportions are significantly different. Thus, in the above problem, the two proportions significantly differ.

SECTION 5.3

The Mann-Whitney *U*-Test for Differences Between Independent Samples

Occasionally the data collected by an experimenter are badly skewed. For this reason, he might choose to use the Mann-Whitney *U*-test rather than the *t*-test for independent groups.

The basic formula for the Mann-Whitney *U*-test is

$$U = n_1n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

or

$$U' = n_1n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

where n_1 = size of the smaller sample
n_2 = size of the larger sample
R_1 = sum of the ranks of the smaller sample
R_2 = sum of the ranks of the larger sample

When either U or U' has been computed, the other can be quickly found by

$$U = n_1 n_2 - U'$$

or

$$U' = n_1 n_2 - U$$

Therefore, it is necessary to compute only one; the other is then obtained by subtraction.

EXAMPLE

Suppose that the following data have been collected representing weekly incomes for stenographers who are members of two different ethnic groups. The experimenter wishes to determine whether there is a significant difference between these incomes.

Group 1		Group 2	
Subject	Weekly Income (dollars)	Subject	Weekly Income (dollars)
S_1	87	S_{10}	131
S_2	72	S_{11}	94
S_3	65	S_{12}	77
S_4	54	S_{13}	88
S_5	67	S_{14}	116
S_6	76	S_{15}	90
S_7	73	S_{16}	87
S_8	82	S_{17}	76
S_9	104	S_{18}	95
		S_{19}	164
		S_{20}	127
		S_{21}	77

Step 1. Rank *all* the numbers in *both* groups taken together according to the size of the number, beginning with the smallest. (*Note:* When there are equal measures, the same rank is assigned to each. But notice that the rank assigned is the mean value of the tied ranks. For example, there are two incomes of 76 dollars, which are tied for the sixth and seventh ranking positions. Therefore, they are both assigned the rank of 6.5.)

Group 1		Group 2	
Income	Rank	Income	Rank
87	11.5	131	20
72	4	94	15
65	2	77	8.5
54	1	88	13
67	3	116	18
76	6.5	90	14
73	5	87	11.5
82	10	76	6.5
104	17	95	16
		164	21
		127	19
		77	8.5

Step 2. Count the number of measures in each group, and then multiply these numbers. (Here, there are 9 measures in Group 1 and 12 in Group 2.)

$$9 \times 12 = \mathbf{108}$$

Step 3. Add 1 to the number of measures in Group 1; then multiply that sum by the original number (9 in this example), and divide the product by 2.

$$\frac{(9+1)9}{2} = \frac{(10)\,9}{2} = \frac{90}{2} = \mathbf{45}$$

Step 4. Add the ranks for Group 1.

$$11.5 + 4 + \cdots + 17 = \mathbf{60}$$

Step 5. Add the result of Step 2 to the result of Step 3, and subtract the sum of Step 4. This yields the value of U.

$$U = 108 + 45 - 60 = \mathbf{93}$$

Step 6. Subtract the result of Step 5 from the result of Step 2. This yields the value of U'.

$$U' = 108 - 93 = \mathbf{15}$$

If the number of measures in the larger group is greater than 8, and the number of measures in either group is less than 21, take whichever answer, from Steps 5 and 6, is smaller, and look in tables of U values to check for significance (see Appendix G). In the present example, $n_1 = 9$ and $n_2 = 12$ so both requirements are satisfied. Thus, go to Appendix G, look in the table for the appropriate alpha level, and see whether the value is smaller than would be expected by chance. For this example, using a two-tailed test, if alpha is .05, we see that if the value is 26 or less, the difference is significant.

If the number of measures in *either* group is greater than 20 carry out the following steps.

Step 7. Divide the result of Step 2 by 2.

$$\frac{108}{2} = \mathbf{54}$$

Then, subtract that quotient from the result of Step 5.

$$93 - 54 = \mathbf{39}$$

Step 8. Add together the number of measures in each group; then add the number 1.

$$9 + 12 + 1 = \mathbf{22}$$

Step 9. Multiply the result of Step 2 by the sum of Step 8.

$$108 \times 22 = \mathbf{2376}$$

Step 10. Divide the result of Step 9 by the number 12. (*Note:* The number 12 is always used.)

$$\frac{2376}{12} = \mathbf{198}$$

Then, take the square root of that quotient.

$$\sqrt{198} = \mathbf{14.071}$$

Step 11. Divide the result of Step 7 by the result of Step 10.

$$\frac{39}{14.071} = \mathbf{2.77}$$

This is a z, or critical-ratio, test. A value greater than ±1.96 is significant at the .05 level using a two-tailed test (see Appendix A). The hypothesis tested by the Mann-Whitney analysis is that the *medians* of the two groups are equal. A z value that is large enough so that the hypothesis is rejected tells us that the chance of the medians being the same is very small. It is concluded, then, that there is a significant difference between the median incomes of stenographers in the two ethnic groups.

SECTION 5.4

A Sign Test (Wilcoxon) for Differences Between Related Samples

The Mann-Whitney U-test presented in Section 5.3 is a nonparametric test for *independent* samples. The Wilcoxon signed-ranks test is applicable where the samples are related (i.e., not independent).

EXAMPLE

Suppose we have collected the reading-rate scores for a group of ten people before and after they enrolled in a course in speed reading. We wish to determine if the course made any significant difference in reading rate.

Step 1. Table the data as follows. Since these are related measures, they must be tabled in appropriate pairs.

Subject	Rate Before	Rate After
A	150	145
B	135	138
C	102	121
D	96	115
E	127	134
F	118	132
G	132	138
H	124	145
I	115	126
J	103	94

Step 2. Subtract each number in the first column from its paired number in the second column, and record the differences.

Subject	After	Before	Difference
A	145	150	−5
B	138	135	3
C	121	102	19
D	115	96	19
E	134	127	7
F	132	118	14
G	138	132	6
H	145	124	21
I	126	115	11
J	94	103	−9

Step 3. Rank the difference values (Step 2) according to the size of the number, beginning with the smallest. Ignore the sign (i.e., use absolute values).

Difference	Rank
-5	2
3	1
19	8.5
19	8.5
7	4
14	7
6	3
21	10
11	6
-9	5

Step 4. Add the ranks for all the difference values that are negative (see Step 3). (In this example, the two negative differences, -5 and -9, are ranked 2 and 5, respectively.)

$$2 + 5 = \mathbf{7}$$

Note: Zero differences are not used in the computation. The N which is used in consulting the table (Appendix J) must be reduced by the number of zero difference cases.

Step 5. Add the ranks for all the difference values that are positive (see Step 3).

$$1 + 8.5 + 8.5 + 4 + 7 + 3 + 10 + 6 = \mathbf{48}$$

Step 6. The results of Steps 4 and 5 are the signed-rank values. Take the result that is smaller (7 in this example) and look in a table of Wilcoxon's signed-rank probabilities (see Appendix J) to check for significance. When the number of pairs is 10, as it is in this example, a value of 8 or less is significant at the .05 level using a two-tailed test. Since the result of Step 4 (i.e., 7) is smaller than 8, we have a significant difference. We conclude that the reading rate *after* training was significantly faster than *before* training.

SECTION 5.5

Simple Chi-Square and the *Phi* Coefficient

When you have frequency data comparing the effects of two variables and there are two groups on both variables, the *phi* coefficient can be quickly computed to show the degree of the relationship between the two variables. Then, the simple chi-square can be easily computed from the *phi* coefficient. The chi-square test will establish whether the variables are related. A significant chi-square is interpreted as showing a relationship between the two variables. The *phi* coefficient gives a numerical value, ranging from 0 to +1, for that relationship.

EXAMPLE

Consider data collected on the two variables (1) gum-chewing activity and (2) educational background. The experimenter simply sampled two groups with differing educational backgrounds. Within each group, the subjects were classified according to their gum-chewing behavior.

The basic computational formula for *phi* (ϕ) is

$$phi = \frac{AD - BC}{\sqrt{(A + B)(C + D)(A + C)(B + D)}}$$

where the numbers represented by the letters A, B, C, and D come from the contingency table

A	B
C	D

And the formula for chi-square is

$$\chi^2 = N\phi^2$$

where N = the total of all the values in the contingency table. Or, as it is usually stated,

$$\chi^2 = \frac{N(AD - BC)^2}{(A + B)(C + D)(A + C)(B + D)}$$

Step 1. Organize the data into a contingency table, as follows. The values in the table represent the number of persons in each category.

		Gum Behavior	
		Chews	Eschews
Educational History	High-School Dropout	90	50
	College Graduate	20	30

Step 2. Add all the numbers in the table.

$$90 + 50 + 20 + 30 = \mathbf{190}$$

Step 3. Add the numbers in each row to get the row sums.

$$90 + 50 = \mathbf{140} = \text{sum for Row 1}$$
$$20 + 30 = \mathbf{50} = \text{sum for Row 2}$$

Then, add the numbers in each column to get the column sums.

	Column 1	Column 2
	90	50
	+ 20	+ 30
Sums:	**110**	**80**

Step 4. Multiply together the four sums from Step 3.

$$140 \times 50 \times 110 \times 80 = \mathbf{61{,}600{,}000}$$

Step 5. Take the square root of the result of Step 4.

$$\sqrt{61{,}600{,}000} = \mathbf{7848.567}$$

Step 6. Multiply the number from the upper-left square by the number from the lower-right square (Step 1).

$$90 \times 30 = \mathbf{2700}$$

Step 7. Multiply the number from the upper-right square by the number from the lower-left square (Step 1).

$$50 \times 20 = \mathbf{1000}$$

Step 8. Get the results of Step 6 and 7, and subtract the smaller from the larger.

$$2700 - 1000 = \mathbf{1700}$$

Step 9. Computation of *phi*: Divide the result of Step 8 by the result of Step 5.

$$phi = \frac{1700}{7848.567} = \mathbf{.217}$$

This value of *phi* is an indicator of the degree of relationship between the two variables in this example; it is similar in meaning to a correlation coefficient.

Step 10. Square the result of Step 9, and multiply it by the result of Step 2. This yields the value of chi-square.

$$\chi^2 = .217^2 \times 190 = .047 \times 190 = \mathbf{8.93}$$

The number of degrees of freedom for this simple chi-square will always be 1. A chi-square value larger than 3.8, with one *df*, is significant at the .05 level (see Appendix C). It is concluded, therefore, that educational background and gum-chewing behavior are significantly related.

The preceding chi-square test is strictly valid only when the frequency counts in all the cells of Step 1 are at least 5.

SECTION 5.6

Complex Chi-Square and the Contingency Coefficient (*C*)

When you have frequency data comparing the effects of two variables and there are more than two groups on either of the two variables, the complex chi-square can be used to test the hypothesis of no relationship between the variables. If the chi-square test shows that there is most likely a relationship between the variables, then the contingency coefficient can be computed to give an indication of the degree of the relationship.

EXAMPLE

Suppose we gather income information for three groups of people: (1) those under 26 years of age, (2) those between 26 and 60 years of age, (3) those 61 years of age and older. The incomes are categorized according to three-thousand-dollar ranges, and a frequency count is made for each category.

The basic formula for chi-square to be used here is

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

where O = the *observed* frequency for a particular cell of the contingency table (see Step 1)
E = the *expected* frequency for a cell, based upon marginal totals (see Steps 2 through 6 for a complete definition)

The relationship between the contingency coefficient (C) and chi-square is

$$C = \sqrt{\frac{\chi^2}{\chi^2 + N}}$$

where χ^2 = the chi-square value
N = the total of all the values in the contingency table

Step 1. Organize the data into a contingency table, as follows. As indicated above, the values in the table represent the number of persons falling into each category.

		Income Groups (dollars)			
		Under 3,000	3,000–6,000	6,000–9,000	Over 9,000
Age Group	Under 26	40	30	10	20
	26 through 60	50	60	20	70
	Over 60	10	20	20	10

Step 2. Add all the numbers in the table to obtain a grand total.

$$40 + 30 + \cdots + 20 + 50 + \cdots + 70 + 10 + \cdots + 10 = \mathbf{360}$$

Step 3. Add the numbers in each row to obtain the row sums.

$$40 + 30 + 10 + 20 = \mathbf{100} = \text{sum for Row 1}$$
$$50 + 60 + 20 + 70 = \mathbf{200} = \text{sum for Row 2}$$
$$10 + 20 + 20 + 10 = \mathbf{60} = \text{sum for Row 3}$$

Step 4. Add the numbers in each column to obtain the column sums.

	Column 1	Column 2	Column 3	Column 4
	40	30	10	20
	50	60	20	70
	+ 10	+ 20	+ 20	+ 10
Sums:	**100**	**110**	**50**	**100**

Step 5. Make a new table similar to the one in Step 1, but get the new numbers as follows.

Compute the new number for each square in the table by using the row sum from Step 3 and the column sum from Step 4 of the row and column that intersect at the square in which you are interested. For example, to compute the new number that goes into the upper-left square, get the sum for Row 1 from Step 3 and the sum for Column 1 from Step 4; multiply these numbers together, and divide by the result of Step 2.

$$\frac{100 \times 100}{360} = \frac{10{,}000}{360} = \mathbf{27.778}$$

Continue this operation for each square in the new table.

$$\text{Row 1} \times \text{Column 2:}\ \frac{100 \times 110}{360} = \mathbf{30.556}$$

$$\text{Row 1} \times \text{Column 3:}\ \frac{100 \times 50}{360} = \mathbf{13.889}$$

$$\text{Row 1} \times \text{Column 4:}\ \frac{100 \times 100}{360} = \mathbf{27.778}$$

$$\text{Row 2} \times \text{Column 1:}\ \frac{200 \times 100}{360} = \mathbf{55.556}$$

$$\text{Row 2} \times \text{Column 2:}\ \frac{200 \times 110}{360} = \mathbf{61.111}$$

$$\text{Row 2} \times \text{Column 3:}\ \frac{200 \times 50}{360} = \mathbf{27.778}$$

$$\text{Row 2} \times \text{Column 4:}\ \frac{200 \times 100}{360} = \mathbf{55.556}$$

$$\text{Row 3} \times \text{Column 1:}\ \frac{60 \times 100}{360} = \mathbf{16.667}$$

$$\text{Row 3} \times \text{Column 2:}\ \frac{60 \times 110}{360} = \mathbf{18.333}$$

$$\text{Row 3} \times \text{Column 3:}\ \frac{60 \times 50}{360} = \mathbf{8.333}$$

$$\text{Row 3} \times \text{Column 4:}\ \frac{60 \times 100}{360} = \mathbf{16.667}$$

Step 6. Put the numbers from Step 5 into the same tabular form as that of Step 1.

		Income Groups (dollars)			
		Under 3,000	3,000–6,000	6,000–9,000	Over 9,000
Age Group	Under 26	27.778	30.556	13.889	27.778
	26 through 60	55.556	61.111	27.778	55.556
	Over 60	16.667	18.333	8.333	16.667

Step 7. The chi-square values can now be computed using the two tables of Step 1 and Step 6. To get these values, take the numbers from the same squares of the two tables, subtract the smaller from the larger, and square that value; then divide by the number that came from the table of Step 6.

For example, the number in the upper-left square of the table of Step 1 is 40, the number from the same square in the table of Step 6 is 27.778. To get the first number for the chi-square, subtract 27.778 from 40, square that difference, and then divide the square by 27.778.

$$\frac{(40 - 27.778)^2}{27.778} = \frac{12.222^2}{27.778} = \frac{149.377}{27.778} = \mathbf{5.378}$$

Continue for each of the other values.

Row 1:

$$\frac{(30.556 - 30)^2}{30.556} = \frac{.556^2}{30.556} = \frac{.309}{30.556} = \mathbf{.01}$$

$$\frac{(13.889 - 10)^2}{13.889} = \frac{3.889^2}{13.889} = \frac{15.124}{13.889} = \mathbf{1.089}$$

$$\frac{(27.778 - 20)^2}{27.778} = \frac{7.778^2}{27.778} = \frac{60.497}{27.778} = \mathbf{2.178}$$

Row 2:

$$\frac{(55.556 - 50)^2}{55.556} = \frac{5.556^2}{55.556} = \frac{30.869}{55.556} = \mathbf{.556}$$

$$\frac{(61.111 - 60)^2}{61.111} = \frac{1.111^2}{61.111} = \frac{1.234}{61.111} = \mathbf{.02}$$

$$\frac{(27.778 - 20)^2}{27.778} = \frac{7.778^2}{27.778} = \frac{60.497}{27.778} = \mathbf{2.178}$$

$$\frac{(70 - 55.556)^2}{55.556} = \frac{14.444^2}{55.556} = \frac{208.629}{55.556} = \mathbf{3.755}$$

Row 3:

$$\frac{(16.667 - 10)^2}{16.667} = \frac{6.667^2}{16.667} = \frac{44.449}{16.667} = \mathbf{2.667}$$

$$\frac{(20 - 18.333)^2}{18.333} = \frac{1.667^2}{18.333} = \frac{2.779}{18.333} = \mathbf{.152}$$

$$\frac{(20 - 8.333)^2}{8.333} = \frac{11.667^2}{8.333} = \frac{136.119}{8.333} = \mathbf{16.335}$$

$$\frac{(16.667 - 10)^2}{16.667} = \frac{6.667^2}{16.667} = \frac{44.449}{16.667} = \mathbf{2.667}$$

Step 8. Chi-square is computed by adding all the values that were computed in Step 7.

$$\chi^2 = 5.378 + .01 + \cdots + 2.667 = \mathbf{36.985}$$

Step 9. The degrees of freedom for this chi-square will always be the number of rows minus 1, multiplied by the number of columns minus 1. (In the present example, there are 3 rows and 4 columns.)

$$df = (3 - 1)(4 - 1) = 2 \times 3 = \mathbf{6}$$

A chi-square value larger than 12.6, with 6 degrees of freedom, is significant at the .05 level (see Appendix C).

Step 10. The extent of the relationship between the two variables can be found by first adding the result of Step 8 to the result of Step 2.

$$36.985 + 360 = \mathbf{396.985}$$

Step 11. Divide the result of Step 8 by the answer of Step 10.

$$\frac{36.985}{396.985} = \mathbf{.093}$$

Then, take the square root of that quotient. This yields the contingency coefficient.

$$C = \sqrt{.093} = \mathbf{.305}$$

This contingency coefficient, which is similar in meaning to the correlation coefficient, has already been shown to be significantly different from zero by the test concluded in Step 9. Thus, it is concluded that age and income are significantly related.

APPENDIXES

APPENDIX A

Normal-Curve Areas

To read this table, first find the z value in which you are interested by looking down the first column for the units and tenths values, and across the first row for the hundredths value. The sign of z, whether positive or negative, is not important for reading the table since it merely indicates whether the z lies to the left or right of the mean. After locating the z value of interest, the number in the body of the table represents the proportional part of the area from the mean ($z = .000$) to your particular z value. For example, if the z of interest were -1.24, the proportion would be .3925. To compute the proportional part of the area in one half of the distribution that lies beyond the z value of interest, subtract the number found above from .5000. Thus, the area below a z of -1.24 would be $.5000 - .3925 = .1075$.

Appendix A. Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.49903									
3.2	.49931									
3.3	.49952									
3.4	.49966									
3.5	.49977									
3.6	.49984									
3.7	.49989									
3.8	.49993									
3.9	.49995									
4.0	.50000									

APPENDIX B

Critical Values of "Student's" *t* Statistic

To read this table, first find the degrees of freedom (*df*) in the column at the left. The minimum *t* values for the several alpha levels of significance are given in the columns to the right of the *df* column. Note that the alpha levels are given for both directional (one-tailed) and nondirectional (two-tailed) tests of significance. Nearly all tests of significance are, however, nondirectional.

To demonstrate the use of this table, assume that a computed *t* value of 3.25 was obtained with *df* equal to 15. Entering the table at the row headed by 15 *df*, read to the right. Note that the *t* of 3.25 is larger than the .01 value (2.947) for a two-tailed test but less than the .001 value (4.073). Therefore, it is concluded that this *t* value is significant beyond the .01 level.

Appendix B. *t* Statistic

df	Alpha level of significance for directional (one-tailed) tests					
	.25	.05	.025	.01	.005	.0005
	Alpha level of significance for nondirectional (two-tailed) tests					
	.50	.10	.05	.02	.01	.001
1	1.000	6.314	12.706	31.821	63.657	636.619
2	.816	2.920	4.303	6.965	9.925	31.598
3	.765	2.353	3.182	4.541	5.841	12.941
4	.741	2.132	2.776	3.747	4.604	8.610
5	.727	2.015	2.571	3.365	4.032	6.859
6	.718	1.943	2.447	3.143	3.707	5.959
7	.711	1.895	2.365	2.998	3.499	5.405
8	.706	1.860	2.306	2.896	3.355	5.041
9	.703	1.833	2.262	2.821	3.250	4.781
10	.700	1.812	2.228	2.764	3.169	4.587
11	.697	1.796	2.201	2.718	3.106	4.437
12	.695	1.782	2.179	2.681	3.055	4.318
13	.694	1.771	2.160	2.650	3.012	4.221
14	.692	1.761	2.145	2.624	2.977	4.140
15	.691	1.753	2.131	2.602	2.947	4.073
16	.690	1.746	2.120	2.583	2.921	4.015
17	.689	1.740	2.110	2.567	2.898	3.965
18	.688	1.734	2.101	2.552	2.878	3.922
19	.688	1.729	2.093	2.539	2.861	3.883
20	.687	1.725	2.086	2.528	2.845	3.850
21	.686	1.721	2.080	2.518	2.831	3.819
22	.686	1.717	2.074	2.508	2.819	3.792
23	.685	1.714	2.069	2.500	2.807	3.767
24	.685	1.711	2.064	2.492	2.797	3.745
25	.684	1.708	2.060	2.485	2.787	3.725
26	.684	1.706	2.056	2.479	2.779	3.707
27	.684	1.703	2.052	2.473	2.771	3.690
28	.683	1.701	2.048	2.467	2.763	3.674
29	.683	1.699	2.045	2.462	2.756	3.659
30	.683	1.697	2.042	2.457	2.750	3.646
40	.681	1.684	2.021	2.423	2.704	3.551
60	.679	1.671	2.000	2.390	2.660	3.460
120	.677	1.658	1.980	2.358	2.617	3.373
∞	.674	1.645	1.960	2.326	2.576	3.291

SOURCE: Appendix B is taken from Table III of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research*, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers. This abridgment is reproduced from John G. Peatman, *Introduction to Applied Statistics*. New York: Harper & Row, Publishers, 1953. Reprinted by permission.

APPENDIX C

Centile Values of the Chi-Square Statistic

To read this table, first find the degrees of freedom (*df*) in the column at the left. The minimum chi-square values for the several alpha levels are given in the columns to the right of the *df* column. The subscript values for each chi-square column indicate the probability of a real difference existing. If you wish to find the probability that the obtained difference is due to chance, simply subtract the given subscript value from 1.00, e.g., $1.00 - .95 = .05$.

To demonstrate the use of this table, assume that a computed chi-square of 25.0 was obtained with 15 *df*. Entering the table at the row headed by 15 *df*, read to the right. The value of 25.0 is listed in the .95 column. Thus, it is concluded that the probability is .95 that the difference is real, or that the probability is .05 that the difference is due to chance alone.

Appendix C. Chi-Square Statistic

df	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.25}$	$\chi^2_{.50}$	$\chi^2_{.75}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$	$\chi^2_{.999}$
1	—	—	—	—	.02	.10	.45	1.3	2.7	3.8	5.0	6.6	7.9	10.8
2	.01	.02	.05	.10	.21	.58	1.4	2.8	4.6	6.0	7.4	9.2	10.6	13.8
3	.07	.11	.22	.35	.58	1.21	2.4	4.1	6.3	7.8	9.4	11.3	12.8	16.3
4	.21	.30	.48	.71	1.1	1.92	3.4	5.4	7.8	9.5	11.1	13.3	14.9	18.5
5	.41	.55	.83	1.1	1.6	2.7	4.4	6.6	9.2	11.1	12.8	15.1	16.7	20.5
6	.68	.87	1.2	1.6	2.2	3.5	5.4	7.8	10.6	12.6	14.4	16.8	18.5	22.5
7	.99	1.24	1.7	2.2	2.8	4.3	6.4	9.0	12.0	14.1	16.0	18.5	20.3	24.3
8	1.3	1.65	2.2	2.7	3.5	5.1	7.3	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	1.7	2.09	2.7	3.3	4.2	5.9	8.3	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	2.2	2.56	3.2	3.9	4.9	6.7	9.3	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	2.6	3.05	3.8	4.6	5.6	7.6	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	3.1	3.57	4.4	5.2	6.3	8.4	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	3.6	4.11	5.0	5.9	7.0	9.3	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	4.1	4.66	5.6	6.6	7.8	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	4.6	5.23	6.3	7.3	8.5	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	5.1	5.81	6.9	8.0	9.3	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3
17	5.7	6.41	7.6	8.7	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8
18	6.3	7.01	8.2	9.4	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3
19	6.8	7.63	8.9	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8
20	7.4	8.26	9.6	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3
21	8.0	8.9	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8
22	8.6	9.5	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3
23	9.3	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7
24	9.9	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.0
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.3	59.7

Note that median values of chi-square are in $\chi^2_{.50}$ column.

SOURCE: Abridged from Table 8 of E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 1, 2nd ed., 1958, published by the Syndics of the Cambridge University Press, London; used by permission of the authors and publishers. This abridgment is reproduced from John G. Peatman, *Introduction to Applied Statistics*. New York: Harper & Row, Publishers, 1953. Reprinted by permission.

APPENDIX D

Per Cent Points in the *F* Distribution

To read this table, the degrees of freedom (*df*) for both mean-square values of the *F* ratio must be located in the table. The *df* for the numerator is located in the top row (df_1). The *df* for the denominator of the *F* ratio is located in the column to the far left of the table (df_2). The values in the block where the row and column intersect represent the *F* values at different alpha levels of significance. Note that in this table, the alpha levels are listed as percentages. To convert to probabilities, simply move the decimal point two places to the left. For example, 2.5% is equal to .025.

To demonstrate the use of this table, assume that a computed *F* ratio of 3.54 was obtained with *df* equal to 5 and 15. Entering the table by the column headed by df_1 equal to 5 and by the row headed by df_2 equal to 15, we find that the values range from 1.68 for the 20% level (.20) to 7.57 for the .1% level (.001). The *F* ratio of 3.54 falls just short of the value required for the 2.5% (.025) level. It is concluded, then, that the *F* of 3.54 with *df* equal to 5 and 15 is significant at the 5% (.05) level.

df_1 df_2		1	2	3	4	5	6	8	12	24	∞
1	0.1%	405284	500000	540379	562500	576405	585937	598144	610667	623497	636619
	0.5%	16211	20000	21615	22500	23056	23437	23925	24426	24940	25465
	1 %	4052	4999	5403	5625	5764	5859	5981	6106	6234	6366
	2.5%	647.79	799.50	864.16	899.58	921.85	937.11	956.66	976.71	997.25	1018.30
	5 %	161.45	199.50	215.71	224.58	230.16	233.99	238.88	243.91	249.05	254.32
	10 %	39.86	49.50	53.59	55.83	57.24	58.20	59.44	60.70	62.00	63.33
	20 %	9.47	12.00	13.06	13.73	14.01	14.26	14.59	14.90	15.24	15.58
2	0.1	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.5	999.5
	0.5	198.50	199.00	199.17	199.25	199.30	199.33	199.37	199.42	199.46	199.51
	1	98.49	99.00	99.17	99.25	99.30	99.33	99.36	99.42	99.46	99.50
	2.5	38.51	39.00	39.17	39.25	39.30	39.33	39.37	39.42	39.46	39.50
	5	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.45	19.50
	10	8.53	9.00	9.16	9.24	9.29	9.33	9.37	9.41	9.45	9.49
	20	3.56	4.00	4.16	4.24	4.28	4.32	4.36	4.40	4.44	4.48
3	0.1	167.5	148.5	141.1	137.1	134.6	132.8	130.6	128.3	125.9	123.5
	0.5	55.55	49.80	47.47	46.20	45.39	44.84	44.13	43.39	42.62	41.83
	1	34.12	30.81	29.46	28.71	28.24	27.91	27.49	27.05	26.60	26.12
	2.5	17.44	16.04	15.44	15.10	14.89	14.74	14.54	14.34	14.12	13.90
	5	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.64	8.53
	10	5.54	5.46	5.39	5.34	5.31	5.28	5.25	5.22	5.18	5.13
	20	2.68	2.89	2.94	2.96	2.97	2.97	2.98	2.98	2.98	2.98
4	0.1	74.14	61.25	56.18	53.44	51.71	50.53	49.00	47.41	45.77	44.05
	0.5	31.33	26.28	24.26	23.16	22.46	21.98	21.35	20.71	20.03	19.33
	1	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	13.93	13.46
	2.5	12.22	10.65	9.98	9.60	9.36	9.20	8.98	8.75	8.51	8.26
	5	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.77	5.63
	10	4.54	4.32	4.19	4.11	4.05	4.01	3.95	3.90	3.83	3.76
	20	2.35	2.47	2.48	2.48	2.48	2.47	2.47	2.46	2.44	2.43
5	0.1	47.04	36.61	33.20	31.09	29.75	28.84	27.64	26.42	25.14	23.78
	0.5	22.79	18.31	16.53	15.56	14.94	14.51	13.96	13.38	12.78	12.14
	1	16.26	13.27	12.06	11.39	10.97	10.67	10.29	9.89	9.47	9.02
	2.5	10.01	8.43	7.76	7.39	7.15	6.98	6.76	6.52	6.28	6.02
	5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.53	4.36
	10	4.06	3.78	3.62	3.52	3.45	3.40	3.34	3.27	3.19	3.10
	20	2.18	2.26	2.25	2.24	2.23	2.22	2.20	2.18	2.16	2.13

SOURCE: Appendix D is taken from Table V of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research*, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers. This abridgment is reproduced from E. F. Lindquist, *Design and Analysis of Experiments in Psychology and Education*. Boston: Houghton Mifflin Company, 1953. Reprinted by permission.

Appendix D. *F* Distribution (*cont.*)

df_1 df_2		1	2	3	4	5	6	8	12	24	∞
6	0.1%	35.51	27.00	23.70	21.90	20.81	20.03	19.03	17.99	16.89	15.75
	0.5%	18.64	14.54	12.92	12.03	11.46	11.07	10.57	10.03	9.47	8.88
	1 %	13.74	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.31	6.88
	2.5%	8.81	7.26	6.60	6.23	5.99	5.82	5.60	5.37	5.12	4.85
	5 %	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.84	3.67
	10 %	3.78	3.46	3.29	3.18	3.11	3.05	2.98	2.90	2.82	2.72
	20 %	2.07	2.13	2.11	2.09	2.08	2.06	2.04	2.02	1.99	1.95
7	0.1	29.22	21.69	18.77	17.19	16.21	15.52	14.63	13.71	12.73	11.69
	0.5	16.24	12.40	10.88	10.05	9.52	9.16	8.68	8.18	7.65	7.08
	1	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.07	5.65
	2.5	8.07	6.54	5.89	5.52	5.29	5.12	4.90	4.67	4.42	4.14
	5	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.41	3.23
	10	3.59	3.26	3.07	2.96	2.88	2.83	2.75	2.67	2.58	2.47
	20	2.00	2.04	2.02	1.99	1.97	1.96	1.93	1.91	1.87	1.83
8	0.1	25.42	18.49	15.83	14.39	13.49	12.86	12.04	11.19	10.30	9.34
	0.5	14.69	11.04	9.60	8.81	8.30	7.95	7.50	7.01	6.50	5.95
	1	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.28	4.86
	2.5	7.57	6.06	5.42	5.05	4.82	4.65	4.43	4.20	3.95	3.67
	5	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.12	2.93
	10	3.46	3.11	2.92	2.81	2.73	2.67	2.59	2.50	2.40	2.29
	20	1.95	1.98	1.95	1.92	1.90	1.88	1.86	1.83	1.79	1.74
9	0.1	22.86	16.39	13.90	12.56	11.71	11.13	10.37	9.57	8.72	7.81
	0.5	13.61	10.11	8.72	7.96	7.47	7.13	6.69	6.23	5.73	5.19
	1	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.73	4.31
	2.5	7.21	5.71	5.08	4.72	4.48	4.32	4.10	3.87	3.61	3.33
	5	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.90	2.71
	10	3.36	3.01	2.81	2.69	2.61	2.55	2.47	2.38	2.28	2.16
	20	1.91	1.94	1.90	1.87	1.85	1.83	1.80	1.76	1.72	1.67
10	0.1	21.04	14.91	12.55	11.28	10.48	9.92	9.20	8.45	7.64	6.76
	0.5	12.83	9.43	8.08	7.34	6.87	6.54	6.12	5.66	5.17	4.64
	1	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.33	3.91
	2.5	6.94	5.46	4.83	4.47	4.24	4.07	3.85	3.62	3.37	3.08
	5	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.74	2.54
	10	3.28	2.92	2.73	2.61	2.52	2.46	2.38	2.28	2.18	2.06
	20	1.88	1.90	1.86	1.83	1.80	1.78	1.75	1.72	1.67	1.62
11	0.1	19.69	13.81	11.56	10.35	9.58	9.05	8.35	7.63	6.85	6.00
	0.5	12.23	8.91	7.60	6.88	6.42	6.10	5.68	5.24	4.76	4.23
	1	9.65	7.20	6.22	5.67	5.32	5.07	4.74	4.40	4.02	3.60
	2.5	6.72	5.26	4.63	4.28	4.04	3.88	3.66	3.43	3.17	2.88
	5	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.61	2.40
	10	3.23	2.86	2.66	2.54	2.45	2.39	2.30	2.21	2.10	1.97
	20	1.86	1.87	1.83	1.80	1.77	1.75	1.72	1.68	1.63	1.57
12	0.1	18.64	12.97	10.80	9.63	8.89	8.38	7.71	7.00	6.25	5.42
	0.5	11.75	8.51	7.23	6.52	6.07	5.76	5.35	4.91	4.43	3.90
	1	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	3.78	3.36
	2.5	6.55	5.10	4.47	4.12	3.89	3.73	3.51	3.28	3.02	2.72
	5	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.50	2.30
	10	3.18	2.81	2.61	2.48	2.39	2.33	2.24	2.15	2.04	1.90
	20	1.84	1.85	1.80	1.77	1.74	1.72	1.69	1.65	1.60	1.54
13	0.1	17.81	12.31	10.21	9.07	8.35	7.86	7.21	6.52	5.78	4.97
	0.5	11.37	8.19	6.93	6.23	5.79	5.48	5.08	4.64	4.17	3.65
	1	9.07	6.70	5.74	5.20	4.86	4.62	4.30	3.96	3.59	3.16
	2.5	6.41	4.97	4.35	4.00	3.77	3.60	3.39	3.15	2.89	2.60
	5	4.67	3.80	3.41	3.18	3.02	2.92	2.77	2.60	2.42	2.21
	10	3.14	2.76	2.56	2.43	2.35	2.28	2.20	2.10	1.98	1.85
	20	1.82	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.57	1.51
14	0.1	17.14	11.78	9.73	8.62	7.92	7.43	6.80	6.13	5.41	4.60
	0.5	11.06	7.92	6.68	6.00	5.56	5.26	4.86	4.43	3.96	3.44
	1	8.86	6.51	5.56	5.03	4.69	4.46	4.14	3.80	3.43	3.00
	2.5	6.30	4.86	4.24	3.89	3.66	3.50	3.29	3.05	2.79	2.49
	5	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.35	2.13
	10	3.10	2.73	2.52	2.39	2.31	2.24	2.15	2.05	1.94	1.80
	20	1.81	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.55	1.48
15	0.1	16.59	11.34	9.34	8.25	7.57	7.09	6.47	5.81	5.10	4.31
	0.5	10.80	7.70	6.48	5.80	5.37	5.07	4.67	4.25	3.79	3.26
	1	8.68	6.36	5.42	4.89	4.56	4.32	4.00	3.67	3.29	2.87
	2.5	6.20	4.77	4.15	3.80	3.58	3.41	3.20	2.96	2.70	2.40
	5	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.29	2.07
	10	3.07	2.70	2.49	2.36	2.27	2.21	2.12	2.02	1.90	1.76
	20	1.80	1.79	1.75	1.71	1.68	1.66	1.62	1.58	1.53	1.46

Appendix D. *F* Distribution (*cont.*)

df_1 / df_2		1	2	3	4	5	6	8	12	24	∞
16	0.1%	16.12	10.97	9.00	7.94	7.27	6.81	6.19	5.55	4.85	4.06
	0.5%	10.58	7.51	6.30	5.64	5.21	4.91	4.52	4.10	3.64	3.11
	1 %	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.18	2.75
	2.5%	6.12	4.69	4.08	3.73	3.50	3.34	3.12	2.89	2.63	2.32
	5 %	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.24	2.01
	10 %	3.05	2.67	2.46	2.33	2.24	2.18	2.09	1.99	1.87	1.72
	20 %	1.79	1.78	1.74	1.70	1.67	1.64	1.61	1.56	1.51	1.43
17	0.1	15.72	10.66	8.73	7.68	7.02	6.56	5.96	5.32	4.63	3.85
	0.5	10.38	7.35	6.16	5.50	5.07	4.78	4.39	3.97	3.51	2.98
	1	8.40	6.11	5.18	4.67	4.34	4.10	3.79	3.45	3.08	2.65
	2.5	6.04	4.62	4.01	3.66	3.44	3.28	3.06	2.82	2.56	2.25
	5	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.19	1.96
	10	3.03	2.64	2.44	2.31	2.22	2.15	2.06	1.96	1.84	1.69
	20	1.78	1.77	1.72	1.68	1.65	1.63	1.59	1.55	1.49	1.42
18	0.1	15.38	10.39	8.49	7.46	6.81	6.35	5.76	5.13	4.45	3.67
	0.5	10.22	7.21	6.03	5.37	4.96	4.66	4.28	3.86	3.40	2.87
	1	8.28	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.00	2.57
	2.5	5.98	4.56	3.95	3.61	3.38	3.22	3.01	2.77	2.50	2.19
	5	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.15	1.92
	10	3.01	2.62	2.42	2.29	2.20	2.13	2.04	1.93	1.81	1.66
	20	1.77	1.76	1.71	1.67	1.64	1.62	1.58	1.53	1.48	1.40
19	0.1	15.08	10.16	8.28	7.26	6.61	6.18	5.59	4.97	4.29	3.52
	0.5	10.07	7.09	5.92	5.27	4.85	4.56	4.18	3.76	3.31	2.78
	1	8.18	5.93	5.01	4.50	4.17	3.94	3.63	3.30	2.92	2.49
	2.5	5.92	4.51	3.90	3.56	3.33	3.17	2.96	2.72	2.45	2.13
	5	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.11	1.88
	10	2.99	2.61	2.40	2.27	2.18	2.11	2.02	1.91	1.79	1.63
	20	1.76	1.75	1.70	1.66	1.63	1.61	1.57	1.52	1.46	1.39
20	0.1	14.82	9.95	8.10	7.10	6.46	6.02	5.44	4.82	4.15	3.38
	0.5	9.94	6.99	5.82	5.17	4.76	4.47	4.09	3.68	3.22	2.69
	1	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	2.86	2.42
	2.5	5.87	4.46	3.86	3.51	3.29	3.13	2.91	2.68	2.41	2.09
	5	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.08	1.84
	10	2.97	2.59	2.38	2.25	2.16	2.09	2.00	1.89	1.77	1.61
	20	1.76	1.75	1.70	1.65	1.62	1.60	1.56	1.51	1.45	1.37
21	0.1	14.59	9.77	7.94	6.95	6.32	5.88	5.31	4.70	4.03	3.26
	0.5	9.83	6.89	5.73	5.09	4.68	4.39	4.01	3.60	3.15	2.61
	1	8.02	5.78	4.87	4.37	4.04	3.81	3.51	3.17	2.80	2.36
	2.5	5.83	4.42	3.82	3.48	3.25	3.09	2.87	2.64	2.37	2.04
	5	4.32	3.47	3.07	2.84	2.68	2.57	2.42	2.25	2.05	1.81
	10	2.96	2.57	2.36	2.23	2.14	2.08	1.98	1.88	1.75	1.59
	20	1.75	1.74	1.69	1.65	1.61	1.59	1.55	1.50	1.44	1.36
22	0.1	14.38	9.61	7.80	6.81	6.19	5.76	5.19	4.58	3.92	3.15
	0.5	9.73	6.81	5.65	5.02	4.61	4.32	3.94	3.54	3.08	2.55
	1	7.94	5.72	4.82	4.31	3.99	3.76	3.45	3.12	2.75	2.31
	2.5	5.79	4.38	3.78	3.44	3.22	3.05	2.84	2.60	2.33	2.00
	5	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.03	1.78
	10	2.95	2.56	2.35	2.22	2.13	2.06	1.97	1.86	1.73	1.57
	20	1.75	1.73	1.68	1.64	1.61	1.58	1.54	1.49	1.43	1.35
23	0.1	14.19	9.47	7.67	6.69	6.08	5.65	5.09	4.48	3.82	3.05
	0.5	9.63	6.73	5.58	4.95	4.54	4.26	3.88	3.47	3.02	2.48
	1	7.88	5.66	4.76	4.26	3.94	3.71	3.41	3.07	2.70	2.26
	2.5	5.75	4.35	3.75	3.41	3.18	3.02	2.81	2.57	2.30	1.97
	5	4.28	3.42	3.03	2.80	2.64	2.53	2.38	2.20	2.00	1.76
	10	2.94	2.55	2.34	2.21	2.11	2.05	1.95	1.84	1.72	1.55
	20	1.74	1.73	1.68	1.63	1.60	1.57	1.53	1.49	1.42	1.34
24	0.1	14.03	9.34	7.55	6.59	5.98	5.55	4.99	4.39	3.74	2.97
	0.5	9.55	6.66	5.52	4.89	4.49	4.20	3.83	3.42	2.97	2.43
	1	7.82	5.61	4.72	4.22	3.90	3.67	3.36	3.03	2.66	2.21
	2.5	5.72	4.32	3.72	3.38	3.15	2.99	2.78	2.54	2.27	1.94
	5	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	1.98	1.73
	10	2.93	2.54	2.33	2.19	2.10	2.04	1.94	1.83	1.70	1.53
	20	1.74	1.72	1.67	1.63	1.59	1.57	1.53	1.48	1.42	1.33
25	0.1	13.88	9.22	7.45	6.49	5.88	5.46	4.91	4.31	3.66	2.89
	0.5	9.48	6.60	5.46	4.84	4.43	4.15	3.78	3.37	2.92	2.38
	1	7.77	5.57	4.68	4.18	3.86	3.63	3.32	2.99	2.62	2.17
	2.5	5.69	4.29	3.69	3.35	3.13	2.97	2.75	2.51	2.24	1.91
	5	4.24	3.38	2.99	2.76	2.60	2.49	2.34	2.16	1.96	1.71
	10	2.92	2.53	2.32	2.18	2.09	2.02	1.93	1.82	1.69	1.52
	20	1.73	1.72	1.66	1.62	1.59	1.56	1.52	1.47	1.41	1.32

Appendix D. *F* Distribution (*cont.*)

df_1 df_2		1	2	3	4	5	6	8	12	24	∞
26	0.1%	13.74	9.12	7.36	6.41	5.80	5.38	4.83	4.24	3.59	2.82
	0.5%	9.41	6.54	5.41	4.79	4.38	4.10	3.73	3.33	2.87	2.33
	1 %	7.72	5.53	4.64	4.14	3.82	3.59	3.29	2.96	2.58	2.13
	2.5%	5.66	4.27	3.67	3.33	3.10	2.94	2.73	2.49	2.22	1.88
	5 %	4.22	3.37	2.98	2.74	2.59	2.47	2.32	2.15	1.95	1.69
	10 %	2.91	2.52	2.31	2.17	2.08	2.01	1.92	1.81	1.68	1.50
	20 %	1.73	1.71	1.66	1.62	1.58	1.56	1.52	1.47	1.40	1.31
27	0.1	13.61	9.02	7.27	6.33	5.73	5.31	4.76	4.17	3.52	2.75
	0.5	9.34	6.49	5.36	4.74	4.34	4.06	3.69	3.28	2.83	2.29
	1	7.68	5.49	4.60	4.11	3.78	3.56	3.26	2.93	2.55	2.10
	2.5	5.63	4.24	3.65	3.31	3.08	2.92	2.71	2.47	2.19	1.85
	5	4.21	3.35	2.96	2.73	2.57	2.46	2.30	2.13	1.93	1.67
	10	2.90	2.51	2.30	2.17	2.07	2.00	1.91	1.80	1.67	1.49
	20	1.73	1.71	1.66	1.61	1.58	1.55	1.51	1.46	1.40	1.30
28	0.1	13.50	8.93	7.19	6.25	5.66	5.24	4.69	4.11	3.46	2.70
	0.5	9.28	6.44	5.32	4.70	4.30	4.02	3.65	3.25	2.79	2.25
	1	7.64	5.45	4.57	4.07	3.75	3.53	3.23	2.90	2.52	2.06
	2.5	5.61	4.22	3.63	3.29	3.06	2.90	2.69	2.45	2.17	1.83
	5	4.20	3.34	2.95	2.71	2.56	2.44	2.29	2.12	1.91	1.65
	10	2.89	2.50	2.29	2.16	2.06	2.00	1.90	1.79	1.66	1.48
	20	1.72	1.71	1.65	1.61	1.57	1.55	1.51	1.46	1.39	1.30
29	0.1	13.39	8.85	7.12	6.19	5.59	5.18	4.64	4.05	3.41	2.64
	0.5	9.23	6.40	5.28	4.66	4.26	3.98	3.61	3.21	2.76	2.21
	1	7.60	5.42	4.54	4.04	3.73	3.50	3.20	2.87	2.49	2.03
	2.5	5.59	4.20	3.61	3.27	3.04	2.88	2.67	2.43	2.15	1.81
	5	4.18	3.33	2.93	2.70	2.54	2.43	2.28	2.10	1.90	1.64
	10	2.89	2.50	2.28	2.15	2.06	1.99	1.89	1.78	1.65	1.47
	20	1.72	1.70	1.65	1.60	1.57	1.54	1.50	1.45	1.39	1.29
30	0.1	13.29	8.77	7.05	6.12	5.53	5.12	4.58	4.00	3.36	2.59
	0.5	9.18	6.35	5.24	4.62	4.23	3.95	3.58	3.18	2.73	2.18
	1	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.47	2.01
	2.5	5.57	4.18	3.59	3.25	3.03	2.87	2.65	2.41	2.14	1.79
	5	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.89	1.62
	10	2.88	2.49	2.28	2.14	2.05	1.98	1.88	1.77	1.64	1.46
	20	1.72	1.70	1.64	1.60	1.57	1.54	1.50	1.45	1.38	1.28
40	0.1	12.61	8.25	6.60	5.70	5.13	4.73	4.21	3.64	3.01	2.23
	0.5	8.83	6.07	4.98	4.37	3.99	3.71	3.35	2.95	2.50	1.93
	1	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.29	1.80
	2.5	5.42	4.05	3.46	3.13	2.90	2.74	2.53	2.29	2.01	1.64
	5	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.79	1.51
	10	2.84	2.44	2.23	2.09	2.00	1.93	1.83	1.71	1.57	1.38
	20	1.70	1.68	1.62	1.57	1.54	1.51	1.47	1.41	1.34	1.24
60	0.1	11.97	7.76	6.17	5.31	4.76	4.37	3.87	3.31	2.69	1.90
	0.5	8.49	5.80	4.73	4.14	3.76	3.49	3.13	2.74	2.29	1.69
	1	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.12	1.60
	2.5	5.29	3.93	3.34	3.01	2.79	2.63	2.41	2.17	1.88	1.48
	5	4.00	3.15	2.76	2.52	2.37	2.25	2.10	1.92	1.70	1.39
	10	2.79	2.39	2.18	2.04	1.95	1.87	1.77	1.66	1.51	1.29
	20	1.68	1.65	1.59	1.55	1.51	1.48	1.44	1.38	1.31	1.18
120	0.1	11.38	7.31	5.79	4.95	4.42	4.04	3.55	3.02	2.40	1.56
	0.5	8.18	5.54	4.50	3.92	3.55	3.28	2.93	2.54	2.09	1.43
	1	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.34	1.95	1.38
	2.5	5.15	3.80	3.23	2.89	2.67	2.52	2.30	2.05	1.76	1.31
	5	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.83	1.61	1.25
	10	2.75	2.35	2.13	1.99	1.90	1.82	1.72	1.60	1.45	1.19
	20	1.66	1.63	1.57	1.52	1.48	1.45	1.41	1.35	1.27	1.12
∞	0.1	10.83	6.91	5.42	4.62	4.10	3.74	3.27	2.74	2.13	1.00
	0.5	7.88	5.30	4.28	3.72	3.35	3.09	2.74	2.36	1.90	1.00
	1	6.64	4.60	3.78	3.32	3.02	2.80	2.51	2.18	1.79	1.00
	2.5	5.02	3.69	3.12	2.79	2.57	2.41	2.19	1.94	1.64	1.00
	5	3.84	2.99	2.60	2.37	2.21	2.09	1.94	1.75	1.52	1.00
	10	2.71	2.30	2.08	1.94	1.85	1.77	1.67	1.55	1.38	1.00
	20	1.64	1.61	1.55	1.50	1.46	1.43	1.38	1.32	1.23	1.00

APPENDIX E

Fisher's z-Transformation Function for Pearson's r Correlation Coefficient: $z = \frac{1}{2}[\log_e(1 + r) - \log_e(1 - r)]$

To read this table, simply find the correlation coefficient value in the r column and then read the corresponding Z value from the adjacent column. For example, if the r value were .46, the Z would be .497.

Appendix E. Fisher's *z* Transformation

r	*Z*	*r*	*Z*	*r*	*Z*	*r*	*Z*	*r*	*Z*
.000	.000	.200	.203	.400	.424	.600	.693	.800	1.099
.005	.005	.205	.208	.405	.430	.605	.701	.805	1.113
.010	.010	.210	.213	.410	.436	.610	.709	.810	1.127
.015	.015	.215	.218	.415	.442	.615	.717	.815	1.142
.020	.020	.220	.224	.420	.448	.620	.725	.820	1.157
.025	.025	.225	.229	.425	.454	.625	.733	.825	1.172
.030	.030	.230	.234	.430	.460	.630	.741	.830	1.188
.035	.035	.235	.239	.435	.466	.635	.750	.835	1.204
.040	.040	.240	.245	.440	.472	.640	.758	.840	1.221
.045	.045	.245	.250	.445	.478	.645	.767	.845	1.238
.050	.050	.250	.255	.450	.485	.650	.775	.850	1.256
.055	.055	.255	.261	.455	.491	.655	.784	.855	1.274
.060	.060	.260	.266	.460	.497	.660	.793	.860	1.293
.065	.065	.265	.271	.465	.504	.665	.802	.865	1.313
.070	.070	.270	.277	.470	.510	.670	.811	.870	1.333
.075	.075	.275	.282	.475	.517	.675	.820	.875	1.354
.080	.080	.280	.288	.480	.523	.680	.829	.880	1.376
.085	.085	.285	.293	.485	.530	.685	.838	.885	1.398
.090	.090	.290	.299	.490	.536	.690	.848	.890	1.422
.095	.095	.295	.304	.495	.543	.695	.858	.895	1.447
.100	.100	.300	.310	.500	.549	.700	.867	.900	1.472
.105	.105	.305	.315	.505	.556	.705	.877	.905	1.499
.110	.110	.310	.321	.510	.563	.710	.887	.910	1.528
.115	.116	.315	.326	.515	.570	.715	.897	.915	1.557
.120	.121	.320	.332	.520	.576	.720	.908	.920	1.589
.125	.126	.325	.337	.525	.583	.725	.918	.925	1.623
.130	.131	.330	.343	.530	.590	.730	.929	.930	1.658
.135	.136	.335	.348	.535	.597	.735	.940	.935	1.697
.140	.141	.340	.354	.540	.604	.740	.950	.940	1.738
.145	.146	.345	.360	.545	.611	.745	.962	.945	1.783
.150	.151	.350	.365	.550	.618	.750	.973	.950	1.832
.155	.156	.355	.371	.555	.626	.755	.984	.955	1.886
.160	.161	.360	.377	.560	.633	.760	.996	.960	1.946
.165	.167	.365	.383	.565	.640	.765	1.008	.965	2.014
.170	.172	.370	.388	.570	.648	.770	1.020	.970	2.092
.175	.177	.375	.394	.575	.655	.775	1.033	.975	2.185
.180	.182	.380	.400	.580	.662	.780	1.045	.980	2.298
.185	.187	.385	.406	.585	.670	.785	1.058	.985	2.443
.190	.192	.390	.412	.590	.678	.790	1.071	.990	2.647
.195	.198	.395	.418	.595	.685	.795	1.085	.995	2.994

SOURCE: From *Statistical Methods* by Allen L. Edwards. Copyright 1954, © 1967 by Allen L. Edwards. Reprinted by permission of Holt, Rinehart & Winston, Inc.

APPENDIX F

Critical Values of Pearson's *r* Correlation Coefficient for Five Alpha Significance Levels

To read this table, find the appropriate degrees of freedom for the correlation coefficient in the column headed by $n' - 2$. When this value has been located, read to the right to find the values that represent the minimum r values for significance at the .10, .05, .02, .01, and .001 alpha levels of significance. For example, if the degrees of freedom were 25, an r of at least .3233 would be needed for significance at the .10 level, .3809 at the .05 level, etc.

Appendix F. Pearson's *r*

$n' - 2$	.10	.05	.02	.01	.001
1	.98769	.99692	.999507	.999877	.9999988
2	.90000	.95000	.98000	.990000	.99900
3	.8054	.8783	.93433	.95873	.99116
4	.7293	.8114	.8822	.91720	.97406
5	.6694	.7545	.8329	.8745	.95074
6	.6215	.7067	.7887	.8343	.92493
7	.5822	.6664	.7498	.7977	.8982
8	.5494	.6319	.7155	.7646	.8721
9	.5214	.6021	.6851	.7348	.8471
10	.4973	.5760	.6581	.7079	.8233
11	.4762	.5529	.6339	.6835	.8010
12	.4575	.5324	.6120	.6614	.7800
13	.4409	.5139	.5923	.6411	.7603
14	.4259	.4973	.5742	.6226	.7420
15	.4124	.4821	.5577	.6055	.7246
16	.4000	.4683	.5425	.5897	.7084
17	.3887	.4555	.5285	.5751	.6932
18	.3783	.4438	.5155	.5614	.6787
19	.3687	.4329	.5034	.5487	.6652
20	.3598	.4227	.4921	.5368	.6524
25	.3233	.3809	.4451	.4869	.5974
30	.2960	.3494	.4093	.4487	.5541
35	.2746	.3246	.3810	.4182	.5189
40	.2573	.3044	.3578	.3932	.4896
45	.2428	.2875	.3384	.3721	.4648
50	.2306	.2732	.3218	.3541	.4433
60	.2108	.2500	.2948	.3248	.4078
70	.1954	.2319	.2737	.3017	.3799
80	.1829	.2172	.2565	.2830	.3568
90	.1726	.2050	.2422	.2673	.3375
100	.1638	.1946	.2301	.2540	.3211

SOURCE: Appendix F is taken from Table VI of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research*, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers.

APPENDIX G

Critical Values of the *U* Statistic of the Mann-Whitney Test

To read this table, the number of cases in both groups must be located in the particular subtable that lists the level of significance desired with either a one-tailed or a two-tailed test. Assume that you wish to find the critical value using a two-tailed test at the .05 level of significance. To do this, the first step is to locate n_1 and n_2 in subtable "c." The point at which the n_1 row and the n_2 column intersect represents the maximum critical value for significance at the .05 level. If, for example, n_1 equals 10, n_2 equals 15, and the computed value equals 23, it is concluded that the difference between the groups is significant—i.e., the tabled critical value for n_1 = 10 and n_2 = 15 is 39, and 23 is obviously smaller than 39.

(*a*) Critical Values of *U* for a One-Tailed Test at .001 or for a Two-Tailed Test at .002

n_2 / n_1	9	10	11	12	13	14	15	16	17	18	19	20
1												
2												
3									0	0	0	0
4		0	0	0	1	1	1	2	2	3	3	3
5	1	1	2	2	3	3	4	5	5	6	7	7
6	2	3	4	4	5	6	7	8	9	10	11	12
7	3	5	6	7	8	9	10	11	13	14	15	16
8	5	6	8	9	11	12	14	15	17	18	20	21
9	7	8	10	12	14	15	17	19	21	23	25	26
10	8	10	12	14	17	19	21	23	25	27	29	32
11	10	12	15	17	20	22	24	27	29	32	34	37
12	12	14	17	20	23	25	28	31	34	37	40	42
13	14	17	20	23	26	29	32	35	38	42	45	48
14	15	19	22	25	29	32	36	39	43	46	50	54
15	17	21	24	28	32	36	40	43	47	51	55	59
16	19	23	27	31	35	39	43	48	52	56	60	65
17	21	25	29	34	38	43	47	52	57	61	66	70
18	23	27	32	37	42	46	51	56	61	66	71	76
19	25	29	34	40	45	50	55	60	66	71	77	82
20	26	32	37	42	48	54	59	65	70	76	82	88

SOURCE: Adapted and abridged from Tables 1, 3, 5, and 7 of D. Aube, "Extended Tables for the Mann-Whitney Statistic," *Bulletin of the Institute of Educational Research at Indiana University*, 1953, **1**, No. 2. Reproduced from S. Siegel, *Nonparametric Statistics for the Behavioral Sciences.* New York: McGraw-Hill Book Company, 1956. Reprinted by permission of the Institute of Educational Research and McGraw-Hill Book Company.

Appendix G. *U* Statistic (*cont.*)

(*b*) Critical Values of *U* for a One-Tailed Test at .01 or for a Two-Tailed Test at .02

n_2 / n_1	9	10	11	12	13	14	15	16	17	18	19	20
1												
2					0	0	0	0	0	0	1	1
3	1	1	1	2	2	2	3	3	4	4	4	5
4	3	3	4	5	5	6	7	7	8	9	9	10
5	5	6	7	8	9	10	11	12	13	14	15	16
6	7	8	9	11	12	13	15	16	18	19	20	22
7	9	11	12	14	16	17	19	21	23	24	26	28
8	11	13	15	17	20	22	24	26	28	30	32	34
9	14	16	18	21	23	26	28	31	33	36	38	40
10	16	19	22	24	27	30	33	36	38	41	44	47
11	18	22	25	28	31	34	37	41	44	47	50	53
12	21	24	28	31	35	38	42	46	49	53	56	60
13	23	27	31	35	39	43	47	51	55	59	63	67
14	26	30	34	38	43	47	51	56	60	65	69	73
15	28	33	37	42	47	51	56	61	66	70	75	80
16	31	36	41	46	51	56	61	66	71	76	82	87
17	33	38	44	49	55	60	66	71	77	82	88	93
18	36	41	47	53	59	65	70	76	82	88	94	100
19	38	44	50	56	63	69	75	82	88	94	101	107
20	40	47	53	60	67	73	80	87	93	100	107	114

Appendix G. *U* Statistic (*cont.*)

(*c*) Critical Values of *U* for a One-Tailed Test at .025 or for a Two-Tailed Test at .05

n_2 n_1	9	10	11	12	13	14	15	16	17	18	19	20
1												
2	0	0	1	1	1	1	1	1	2	2	2	2
3	2	3	3	4	4	5	5	6	6	7	7	8
4	4	5	6	7	8	9	10	11	11	12	13	13
5	7	8	9	11	12	13	14	15	17	18	19	20
6	10	11	13	14	16	17	19	21	22	24	25	27
7	12	14	16	18	20	22	24	26	28	30	32	34
8	15	17	19	22	24	26	29	31	34	36	38	41
9	17	20	23	26	28	31	34	37	39	42	45	48
10	20	23	26	29	33	36	39	42	45	48	52	55
11	23	26	30	33	37	40	44	47	51	55	58	62
12	26	29	33	37	41	45	49	53	57	61	65	69
13	28	33	37	41	45	50	54	59	63	67	72	76
14	31	36	40	45	50	55	59	64	67	74	78	83
15	34	39	44	49	54	59	64	70	75	80	85	90
16	37	42	47	53	59	64	70	75	81	86	92	98
17	39	45	51	57	63	67	75	81	87	93	99	105
18	42	48	55	61	67	74	80	86	93	99	106	112
19	45	52	58	65	72	78	85	92	99	106	113	119
20	48	55	62	69	76	83	90	98	105	112	119	127

Appendix G. *U* Statistic (*cont.*)

(*d*) Critical Values of *U* for a One-Tailed Test at .05 or for a Two-Tailed Test at .10

n_2 / n_1	9	10	11	12	13	14	15	16	17	18	19	20
1											0	0
2	1	1	1	2	2	2	3	3	3	4	4	4
3	3	4	5	5	6	7	7	8	9	9	10	11
4	6	7	8	9	10	11	12	14	15	16	17	18
5	9	11	12	13	15	16	18	19	20	22	23	25
6	12	14	16	17	19	21	23	25	26	28	30	32
7	15	17	19	21	24	26	28	30	33	35	37	39
8	18	20	23	26	28	31	33	36	39	41	44	47
9	21	24	27	30	33	36	39	42	45	48	51	54
10	24	27	31	34	37	41	44	48	51	55	58	62
11	27	31	34	38	42	46	50	54	57	61	65	69
12	30	34	38	42	47	51	55	60	64	68	72	77
13	33	37	42	47	51	56	61	65	70	75	80	84
14	36	41	46	51	56	61	66	71	77	82	87	92
15	39	44	50	55	61	66	72	77	83	88	94	100
16	42	48	54	60	65	71	77	83	89	95	101	107
17	45	51	57	64	70	77	83	89	96	102	109	115
18	48	55	61	68	75	82	88	95	102	109	116	123
19	51	58	65	72	80	87	94	101	109	116	123	130
20	54	62	69	77	84	92	100	107	115	123	130	138

APPENDIX H

Critical Values for Hartley's Maximum *F*-Ratio Significance Test for Homogeneity of Variances

Since the sample size in all groups is the same, the *df* column represents the value for each group. To use this table, simply locate the appropriate *df* row heading and the column headed by the number of variances that are being compared; then read the value from the point at which the row and column intersect. For example, assume that the *df* of each sample is 20, that six variances were compared, and that the $F_{\max}$ ratio is 4.2. Entering the table at the row headed by 20 *df* and reading across to the $k = 6$ column, it is noted that the .05 value is 3.76 and the .01 value is 4.9. Therefore, the $F_{\max} = 4.2$ is considered significant at the .05 level.

Appendix H. $F_{\max}$ Statistic

Alpha = .05 and *.01* (in italics)

df \ *k*	2	3	4	5	6	7	8	9	10	11	12
2	39.0 *199.*	87.5 *448.*	142. *729.*	202. *1036.*	266. *1362.*	333. *1705.*	403. *2063.*	475. *2432.*	550. *2813.*	626. *3204.*	704. *3605.*
3	15.4 *47.5*	27.8 *85.*	39.2 *120.*	50.7 *151.*	62.0 *184.*	72.9 *216.**	83.5 *249.**	93.9 *281.**	104. *310.**	114. *337.**	124. *361.**
4	9.60 *23.2*	15.5 *37.*	20.6 *49.*	25.2 *59.*	29.5 *69.*	33.6 *79.*	37.5 *89.*	41.1 *97.*	44.6 *106.*	48.0 *113.*	51.4 *120.*
5	7.15 *14.9*	10.8 *22.*	13.7 *28.*	16.3 *33.*	18.7 *38.*	20.8 *42.*	22.9 *46.*	24.7 *50.*	26.5 *54.*	28.2 *57.*	29.9 *60.*
6	5.82 *11.1*	8.38 *15.5*	10.4 *19.1*	12.1 *22.*	13.7 *25.*	15.0 *27.*	16.3 *30.*	17.5 *32.*	18.6 *34.*	19.7 *36.*	20.7 *37.*
7	4.99 *8.89*	6.94 *12.1*	8.44 *14.5*	9.70 *16.5*	10.8 *18.4*	11.8 *20.*	12.7 *22.*	13.5 *23.*	14.3 *24.*	15.1 *26.*	15.8 *27.*
8	4.43 *7.50*	6.00 *9.9*	7.18 *11.7*	8.12 *13.2*	9.03 *14.5*	9.78 *15.8*	10.5 *16.9*	11.1 *17.9*	11.7 *18.9*	12.2 *19.8*	12.7 *21.*
9	4.03 *6.54*	5.34 *8.5*	6.31 *9.9*	7.11 *11.1*	7.80 *12.1*	8.41 *13.1*	8.95 *13.9*	9.45 *14.7*	9.91 *15.3*	10.3 *16.0*	10.7 *16.6*
10	3.72 *5.85*	4.85 *7.4*	5.67 *8.6*	6.34 *9.6*	6.92 *10.4*	7.42 *11.1*	7.87 *11.8*	8.28 *12.4*	8.66 *12.9*	9.01 *13.4*	9.34 *13.9*
12	3.28 *4.91*	4.16 *6.1*	4.79 *6.9*	5.30 *7.6*	5.72 *8.2*	6.09 *8.7*	6.42 *9.1*	6.72 *9.5*	7.00 *9.9*	7.25 *10.2*	7.48 *10.6*
15	2.86 *4.07*	3.54 *4.9*	4.01 *5.5*	4.37 *6.0*	4.68 *6.4*	4.95 *6.7*	5.19 *7.1*	5.40 *7.3*	5.59 *7.5*	5.77 *7.8*	5.93 *8.0*
20	2.46 *3.32*	2.95 *3.8*	3.29 *4.3*	3.54 *4.6*	3.76 *4.9*	3.94 *5.1*	4.10 *5.3*	4.24 *5.5*	4.37 *5.6*	4.49 *5.8*	4.59 *5.9*
30	2.07 *2.63*	2.40 *3.0*	2.61 *3.3*	2.78 *3.4*	2.91 *3.6*	3.02 *3.7*	3.12 *3.8*	3.21 *3.9*	3.29 *4.0*	3.36 *4.1*	3.39 *4.2*
60	1.67 *1.96*	1.85 *2.2*	1.96 *2.3*	2.04 *2.4*	2.11 *2.4*	2.17 *2.5*	2.22 *2.5*	2.26 *2.6*	2.30 *2.6*	2.33 *2.7*	2.36 *2.7*
∞	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*	1.00 *1.00*

* Values in the column $k = 2$ and in the rows $df = 2$ and ∞ are exact. Elsewhere the third digit may be in error by a few units for F_{95} and several units for $F_{.99}$. The third digit figures of values marked by an asterisk are the most uncertain.

SOURCE: Reproduced from Table 31 of E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 1, 2nd ed., 1958, published by the Syndics of the Cambridge University Press, London; used by permission of the authors and publishers.

APPENDIX I

Significant Studentized Ranges for Duncan's New Multiple-Range Test

To use these tables, the *df* and the range of the pairs of groups being compared must be located in the particular subtable which lists the level of significance desired. Assume that you wish to find the critical value using a two-tailed test at the .05 level of significance. To do this, first locate the *df* and *k* values in subtable "b," for alpha equal to .05. The point at which the *df* row and the *k* column intersect represents the minimum critical value for significance at the .05 level. If, for example, the *df* were equal to 15, the range of the pair being compared equal to 6, and the computed critical value equal to 4.53, it would be concluded that the difference between the groups was significant—i.e., the critical value for $df = 15$ and $k = 6$ is 3.356; 4.53 is larger and, therefore, significant beyond the .05 level.

Appendix I. Duncan's Multiple Range

(*a*) Alpha = .10*

k *df*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	4.130																	
3	3.328	3.330																
4	3.015	3.074	3.081															
5	2.850	2.934	2.964	2.970														
6	2.748	2.846	2.890	2.908	2.911													
7	2.680	2.785	2.838	2.864	2.876	2.878												
8	2.630	2.742	2.800	2.832	2.849	2.857	2.858											
9	2.592	2.708	2.771	2.808	2.829	2.840	2.845	2.847										
10	2.563	2.682	2.748	2.788	2.813	2.827	2.835	2.839	2.839									
11	2.540	2.660	2.730	2.772	2.799	2.817	2.827	2.833	2.835	2.835								
12	2.521	2.643	2.714	2.759	2.789	2.808	2.821	2.828	2.832	2.833	2.833							
13	2.505	2.628	2.701	2.748	2.779	2.800	2.815	2.824	2.829	2.832	2.832	2.832						
14	2.491	2.616	2.690	2.739	2.771	2.794	2.810	2.820	2.827	2.831	2.832	2.833	2.833					
15	2.479	2.605	2.681	2.731	2.765	2.789	2.805	2.817	2.825	2.830	2.833	2.834	2.834	2.834				
16	2.469	2.596	2.673	2.723	2.759	2.784	2.802	2.815	2.824	2.829	2.833	2.835	2.836	2.836	2.836			
17	2.460	2.588	2.665	2.717	2.753	2.780	2.798	2.812	2.822	2.829	2.834	2.836	2.838	2.838	2.838	2.838		
18	2.452	2.580	2.659	2.712	2.749	2.776	2.796	2.810	2.821	2.828	2.834	2.838	2.840	2.840	2.840	2.840	2.840	
19	2.445	2.574	2.653	2.707	2.745	2.773	2.793	2.808	2.820	2.828	2.834	2.839	2.841	2.842	2.843	2.843	2.843	2.843
20	2.439	2.568	2.648	2.702	2.741	2.770	2.791	2.807	2.819	2.828	2.834	2.839	2.843	2.845	2.845	2.845	2.845	2.845
24	2.420	2.550	2.632	2.688	2.729	2.760	2.783	2.801	2.816	2.827	2.835	2.842	2.848	2.851	2.854	2.856	2.857	2.857
30	2.400	2.532	2.615	2.674	2.717	2.750	2.776	2.796	2.813	2.826	2.837	2.846	2.853	2.859	2.863	2.867	2.869	2.871
40	2.381	2.514	2.600	2.660	2.705	2.741	2.769	2.791	2.810	2.825	2.838	2.849	2.858	2.866	2.873	2.878	2.883	2.887
60	2.363	2.497	2.584	2.646	2.694	2.731	2.761	2.786	2.807	2.825	2.839	2.853	2.864	2.874	2.883	2.890	2.897	2.903
120	2.344	2.479	2.568	2.632	2.682	2.722	2.754	2.781	2.804	2.824	2.842	2.857	2.871	2.883	2.893	2.903	2.912	2.920
∞	2.326	2.462	2.552	2.619	2.670	2.712	2.746	2.776	2.801	2.824	2.844	2.861	2.877	2.892	2.905	2.918	2.929	2.939

SOURCE: The entries in this table were tabulated and made available by H. Leon Harter of the Aerospace Research Laboratories, Office of Aerospace Research, U.S. Air Force.

Appendix I. Duncan's Multiple Range (*cont.*)

(*b*) Alpha = .05*

k / *df*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	6.085																	
3	4.501	4.516																
4	3.927	4.013	4.033															
5	3.635	3.749	3.797	3.814														
6	3.461	3.587	3.649	3.680	3.694													
7	3.344	3.477	3.548	3.588	3.611	3.622												
8	3.261	3.399	3.475	3.521	3.549	3.566	3.575											
9	3.199	3.339	3.420	3.470	3.502	3.523	3.536	3.544										
10	3.151	3.293	3.376	3.430	3.465	3.489	3.505	3.516	3.522									
11	3.113	3.256	3.342	3.397	3.435	3.462	3.480	3.493	3.501	3.506								
12	3.082	3.225	3.313	3.370	3.410	3.439	3.459	3.474	3.484	3.491	3.496							
13	3.055	3.200	3.289	3.348	3.389	3.419	3.442	3.458	3.470	3.478	3.484	3.488						
14	3.033	3.178	3.268	3.329	3.372	3.403	3.426	3.444	3.457	3.467	3.474	3.479	3.482					
15	3.014	3.160	3.250	3.312	3.356	3.389	3.413	3.432	3.446	3.457	3.465	3.471	3.476	3.478				
16	2.998	3.144	3.235	3.298	3.343	3.376	3.402	3.422	3.437	3.449	3.458	3.465	3.470	3.473	3.477			
17	2.984	3.130	3.222	3.285	3.331	3.366	3.392	3.412	3.429	3.441	3.451	3.459	3.465	3.469	3.473	3.475		
18	2.971	3.118	3.210	3.274	3.321	3.356	3.383	3.405	3.421	3.435	3.445	3.454	3.460	3.465	3.470	3.472	3.474	
19	2.960	3.107	3.199	3.264	3.311	3.347	3.375	3.397	3.415	3.429	3.440	3.449	3.456	3.462	3.467	3.470	3.472	3.473
20	2.950	3.097	3.190	3.255	3.303	3.339	3.368	3.391	3.409	3.424	3.436	3.445	3.453	3.459	3.464	3.467	3.470	3.472
24	2.919	3.066	3.160	3.226	3.276	3.315	3.345	3.370	3.390	3.406	3.420	3.432	3.441	3.449	3.456	3.461	3.465	3.469
30	2.888	3.035	3.131	3.199	3.250	3.290	3.322	3.349	3.371	3.389	3.405	3.418	3.430	3.439	3.447	3.454	3.460	3.466
40	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	3.373	3.390	3.405	3.418	3.429	3.439	3.448	3.456	3.463
60	2.829	2.976	3.073	3.143	3.198	3.241	3.277	3.307	3.333	3.355	3.374	3.391	3.406	3.419	3.431	3.442	3.451	3.460
120	2.800	2.947	3.045	3.116	3.172	3.217	3.254	3.287	3.314	3.337	3.359	3.377	3.394	3.409	3.423	3.435	3.446	3.457
∞	2.772	2.918	3.017	3.089	3.146	3.193	3.232	3.265	3.294	3.320	3.343	3.363	3.382	3.399	3.414	3.428	3.442	3.454

Appendix I. Duncan's Multiple Range (*cont.*)

(*c*) Alpha = .01*

k / *df*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	14.04																	
3	8.261	8.321																
4	6.512	6.677	6.740															
5	5.702	5.893	5.989	6.040														
6	5.243	5.439	5.549	5.614	5.655													
7	4.949	5.145	5.260	5.334	5.383	5.416												
8	4.746	4.939	5.057	5.135	5.189	5.227	5.256											
9	4.596	4.787	4.906	4.986	5.043	5.086	5.118	5.142										
10	4.482	4.671	4.790	4.871	4.931	4.975	5.010	5.037	5.058									
11	4.392	4.579	4.697	4.780	4.841	4.887	4.924	4.952	4.975	4.994								
12	4.320	4.504	4.622	4.706	4.767	4.815	4.852	4.883	4.907	4.927	4.944							
13	4.260	4.442	4.560	4.644	4.706	4.755	4.793	4.824	4.850	4.872	4.889	4.904						
14	4.210	4.391	4.508	4.591	4.654	4.704	4.743	4.775	4.802	4.824	4.843	4.859	4.872					
15	4.168	4.347	4.463	4.547	4.610	4.660	4.700	4.733	4.760	4.783	4.803	4.820	4.834	4.846				
16	4.131	4.309	4.425	4.509	4.572	4.622	4.663	4.696	4.724	4.748	4.768	4.786	4.800	4.813	4.825			
17	4.099	4.275	4.391	4.475	4.539	4.589	4.630	4.664	4.693	4.717	4.738	4.756	4.771	4.785	4.797	4.807		
18	4.071	4.246	4.362	4.445	4.509	4.560	4.601	4.635	4.664	4.689	4.711	4.729	4.745	4.759	4.772	4.783	4.792	
19	4.046	4.220	4.335	4.419	4.483	4.534	4.575	4.610	4.639	4.665	4.686	4.705	4.722	4.736	4.749	4.761	4.771	4.780
20	4.024	4.197	4.312	4.395	4.459	4.510	4.552	4.587	4.617	4.642	4.664	4.684	4.701	4.716	4.729	4.741	4.751	4.761
24	3.956	4.126	4.239	4.322	4.386	4.437	4.480	4.516	4.546	4.573	4.596	4.616	4.634	4.651	4.665	4.678	4.690	4.700
30	3.889	4.056	4.168	4.250	4.314	4.366	4.409	4.445	4.477	4.504	4.528	4.550	4.569	4.586	4.601	4.615	4.628	4.640
40	3.825	3.988	4.098	4.180	4.244	4.296	4.339	4.376	4.408	4.436	4.461	4.483	4.503	4.521	4.537	4.553	4.566	4.579
60	3.762	3.922	4.031	4.111	4.174	4.226	4.270	4.307	4.340	4.368	4.394	4.417	4.438	4.456	4.474	4.490	4.504	4.518
120	3.702	3.858	3.965	4.044	4.107	4.158	4.202	4.239	4.272	4.301	4.327	4.351	4.372	4.392	4.410	4.426	4.442	4.456
∞	3.643	3.796	3.900	3.978	4.040	4.091	4.135	4.172	4.205	4.235	4.261	4.285	4.307	4.327	4.345	4.363	4.379	4.394

Appendix I. Duncan's Multiple Range (*cont.*)

(*d*) Alpha = .005*

k / *df*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	19.93																	
3	10.55	10.63																
4	7.916	8.126	8.210															
5	6.751	6.980	7.100	7.167														
6	6.105	6.334	6.466	6.547	6.600													
7	5.699	5.922	6.057	6.145	6.207	6.250												
8	5.420	5.638	5.773	5.864	5.930	5.978	6.014											
9	5.218	5.430	5.565	5.657	5.725	5.776	5.815	5.846										
10	5.065	5.273	5.405	5.498	5.567	5.620	5.662	5.695	5.722									
11	4.945	5.149	5.280	5.372	5.442	5.496	5.539	5.574	5.603	5.626								
12	4.849	5.048	5.178	5.270	5.341	5.396	5.439	5.475	5.505	5.531	5.552							
13	4.770	4.966	5.094	5.186	5.256	5.312	5.356	5.393	5.424	5.450	5.472	5.492						
14	4.704	4.897	5.023	5.116	5.185	5.241	5.286	5.324	5.355	5.382	5.405	5.425	5.442					
15	4.647	4.838	4.964	5.055	5.125	5.181	5.226	5.264	5.297	5.324	5.348	5.368	5.386	5.402				
16	4.599	4.787	4.912	5.003	5.073	5.129	5.175	5.213	5.245	5.273	5.298	5.319	5.338	5.354	5.368			
17	4.557	4.744	4.867	4.958	5.027	5.084	5.130	5.168	5.201	5.229	5.254	5.275	5.295	5.311	5.327	5.340		
18	4.521	4.705	4.828	4.918	4.987	5.043	5.090	5.129	5.162	5.190	5.215	5.237	5.256	5.274	5.289	5.303	5.316	
19	4.488	4.671	4.793	4.883	4.952	5.008	5.054	5.093	5.127	5.156	5.181	5.203	5.222	5.240	5.256	5.270	5.283	5.295
20	4.460	4.641	4.762	4.851	4.920	4.976	5.022	5.061	5.095	5.124	5.150	5.172	5.193	5.210	5.226	5.241	5.254	5.266
24	4.371	4.547	4.666	4.753	4.822	4.877	4.924	4.963	4.997	5.027	5.053	5.076	5.097	5.116	5.133	5.148	5.162	5.175
30	4.285	4.456	4.572	4.658	4.726	4.781	4.827	4.867	4.901	4.931	4.958	4.981	5.003	5.022	5.040	5.056	5.071	5.085
40	4.202	4.369	4.482	4.566	4.632	4.687	4.733	4.772	4.806	4.837	4.864	4.888	4.910	4.930	4.948	4.965	4.980	4.995
60	4.122	4.284	4.394	4.476	4.541	4.595	4.640	4.679	4.713	4.744	4.771	4.796	4.818	4.838	4.857	4.874	4.890	4.905
120	4.045	4.201	4.308	4.388	4.452	4.505	4.550	4.588	4.622	4.652	4.679	4.704	4.726	4.747	4.766	4.784	4.800	4.815
∞	3.970	4.121	4.225	4.303	4.365	4.417	4.461	4.499	4.532	4.562	4.589	4.614	4.636	4.657	4.676	4.694	4.710	4.726

Appendix I. Duncan's Multiple Range (*cont.*)

(*e*) Alpha = .001*

df \ *k*	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	44.69																	
3	18.28	18.45																
4	12.18	12.52	12.67															
5	9.714	10.05	10.24	10.35														
6	8.427	8.743	8.932	9.055	9.139													
7	7.648	7.943	8.127	8.252	8.342	8.409												
8	7.130	7.407	7.584	7.708	7.799	7.869	7.924											
9	6.762	7.024	7.195	7.316	7.407	7.478	7.535	7.582										
10	6.487	6.738	6.902	7.021	7.111	7.182	7.240	7.287	7.327									
11	6.275	6.516	6.676	6.791	6.880	6.950	7.008	7.056	7.097	7.132								
12	6.106	6.340	6.494	6.607	6.695	6.765	6.822	6.870	6.911	6.947	6.978							
13	5.970	6.195	6.346	6.457	6.543	6.612	6.670	6.718	6.759	6.795	6.826	6.854						
14	5.856	6.075	6.223	6.332	6.416	6.485	6.542	6.590	6.631	6.667	6.699	6.727	6.752					
15	5.760	5.974	6.119	6.225	6.309	6.377	6.433	6.481	6.522	6.558	6.590	6.619	6.644	6.666				
16	5.678	5.888	6.030	6.135	6.217	6.284	6.340	6.388	6.429	6.465	6.497	6.525	6.551	6.574	6.595			
17	5.608	5.813	5.953	6.056	6.138	6.204	6.260	6.307	6.348	6.384	6.416	6.444	6.470	6.493	6.514	6.533		
18	5.546	5.748	5.886	5.988	6.068	6.134	6.189	6.236	6.277	6.313	6.345	6.373	6.399	6.422	6.443	6.462	6.480	
19	5.492	5.691	5.826	5.927	6.007	6.072	6.127	6.174	6.214	6.250	6.281	6.310	6.336	6.359	6.380	6.400	6.418	6.434
20	5.444	5.640	5.774	5.873	5.952	6.017	6.071	6.117	6.158	6.193	6.225	6.254	6.279	6.303	6.324	6.344	6.362	6.379
24	5.297	5.484	5.612	5.708	5.784	5.846	5.899	5.945	5.984	6.020	6.051	6.079	6.105	6.129	6.150	6.170	6.188	6.205
30	5.156	5.335	5.457	5.549	5.622	5.682	5.734	5.778	5.817	5.851	5.882	5.910	5.935	5.958	5.980	6.000	6.018	6.036
40	5.022	5.191	5.308	5.396	5.466	5.524	5.574	5.617	5.654	5.688	5.718	5.745	5.770	5.793	5.814	5.834	5.852	5.869
60	4.894	5.055	5.166	5.249	5.317	5.372	5.420	5.461	5.498	5.530	5.559	5.586	5.610	5.632	5.653	5.672	5.690	5.707
120	4.771	4.924	5.029	5.109	5.173	5.226	5.271	5.311	5.346	5.377	5.405	5.431	5.454	5.476	5.496	5.515	5.532	5.549
∞	4.654	4.798	4.898	4.974	5.034	5.085	5.128	5.166	5.199	5.229	5.256	5.280	5.303	5.324	5.343	5.361	5.378	5.394

APPENDIX J

Critical Values of Wilcoxon's T Statistic for the Matched-Pairs Signed-Ranks Test

To use this table, first locate the number of *pairs* of scores in the n' column. The critical values for the several levels of significance are listed in the columns to the right. For example, if n' were 20 and the computed value 25, it would be concluded that since 25 is less than 38, this value is significant beyond the .01 level of significance for a two-tailed test.

n'	Level of significance for one-tailed test		
	.025	.01	.005
	Level of significance for two-tailed test		
	.05	.02	.01
6	1	—	—
7	2	0	—
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	19
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	37
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	90	77	68

Note that n' is the number of matched pairs.

SOURCE: Adapted from Table I of F. Wilcoxon, *Some Rapid Approximate Statistical Procedures*, rev. ed. New York: American Cyanamid Company, 1964. Reproduced from S. Siegel, *Nonparametric Statistics for the Behavioral Sciences*. New York: McGraw-Hill Book Company, 1956. Reprinted by permission of the American Cyanamid Company and McGraw-Hill Book Company.

APPENDIX K

Coefficients for Orthogonal Polynomials

In this table, the column headed by k indicates the number of components being compared. The columns headed by the ten X values indicate the actual coefficients to be used. Thus, if $k = 3$ groups were being analyzed, the coefficients for analysis of the linear component would be −1 for Group 1, 0 for Group 2, and 1 for Group 3. The quadratic coefficients would be 1, −2, and 1 for the three respective groups.

k	Polynomial	$X = 1$	2	3	4	5	6	7	8	9	10
3	Linear	−1	0	1							
	Quadratic	1	−2	1							
4	Linear	−3	−1	1	3						
	Quadratic	1	−1	−1	1						
	Cubic	−1	3	−3	1						
5	Linear	−2	−1	0	1	2					
	Quadratic	2	−1	−2	−1	2					
	Cubic	−1	2	0	−2	1					
	Quartic	1	−4	6	−4	1					
6	Linear	−5	−3	−1	1	3	5				
	Quadratic	5	−1	−4	−4	−1	5				
	Cubic	−5	7	4	−4	−7	5				
	Quartic	1	−3	2	2	−3	1				
7	Linear	−3	−2	−1	0	1	2	3			
	Quadratic	5	0	−3	−4	−3	0	5			
	Cubic	−1	1	1	0	−1	−1	1			
	Quartic	3	−7	1	6	1	−7	3			
8	Linear	−7	−5	−3	−1	1	3	5	7		
	Quadratic	7	1	−3	−5	−5	−3	1	7		
	Cubic	−7	5	7	3	−3	−7	−5	7		
	Quartic	7	−13	−3	9	9	−3	−13	7		
	Quintic	−7	23	−17	−15	15	17	−23	7		
9	Linear	−4	−3	−2	−1	0	1	2	3	4	
	Quadratic	28	7	−8	−17	−20	−17	−8	7	28	
	Cubic	−14	7	13	9	0	−9	−13	−7	14	
	Quartic	14	−21	−11	9	18	9	−11	−21	14	
	Quintic	−4	11	−4	−9	0	9	4	−11	4	
10	Linear	−9	−7	−5	−3	−1	1	3	5	7	9
	Quadratic	6	2	−1	−3	−4	−4	−3	−1	2	6
	Cubic	−42	14	35	31	12	−12	−31	−35	−14	42
	Quartic	18	−22	−17	3	18	18	3	−17	−22	18
	Quintic	−6	14	−1	−11	−6	6	11	1	−14	6

SOURCE: Abridged from Table B.10 of *Statistical Principles in Experimental Design* by B. J. Winer, Copyright 1962 by McGraw-Hill Book Company. Used by permission of McGraw-Hill Book Company.

APPENDIX L

Squares and Square Roots

This table lists values of $n = 1.00$ to 10.00 in the first column, the squares of n in the second column, and the square roots of n in the third column. The fourth column gives the square roots of $10n$, or the square roots of values from 10 (10×1.00) to 100 (10×10.00). The table can also be used to find the squares and square roots of values that have the same sequence of digits as n. In each of these operations, the decimal points in the values of n, n^2, and $\sqrt{n}$ must be adjusted accordingly.

Appendix L. Squares and Square Roots

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
1.00	1.0000	1.00000	3.16228
1.01	1.0201	1.00499	3.17805
1.02	1.0404	1.00995	3.19374
1.03	1.0609	1.01489	3.20936
1.04	1.0816	1.01980	3.22490
1.05	1.1025	1.02470	3.24037
1.06	1.1236	1.02956	3.25576
1.07	1.1449	1.03441	3.27109
1.08	1.1664	1.03923	3.28634
1.09	1.1881	1.04403	3.30151
1.10	1.2100	1.04881	3.31662
1.11	1.2321	1.05357	3.33167
1.12	1.2544	1.05830	3.34664
1.13	1.2769	1.06301	3.36155
1.14	1.2996	1.06771	3.37639
1.15	1.3225	1.07238	3.39116
1.16	1.3456	1.07703	3.40588
1.17	1.3689	1.08167	3.42053
1.18	1.3924	1.08628	3.43511
1.19	1.4161	1.09087	3.44964
1.20	1.4400	1.09545	3.46410
1.21	1.4641	1.10000	3.47851
1.22	1.4884	1.10454	3.49285
1.23	1.5129	1.10905	3.50714
1.24	1.5376	1.11355	3.52136
1.25	1.5625	1.11803	3.53553
1.26	1.5876	1.12250	3.54965
1.27	1.6129	1.12694	3.56371
1.28	1.6384	1.13137	3.57771
1.29	1.6641	1.13578	3.59166
1.30	1.6900	1.14018	3.60555
1.31	1.7161	1.14455	3.61939
1.32	1.7424	1.14891	3.63318
1.33	1.7689	1.15326	3.64692
1.34	1.7956	1.15758	3.66060
1.35	1.8225	1.16190	3.67423
1.36	1.8496	1.16619	3.68782
1.37	1.8769	1.17047	3.70135
1.38	1.9044	1.17473	3.71484
1.39	1.9321	1.17898	3.72827
1.40	1.9600	1.18322	3.74166
1.41	1.9881	1.18743	3.75500
1.42	2.0164	1.19164	3.76829
1.43	2.0449	1.19583	3.78153
1.44	2.0736	1.20000	3.79473
1.45	2.1025	1.20416	3.80789
1.46	2.1316	1.20830	3.82099
1.47	2.1609	1.21244	3.83406
1.48	2.1904	1.21655	3.84708
1.49	2.2201	1.22066	3.86005
1.50	2.2500	1.22474	3.87298
1.51	2.2801	1.22882	3.88587
1.52	2.3104	1.23288	3.89872
1.53	2.3409	1.23693	3.91152
1.54	2.3716	1.24097	3.92428
1.55	2.4025	1.24499	3.93700
1.56	2.4336	1.24900	3.94968
1.57	2.4649	1.25300	3.96232
1.58	2.4964	1.25698	3.97492
1.59	2.5281	1.26095	3.98748
1.60	2.5600	1.26491	4.00000
1.61	2.5921	1.26886	4.01248
1.62	2.6244	1.27279	4.02492
1.63	2.6569	1.27671	4.03733
1.64	2.6896	1.28062	4.04969
1.65	2.7225	1.28452	4.06202
1.66	2.7556	1.28841	4.07431
1.67	2.7889	1.29228	4.08656
1.68	2.8224	1.29615	4.09878
1.69	2.8561	1.30000	4.11096
1.70	2.8900	1.30384	4.12311
1.71	2.9241	1.30767	4.13521
1.72	2.9584	1.31149	4.14729
1.73	2.9929	1.31529	4.15933
1.74	3.0276	1.31909	4.17133
1.75	3.0625	1.32288	4.18330
1.76	3.0976	1.32665	4.19524
1.77	3.1329	1.33041	4.20714
1.78	3.1684	1.33417	4.21900
1.79	3.2041	1.33791	4.23084
1.80	3.2400	1.34164	4.24264
1.81	3.2761	1.34536	4.25441
1.82	3.3124	1.34907	4.26615
1.83	3.3489	1.35277	4.27785
1.84	3.3856	1.35647	4.28952
1.85	3.4225	1.36015	4.30116
1.86	3.4596	1.36382	4.31277
1.87	3.4969	1.36748	4.32435
1.88	3.5344	1.37113	4.33590
1.89	3.5721	1.37477	4.34741
1.90	3.6100	1.37840	4.35890
1.91	3.6481	1.38203	4.37035
1.92	3.6864	1.38564	4.38178
1.93	3.7249	1.38924	4.39318
1.94	3.7636	1.39284	4.40454
1.95	3.8025	1.39642	4.41588
1.96	3.8416	1.40000	4.42719
1.97	3.8809	1.40357	4.43847
1.98	3.9204	1.40712	4.44972
1.99	3.9601	1.41067	4.46094

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
2.00	4.0000	1.41421	4.47214
2.01	4.0401	1.41774	4.48330
2.02	4.0804	1.42127	4.49444
2.03	4.1209	1.42478	4.50555
2.04	4.1616	1.42829	4.51664
2.05	4.2025	1.43178	4.52769
2.06	4.2436	1.43527	4.53872
2.07	4.2849	1.43875	4.54973
2.08	4.3264	1.44222	4.56070
2.09	4.3681	1.44568	4.57165
2.10	4.4100	1.44914	4.58258
2.11	4.4521	1.45258	4.59347
2.12	4.4944	1.45602	4.60435
2.13	4.5369	1.45945	4.61519
2.14	4.5796	1.46287	4.62601
2.15	4.6225	1.46629	4.63681
2.16	4.6656	1.46969	4.64758
2.17	4.7089	1.47309	4.65833
2.18	4.7524	1.47648	4.66905
2.19	4.7961	1.47986	4.67974
2.20	4.8400	1.48324	4.69042
2.21	4.8841	1.48661	4.70106
2.22	4.9284	1.48997	4.71169
2.23	4.9729	1.49332	4.72229
2.24	5.0176	1.49666	4.73286
2.25	5.0625	1.50000	4.74342
2.26	5.1076	1.50333	4.75395
2.27	5.1529	1.50665	4.76445
2.28	5.1984	1.50997	4.77493
2.29	5.2441	1.51327	4.78539
2.30	5.2900	1.51658	4.79583
2.31	5.3361	1.51987	4.80625
2.32	5.3824	1.52315	4.81664
2.33	5.4289	1.52643	4.82701
2.34	5.4756	1.52971	4.83735
2.35	5.5225	1.53297	4.84768
2.36	5.5696	1.53623	4.85798
2.37	5.6169	1.53948	4.86826
2.38	5.6644	1.54272	4.87852
2.39	5.7121	1.54596	4.88876
2.40	5.7600	1.54919	4.89898
2.41	5.8081	1.55242	4.90918
2.42	5.8564	1.55563	4.91935
2.43	5.9049	1.55885	4.92950
2.44	5.9536	1.56205	4.93964
2.45	6.0025	1.56525	4.94975
2.46	6.0516	1.56844	4.95984
2.47	6.1009	1.57162	4.96991
2.48	6.1504	1.57480	4.97996
2.49	6.2001	1.57797	4.98999
2.50	6.2500	1.58114	5.00000
2.51	6.3001	1.58430	5.00999
2.52	6.3504	1.58745	5.01996
2.53	6.4009	1.59060	5.02991
2.54	6.4516	1.59374	5.03984
2.55	6.5025	1.59687	5.04975
2.56	6.5536	1.60000	5.05964
2.57	6.6049	1.60312	5.06952
2.58	6.6564	1.60624	5.07937
2.59	6.7081	1.60935	5.08920
2.60	6.7600	1.61245	5.09902
2.61	6.8121	1.61555	5.10882
2.62	6.8644	1.61864	5.11859
2.63	6.9169	1.62173	5.12835
2.64	6.9696	1.62481	5.13809
2.65	7.0225	1.62788	5.14782
2.66	7.0756	1.63095	5.15752
2.67	7.1289	1.63401	5.16720
2.68	7.1824	1.63707	5.17687
2.69	7.2361	1.64012	5.18652
2.70	7.2900	1.64317	5.19615
2.71	7.3441	1.64621	5.20577
2.72	7.3984	1.64924	5.21536
2.73	7.4529	1.65227	5.22494
2.74	7.5076	1.65529	5.23450
2.75	7.5625	1.65831	5.24404
2.76	7.6176	1.66132	5.25357
2.77	7.6729	1.66433	5.26308
2.78	7.7284	1.66733	5.27257
2.79	7.7841	1.67033	5.28205
2.80	7.8400	1.67332	5.29150
2.81	7.8961	1.67631	5.30094
2.82	7.9524	1.67929	5.31037
2.83	8.0089	1.68226	5.31977
2.84	8.0656	1.68523	5.32917
2.85	8.1225	1.68819	5.33854
2.86	8.1796	1.69115	5.34790
2.87	8.2369	1.69411	5.35724
2.88	8.2944	1.69706	5.36656
2.89	8.3521	1.70000	5.37587
2.90	8.4100	1.70294	5.38516
2.91	8.4681	1.70587	5.39444
2.92	8.5264	1.70880	5.40370
2.93	8.5849	1.71172	5.41295
2.94	8.6436	1.71464	5.42218
2.95	8.7025	1.71756	5.43139
2.96	8.7616	1.72047	5.44059
2.97	8.8209	1.72337	5.44977
2.98	8.8804	1.72627	5.45894
2.99	8.9401	1.72916	5.46809

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
3.00	9.0000	1.73205	5.47723
3.01	9.0601	1.73494	5.48635
3.02	9.1204	1.73781	5.49545
3.03	9.1809	1.74069	5.50454
3.04	9.2416	1.74356	5.51362
3.05	9.3025	1.74642	5.52268
3.06	9.3636	1.74929	5.53173
3.07	9.4249	1.75214	5.54076
3.08	9.4864	1.75499	5.54977
3.09	9.5481	1.75784	5.55878
3.10	9.6100	1.76068	5.56776
3.11	9.6721	1.76352	5.57674
3.12	9.7344	1.76635	5.58570
3.13	9.7969	1.76918	5.59464
3.14	9.8596	1.77200	5.60357
3.15	9.9225	1.77482	5.61249
3.16	9.9856	1.77764	5.62139
3.17	10.0489	1.78045	5.63028
3.18	10.1124	1.78326	5.63915
3.19	10.1761	1.78606	5.64801
3.20	10.2400	1.78885	5.65685
3.21	10.3041	1.79165	5.66569
3.22	10.3684	1.79444	5.67450
3.23	10.4329	1.79722	5.68331
3.24	10.4976	1.80000	5.69210
3.25	10.5625	1.80278	5.70088
3.26	10.6276	1.80555	5.70964
3.27	10.6929	1.80831	5.71839
3.28	10.7584	1.81108	5.72713
3.29	10.8241	1.81384	5.73585
3.30	10.8900	1.81659	5.74456
3.31	10.9561	1.81934	5.75326
3.32	11.0224	1.82209	5.76194
3.33	11.0889	1.82483	5.77062
3.34	11.1556	1.82757	5.77927
3.35	11.2225	1.83030	5.78792
3.36	11.2896	1.83303	5.79655
3.37	11.3569	1.83576	5.80517
3.38	11.4244	1.83848	5.81378
3.39	11.4921	1.84120	5.82237
3.40	11.5600	1.84391	5.83095
3.41	11.6281	1.84662	5.83952
3.42	11.6964	1.84932	5.84808
3.43	11.7649	1.85203	5.85662
3.44	11.8336	1.85472	5.86515
3.45	11.9025	1.85742	5.87367
3.46	11.9716	1.86011	5.88218
3.47	12.0409	1.86279	5.89067
3.48	12.1104	1.86548	5.89915
3.49	12.1801	1.86815	5.90762
3.50	12.2500	1.87083	5.91608
3.51	12.3201	1.87350	5.92453
3.52	12.3904	1.87617	5.93296
3.53	12.4609	1.87883	5.94138
3.54	12.5316	1.88149	5.94979
3.55	12.6025	1.88414	5.95819
3.56	12.6736	1.88680	5.96657
3.57	12.7449	1.88944	5.97495
3.58	12.8164	1.89209	5.98331
3.59	12.8881	1.89473	5.99166
3.60	12.9600	1.89737	6.00000
3.61	13.0321	1.90000	6.00833
3.62	13.1044	1.90263	6.01664
3.63	13.1769	1.90526	6.02495
3.64	13.2496	1.90788	6.03324
3.65	13.3225	1.91050	6.04152
3.66	13.3956	1.91311	6.04979
3.67	13.4689	1.91572	6.05805
3.68	13.5424	1.91833	6.06630
3.69	13.6161	1.92094	6.07454
3.70	13.6900	1.92354	6.08276
3.71	13.7641	1.92614	6.09098
3.72	13.8384	1.92873	6.09918
3.73	13.9129	1.93132	6.10737
3.74	13.9876	1.93391	6.11555
3.75	14.0625	1.93649	6.12372
3.76	14.1376	1.93907	6.13188
3.77	14.2129	1.94165	6.14003
3.78	14.2884	1.94422	6.14817
3.79	14.3641	1.94679	6.15630
3.80	14.4400	1.94936	6.16441
3.81	14.5161	1.95192	6.17252
3.82	14.5924	1.95448	6.18061
3.83	14.6689	1.95704	6.18870
3.84	14.7456	1.95959	6.19677
3.85	14.8225	1.96214	6.20484
3.86	14.8996	1.96469	6.21289
3.87	14.9769	1.96723	6.22093
3.88	15.0544	1.96977	6.22896
3.89	15.1321	1.97231	6.23699
3.90	15.2100	1.97484	6.24500
3.91	15.2881	1.97737	6.25300
3.92	15.3664	1.97990	6.26099
3.93	15.4449	1.98242	6.26897
3.94	15.5236	1.98494	6.27694
3.95	15.6025	1.98746	6.28490
3.96	15.6816	1.98997	6.29285
3.97	15.7609	1.99249	6.30079
3.98	15.8408	1.99499	6.30872
3.99	15.9201	1.99750	6.31664

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
4.00	16.0000	2.00000	6.32456
4.01	16.0801	2.00250	6.33246
4.02	16.1604	2.00499	6.34035
4.03	16.2409	2.00749	6.34823
4.04	16.3216	2.00998	6.35610
4.05	16.4025	2.01246	6.36396
4.06	16.4836	2.01494	6.37181
4.07	16.5649	2.01742	6.37966
4.08	16.6464	2.01990	6.38749
4.09	16.7281	2.02237	6.39531
4.10	16.8100	2.02485	6.40312
4.11	16.8921	2.02731	6.41093
4.12	16.9744	2.02978	6.41872
4.13	17.0569	2.03224	6.42651
4.14	17.1396	2.03470	6.43428
4.15	17.2225	2.03715	6.44205
4.16	17.3056	2.03961	6.44981
4.17	17.3889	2.04206	6.45755
4.18	17.4724	2.04450	6.46529
4.19	17.5561	2.04695	6.47302
4.20	17.6400	2.04939	6.48074
4.21	17.7241	2.05183	6.48845
4.22	17.8084	2.05426	6.49615
4.23	17.8929	2.05670	6.50384
4.24	17.9776	2.05913	6.51153
4.25	18.0625	2.06155	6.51920
4.26	18.1476	2.06398	6.52687
4.27	18.2329	2.06640	6.53452
4.28	18.3184	2.06882	6.54217
4.29	18.4041	2.07123	6.54981
4.30	18.4900	2.07364	6.55744
4.31	18.5761	2.07605	6.56506
4.32	18.6624	2.07846	6.57267
4.33	18.7489	2.08087	6.58027
4.34	18.8356	2.08327	6.58787
4.35	18.9225	2.08567	6.59545
4.36	19.0096	2.08806	6.60303
4.37	19.0969	2.09045	6.61060
4.38	19.1844	2.09284	6.61816
4.39	19.2721	2.09523	6.62571
4.40	19.3600	2.09762	6.63325
4.41	19.4481	2.10000	6.64078
4.42	19.5364	2.10238	6.64831
4.43	19.6249	2.10476	6.65582
4.44	19.7136	2.10713	6.66333
4.45	19.8025	2.10950	6.67083
4.46	19.8916	2.11187	6.67832
4.47	19.9809	2.11424	6.68581
4.48	20.0704	2.11660	6.69328
4.49	20.1601	2.11896	6.70075
4.50	20.2500	2.12132	6.70820
4.51	20.3401	2.12368	6.71565
4.52	20.4304	2.12603	6.72309
4.53	20.5209	2.12838	6.73053
4.54	20.6116	2.13073	6.73795
4.55	20.7025	2.13307	6.74537
4.56	20.7936	2.13542	6.75278
4.57	20.8849	2.13776	6.76018
4.58	20.9764	2.14009	6.76757
4.59	21.0681	2.14243	6.77495
4.60	21.1600	2.14476	6.78233
4.61	21.2521	2.14709	6.78970
4.62	21.3444	2.14942	6.79706
4.63	21.4369	2.15174	6.80441
4.64	21.5296	2.15407	6.81175
4.65	21.6225	2.15639	6.81909
4.66	21.7156	2.15870	6.82642
4.67	21.8089	2.16102	6.83374
4.68	21.9024	2.16333	6.84105
4.69	21.9961	2.16564	6.84836
4.70	22.0900	2.16795	6.85565
4.71	22.1841	2.17025	6.86294
4.72	22.2784	2.17256	6.87023
4.73	22.3729	2.17486	6.87750
4.74	22.4676	2.17715	6.88477
4.75	22.5625	2.17945	6.89202
4.76	22.6576	2.18174	6.89928
4.77	22.7529	2.18403	6.90652
4.78	22.8484	2.18632	6.91375
4.79	22.9441	2.18861	6.92098
4.80	23.0400	2.19089	6.92820
4.81	23.1361	2.19317	6.93542
4.82	23.2324	2.19545	6.94262
4.83	23.3289	2.19773	6.94982
4.84	23.4256	2.20000	6.95701
4.85	23.5225	2.20227	6.96419
4.86	23.6196	2.20454	6.97137
4.87	23.7169	2.20681	6.97854
4.88	23.8144	2.20907	6.98570
4.89	23.9121	2.21133	6.99285
4.90	24.0100	2.21359	7.00000
4.91	24.1081	2.21585	7.00714
4.92	24.2064	2.21811	7.01427
4.93	24.3049	2.22036	7.02140
4.94	24.4036	2.22261	7.02851
4.95	24.5025	2.22486	7.03562
4.96	24.6016	2.22711	7.04273
4.97	24.7009	2.22935	7.04982
4.98	24.8004	2.23159	7.05691
4.99	24.9001	2.23383	7.06399

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n	n^2	$\sqrt{n}$	$\sqrt{10n}$
5.00	25.0000	2.23607	7.07107	5.50	30.2500	2.34521	7.41620
5.01	25.1001	2.23830	7.07814	5.51	30.3601	2.34734	7.42294
5.02	25.2004	2.24054	7.08520	5.52	30.4704	2.34947	7.42967
5.03	25.3009	2.24277	7.09225	5.53	30.5809	2.35160	7.43640
5.04	25.4016	2.24499	7.09930	5.54	30.6916	2.35372	7.44312
5.05	25.5025	2.24722	7.10634	5.55	30.8025	2.35584	7.44983
5.06	25.6036	2.24944	7.11337	5.56	30.9136	2.35797	7.45654
5.07	25.7049	2.25167	7.12039	5.57	31.0249	2.36008	7.46324
5.08	25.8064	2.25389	7.12741	5.58	31.1364	2.36220	7.46994
5.09	25.9081	2.25610	7.13442	5.59	31.2481	2.36432	7.47663
5.10	26.0100	2.25832	7.14143	5.60	31.3600	2.36643	7.48331
5.11	26.1121	2.26053	7.14843	5.61	31.4721	2.36854	7.48999
5.12	26.2144	2.26274	7.15542	5.62	31.5844	2.37065	7.49667
5.13	26.3169	2.26495	7.16240	5.63	31.6969	2.37276	7.50333
5.14	26.4196	2.26716	7.16938	5.64	31.8096	2.37487	7.50999
5.15	26.5225	2.26936	7.17635	5.65	31.9225	2.37697	7.51665
5.16	26.6256	2.27156	7.18331	5.66	32.0356	2.37908	7.52330
5.17	26.7289	2.27376	7.19027	5.67	32.1489	2.38118	7.52994
5.18	26.8324	2.27596	7.19722	5.68	32.2624	2.38328	7.53658
5.19	26.9361	2.27816	7.20417	5.69	32.3761	2.38537	7.54321
5.20	27.0400	2.28035	7.21110	5.70	32.4900	2.38747	7.54983
5.21	27.1441	2.28254	7.21803	5.71	32.6041	2.38956	7.55645
5.22	27.2484	2.28473	7.22496	5.72	32.7184	2.39165	7.56307
5.23	27.3529	2.28692	7.23187	5.73	32.8329	2.39374	7.56968
5.24	27.4576	2.28910	7.23878	5.74	32.9476	2.39583	7.57628
5.25	27.5625	2.29129	7.24569	5.75	33.0625	2.39792	7.58288
5.26	27.6676	2.29347	7.25259	5.76	33.1776	2.40000	7.58947
5.27	27.7729	2.29565	7.25948	5.77	33.2929	2.40208	7.59605
5.28	27.8784	2.29783	7.26636	5.78	33.4084	2.40416	7.60263
5.29	27.9841	2.30000	7.27324	5.79	33.5241	2.40624	7.60920
5.30	28.0900	2.30217	7.28011	5.80	33.6400	2.40832	7.61577
5.31	28.1961	2.30434	7.28697	5.81	33.7561	2.41039	7.62234
5.32	28.3024	2.30651	7.29383	5.82	33.8724	2.41247	7.62889
5.33	28.4089	2.30868	7.30068	5.83	33.9889	2.41454	7.63544
5.34	28.5156	2.31084	7.30753	5.84	34.1056	2.41661	7.64199
5.35	28.6225	2.31301	7.31437	5.85	34.2225	2.41868	7.64853
5.36	28.7296	2.31517	7.32120	5.86	34.3396	2.42074	7.65506
5.37	28.8369	2.31733	7.32803	5.87	34.4569	2.42281	7.66159
5.38	28.9444	2.31948	7.33485	5.88	34.5744	2.42487	7.66812
5.39	29.0521	2.32164	7.34166	5.89	34.6921	2.42693	7.67463
5.40	29.1600	2.32379	7.34847	5.90	34.8100	2.42899	7.68115
5.41	29.2681	2.32594	7.35527	5.91	34.9281	2.43105	7.68765
5.42	29.3764	2.32809	7.36206	5.92	35.0464	2.43311	7.69415
5.43	29.4849	2.33024	7.36885	5.93	35.1649	2.43516	7.70065
5.44	29.5936	2.33238	7.37564	5.94	35.2836	2.43721	7.70714
5.45	29.7025	2.33452	7.38241	5.95	35.4025	2.43926	7.71362
5.46	29.8116	2.33666	7.38918	5.96	35.5216	2.44131	7.72010
5.47	29.9209	2.33880	7.39594	5.97	35.6409	2.44336	7.72658
5.48	30.0304	2.34094	7.40270	5.98	35.7604	2.44540	7.73305
5.49	30.1401	2.34307	7.40945	5.99	35.8801	2.44745	7.73951

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
6.00	36.0000	2.44949	7.74597
6.01	36.1201	2.45153	7.75242
6.02	36.2404	2.45357	7.75887
6.03	36.3609	2.45561	7.76531
6.04	36.4816	2.45764	7.77174
6.05	36.6025	2.45967	7.77817
6.06	36.7236	2.46171	7.78460
6.07	36.8449	2.46374	7.79102
6.08	36.9664	2.46577	7.79744
6.09	37.0881	2.46779	7.80385
6.10	37.2100	2.46982	7.81025
6.11	37.3321	2.47184	7.81665
6.12	37.4544	2.47386	7.82304
6.13	37.5769	2.47588	7.82943
6.14	37.6996	2.47790	7.83582
6.15	37.8225	2.47992	7.84219
6.16	37.9456	2.48193	7.84857
6.17	38.0689	2.48395	7.85493
6.18	38.1924	2.48596	7.86130
6.19	38.3161	2.48797	7.86766
6.20	38.4400	2.48998	7.87401
6.21	38.5641	2.49199	7.88036
6.22	38.6884	2.49399	7.88670
6.23	38.8129	2.49600	7.89303
6.24	38.9376	2.49800	7.89937
6.25	39.0625	2.50000	7.90569
6.26	39.1876	2.50200	7.91202
6.27	39.3129	2.50400	7.91833
6.28	39.4384	2.50599	7.92465
6.29	39.5641	2.50799	7.93095
6.30	39.6900	2.50998	7.93725
6.31	39.8161	2.51197	7.94355
6.32	39.9424	2.51396	7.94984
6.33	40.0689	2.51595	7.95613
6.34	40.1956	2.51794	7.96241
6.35	40.3225	2.51992	7.96869
6.36	40.4496	2.52190	7.97496
6.37	40.5769	2.52389	7.98123
6.38	40.7044	2.52587	7.98749
6.39	40.8321	2.52784	7.99375
6.40	40.9600	2.52982	8.00000
6.41	41.0881	2.53180	8.00625
6.42	41.2164	2.53377	8.01249
6.43	41.3449	2.53574	8.01873
6.44	41.4736	2.53772	8.02496
6.45	41.6025	2.53969	8.03119
6.46	41.7316	2.54165	8.03741
6.47	41.8609	2.54362	8.04363
6.48	41.9904	2.54558	8.04984
6.49	42.1201	2.54755	8.05605
6.50	42.2500	2.54951	8.06226
6.51	42.3801	2.55147	8.06846
6.52	42.5104	2.55343	8.07465
6.53	42.6409	2.55539	8.08084
6.54	42.7716	2.55734	8.08703
6.55	42.9025	2.55930	8.09321
6.56	43.0336	2.56125	8.09938
6.57	43.1649	2.56320	8.10555
6.58	43.2964	2.56515	8.11172
6.59	43.4281	2.56710	8.11788
6.60	43.5600	2.56905	8.12404
6.61	43.6921	2.57099	8.13019
6.62	43.8244	2.57294	8.13634
6.63	43.9569	2.57488	8.14248
6.64	44.0896	2.57682	8.14862
6.65	44.2225	2.57876	8.15475
6.66	44.3556	2.58070	8.16088
6.67	44.4889	2.58263	8.16701
6.68	44.6224	2.58457	8.17313
6.69	44.7561	2.58650	8.17924
6.70	44.8900	2.58844	8.18535
6.71	45.0241	2.59037	8.19146
6.72	45.1584	2.59230	8.19756
6.73	45.2929	2.59422	8.20366
6.74	45.4276	2.59615	8.20975
6.75	45.5625	2.59808	8.21584
6.76	45.6976	2.60000	8.22192
6.77	45.8329	2.60192	8.22800
6.78	45.9684	2.60384	8.23408
6.79	46.1041	2.60576	8.24015
6.80	46.2400	2.60768	8.24621
6.81	46.3761	2.60960	8.25227
6.82	46.5124	2.61151	8.25833
6.83	46.6489	2.61343	8.26438
6.84	46.7856	2.61534	8.27043
6.85	46.9225	2.61725	8.27647
6.86	47.0596	2.61916	8.28251
6.87	47.1969	2.62107	8.28855
6.88	47.3344	2.62298	8.29458
6.89	47.4721	2.62488	8.30060
6.90	47.6100	2.62679	8.30662
6.91	47.7481	2.62869	8.31264
6.92	47.8864	2.63059	8.31865
6.93	48.0249	2.63249	8.32466
6.94	48.1636	2.63439	8.33067
6.95	48.3025	2.63629	8.33667
6.96	48.4416	2.63818	8.34266
6.97	48.5809	2.64008	8.34865
6.98	48.7204	2.64197	8.35464
6.99	48.8601	2.64386	8.36062

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
7.00	49.0000	2.64575	8.36660
7.01	49.1401	2.64764	8.37257
7.02	49.2804	2.64953	8.37854
7.03	49.4209	2.65141	8.38451
7.04	49.5616	2.65330	8.39047
7.05	49.7025	2.65518	8.39643
7.06	49.8436	2.65707	8.40238
7.07	49.9849	2.65895	8.40833
7.08	50.1264	2.66083	8.41427
7.09	50.2681	2.66271	8.42021
7.10	50.4100	2.66458	8.42615
7.11	50.5521	2.66646	8.43208
7.12	50.6944	2.66833	8.43801
7.13	50.8369	2.67021	8.44393
7.14	50.9796	2.67208	8.44985
7.15	51.1225	2.67395	8.45577
7.16	51.2656	2.67582	8.46168
7.17	51.4089	2.67769	8.46759
7.18	51.5524	2.67955	8.47349
7.19	51.6961	2.68142	8.47939
7.20	51.8400	2.68328	8.48528
7.21	51.9841	2.68514	8.49117
7.22	52.1284	2.68701	8.49706
7.23	52.2729	2.68887	8.50294
7.24	52.4176	2.69072	8.50882
7.25	52.5625	2.69258	8.51469
7.26	52.7076	2.69444	8.52056
7.27	52.8529	2.69629	8.52643
7.28	52.9984	2.69815	8.53229
7.29	53.1441	2.70000	8.53815
7.30	53.2900	2.70185	8.54400
7.31	53.4361	2.70370	8.54985
7.32	53.5824	2.70555	8.55570
7.33	53.7289	2.70740	8.56154
7.34	53.8756	2.70924	8.56738
7.35	54.0225	2.71109	8.57321
7.36	54.1696	2.71293	8.57904
7.37	54.3169	2.71477	8.58487
7.38	54.4644	2.71662	8.59069
7.39	54.6121	2.71846	8.59651
7.40	54.7600	2.72029	8.60233
7.41	54.9081	2.72213	8.60814
7.42	55.0564	2.72397	8.61394
7.43	55.2049	2.72580	8.61974
7.44	55.3536	2.72764	8.62554
7.45	55.5025	2.72947	8.63134
7.46	55.6516	2.73130	8.63713
7.47	55.8009	2.73313	8.64292
7.48	55.9504	2.73496	8.64870
7.49	56.1001	2.73679	8.65448
7.50	56.2500	2.73861	8.66025
7.51	56.4001	2.74044	8.66603
7.52	56.5504	2.74226	8.67179
7.53	56.7009	2.74408	8.67756
7.54	56.8516	2.74591	8.68332
7.55	57.0025	2.74773	8.68907
7.56	57.1536	2.74955	8.69483
7.57	57.3049	2.75136	8.70057
7.58	57.4564	2.75318	8.70632
7.59	57.6081	2.75500	8.71206
7.60	57.7600	2.75681	8.71780
7.61	57.9121	2.75862	8.72353
7.62	58.0644	2.76043	8.72926
7.63	58.2169	2.76225	8.73499
7.64	58.3696	2.76405	8.74071
7.65	58.5225	2.76586	8.74643
7.66	58.6756	2.76767	8.75214
7.67	58.8289	2.76948	8.75785
7.68	58.9824	2.77128	8.76356
7.69	59.1361	2.77308	8.76926
7.70	59.2900	2.77489	8.77496
7.71	59.4441	2.77669	8.78066
7.72	59.5984	2.77849	8.78635
7.73	59.7529	2.78029	8.79204
7.74	59.9076	2.78209	8.79773
7.75	60.0625	2.78388	8.80341
7.76	60.2176	2.78568	8.80909
7.77	60.3729	2.78747	8.81476
7.78	60.5284	2.78927	8.82043
7.79	60.6841	2.79106	8.82610
7.80	60.8400	2.79285	8.83176
7.81	60.9961	2.79464	8.83742
7.82	61.1524	2.79643	8.84308
7.83	61.3089	2.79821	8.84873
7.84	61.4656	2.80000	8.85438
7.85	61.6225	2.80179	8.86002
7.86	61.7796	2.80357	8.86566
7.87	61.9369	2.80535	8.87130
7.88	62.0944	2.80713	8.87694
7.89	62.2521	2.80891	8.88257
7.90	62.4100	2.81069	8.88819
7.91	62.5681	2.81247	8.89382
7.92	62.7264	2.81425	8.89944
7.93	62.8849	2.81603	8.90505
7.94	63.0436	2.81780	8.91067
7.95	63.2025	2.81957	8.91628
7.96	63.3616	2.82135	8.92188
7.97	63.5209	2.82312	8.92749
7.98	63.6804	2.82489	8.93308
7.99	63.8401	2.82666	8.93868

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
8.00	64.0000	2.82843	8.94427
8.01	64.1601	2.83019	8.94986
8.02	64.3204	2.83196	8.95545
8.03	64.4809	2.83373	8.96103
8.04	64.6416	2.83549	8.96660
8.05	64.8025	2.83725	8.97218
8.06	64.9636	2.83901	8.97775
8.07	65.1249	2.84077	8.98332
8.08	65.2864	2.84253	8.98888
8.09	65.4481	2.84429	8.99444
8.10	65.6100	2.84605	9.00000
8.11	65.7721	2.84781	9.00555
8.12	65.9344	2.84956	9.01110
8.13	66.0969	2.85132	9.01665
8.14	66.2596	2.85307	9.02219
8.15	66.4225	2.85482	9.02774
8.16	66.5856	2.85657	9.03327
8.17	66.7489	2.85832	9.03881
8.18	66.9124	2.86007	9.04434
8.19	67.0761	2.86182	9.04986
8.20	67.2400	2.86356	9.05539
8.21	67.4041	2.86531	9.06091
8.22	67.5684	2.86705	9.06642
8.23	67.7329	2.86880	9.07193
8.24	67.8976	2.87054	9.07744
8.25	68.0625	2.87228	9.08295
8.26	68.2276	2.87402	9.08845
8.27	68.3929	2.87576	9.09395
8.28	68.5584	2.87750	9.09945
8.29	68.7241	2.87924	9.10494
8.30	68.8900	2.88097	9.11043
8.31	69.0561	2.88271	9.11592
8.32	69.2224	2.88444	9.12140
8.33	69.3889	2.88617	9.12688
8.34	69.5556	2.88791	9.13236
8.35	69.7225	2.88964	9.13783
8.36	69.8896	2.89137	9.14330
8.37	70.0569	2.89310	9.14877
8.38	70.2244	2.89482	9.15423
8.39	70.3921	2.89655	9.15969
8.40	70.5600	2.89828	9.16515
8.41	70.7281	2.90000	9.17061
8.42	70.8964	2.90172	9.17606
8.43	71.0649	2.90345	9.18150
8.44	71.2336	2.90517	9.18695
8.45	71.4025	2.90689	9.19239
8.46	71.5716	2.90861	9.19783
8.47	71.7409	2.91033	9.20326
8.48	71.9104	2.91204	9.20869
8.49	72.0801	2.91376	9.21412
8.50	72.2500	2.91548	9.21954
8.51	72.4201	2.91719	9.22497
8.52	72.5904	2.91890	9.23038
8.53	72.7609	2.92062	9.23580
8.54	72.9316	2.92233	9.24121
8.55	73.1025	2.92404	9.24662
8.56	73.2736	2.92575	9.25203
8.57	73.4449	2.92746	9.25743
8.58	73.6164	2.92916	9.26283
8.59	73.7881	2.93087	9.26823
8.60	73.9600	2.93258	9.27362
8.61	74.1321	2.93428	9.27901
8.62	74.3044	2.93598	9.28440
8.63	74.4769	2.93769	9.28978
8.64	74.6496	2.93939	9.29516
8.65	74.8225	2.94109	9.30054
8.66	74.9956	2.94279	9.30591
8.67	75.1689	2.94449	9.31128
8.68	75.3424	2.94618	9.31665
8.69	75.5161	2.94788	9.32202
8.70	75.6900	2.94958	9.32738
8.71	75.8641	2.95127	9.33274
8.72	76.0384	2.95296	9.33809
8.73	76.2129	2.95466	9.34345
8.74	76.3876	2.95635	9.34880
8.75	76.5625	2.95804	9.35414
8.76	76.7376	2.95973	9.35949
8.77	76.9129	2.96142	9.36483
8.78	77.0884	2.96311	9.37017
8.79	77.2641	2.96479	9.37550
8.80	77.4400	2.96648	9.38083
8.81	77.6161	2.96816	9.38616
8.82	77.7924	2.96985	9.39149
8.83	77.9689	2.97153	9.39681
8.84	78.1456	2.97321	9.40213
8.85	78.3225	2.97489	9.40744
8.86	78.4996	2.97658	9.41276
8.87	78.6769	2.97825	9.41807
8.88	78.8544	2.97993	9.42338
8.89	79.0321	2.98161	9.42868
8.90	79.2100	2.98329	9.43398
8.91	79.3881	2.98496	9.43928
8.92	79.5664	2.98664	9.44458
8.93	79.7449	2.98831	9.44987
8.94	79.9236	2.98998	9.45516
8.95	80.1025	2.99166	9.46044
8.96	80.2816	2.99333	9.46573
8.97	80.4609	2.99500	9.47101
8.98	80.6404	2.99666	9.47629
8.99	80.8201	2.99833	9.48156

Appendix L. Squares and Square Roots (*cont.*)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$
9.00	81.0000	3.00000	9.48683
9.01	81.1801	3.00167	9.49210
9.02	81.3604	3.00333	9.49737
9.03	81.5409	3.00500	9.50263
9.04	81.7216	3.00666	9.50789
9.05	81.9025	3.00832	9.51315
9.06	82.0836	3.00998	9.51840
9.07	82.2649	3.01164	9.52365
9.08	82.4464	3.01330	9.52890
9.09	82.6281	3.01496	9.53415
9.10	82.8100	3.01662	9.53939
9.11	82.9921	3.01828	9.54463
9.12	83.1744	3.01993	9.54987
9.13	83.3569	3.02159	9.55510
9.14	83.5396	3.02324	9.56033
9.15	83.7225	3.02490	9.56556
9.16	83.9056	3.02655	9.57079
9.17	84.0889	3.02820	9.57601
9.18	84.2724	3.02985	9.58123
9.19	84.4561	3.03150	9.58645
9.20	84.6400	3.03315	9.59166
9.21	84.8241	3.03480	9.59687
9.22	85.0084	3.03645	9.60208
9.23	85.1929	3.03809	9.60729
9.24	85.3776	3.03974	9.61249
9.25	85.5625	3.04138	9.61769
9.26	85.7476	3.04302	9.62289
9.27	85.9329	3.04467	9.62808
9.28	86.1184	3.04631	9.63328
9.29	86.3041	3.04795	9.63846
9.30	86.4900	3.04959	9.64365
9.31	86.6761	3.05123	9.64883
9.32	86.8624	3.05287	9.65401
9.33	87.0489	3.05450	9.65919
9.34	87.2356	3.05614	9.66437
9.35	87.4225	3.05778	9.66954
9.36	87.6096	3.05941	9.67471
9.37	87.7969	3.06105	9.67988
9.38	87.9844	3.06268	9.68504
9.39	88.1721	3.06431	9.69020
9.40	88.3600	3.06594	9.69536
9.41	88.5481	3.06757	9.70052
9.42	88.7364	3.06920	9.70567
9.43	88.9249	3.07083	9.71082
9.44	89.1136	3.07246	9.71597
9.45	89.3025	3.07409	9.72111
9.46	89.4916	3.07571	9.72625
9.47	89.6809	3.07734	9.73139
9.48	89.8704	3.07896	9.73653
9.49	90.0601	3.08058	9.74166
9.50	90.2500	3.08221	9.74679
9.51	90.4401	3.08383	9.75192
9.52	90.6304	3.08545	9.75705
9.53	90.8209	3.08707	9.76217
9.54	91.0116	3.08869	9.76729
9.55	91.2025	3.09031	9.77241
9.56	91.3936	3.09192	9.77753
9.57	91.5849	3.09354	9.78264
9.58	91.7764	3.09516	9.78775
9.59	91.9681	3.09677	9.79285
9.60	92.1600	3.09839	9.79796
9.61	92.3521	3.10000	9.80306
9.62	92.5444	3.10161	9.80816
9.63	92.7369	3.10322	9.81326
9.64	92.9296	3.10483	9.81835
9.65	93.1225	3.10644	9.82344
9.66	93.3156	3.10805	9.82853
9.67	93.5089	3.10966	9.83362
9.68	93.7024	3.11127	9.83870
9.69	93.8961	3.11288	9.84378
9.70	94.0900	3.11448	9.84886
9.71	94.2841	3.11609	9.85393
9.72	94.4784	3.11769	9.85901
9.73	94.6729	3.11929	9.86408
9.74	94.8676	3.12090	9.86914
9.75	95.0625	3.12250	9.87421
9.76	95.2576	3.12410	9.87927
9.77	95.4529	3.12570	9.88433
9.78	95.6484	3.12730	9.88939
9.79	95.8441	3.12890	9.89444
9.80	96.0400	3.13050	9.89949
9.81	96.2361	3.13209	9.90454
9.82	96.4324	3.13369	9.90959
9.83	96.6289	3.13528	9.91464
9.84	96.8256	3.13688	9.91968
9.85	97.0225	3.13847	9.92472
9.86	97.2196	3.14006	9.92975
9.87	97.4169	3.14166	9.93479
9.88	97.6144	3.14325	9.93982
9.89	97.8121	3.14484	9.94485
9.90	98.0100	3.14643	9.94987
9.91	98.2081	3.14802	9.95490
9.92	98.4064	3.14960	9.95992
9.93	98.6049	3.15119	9.96494
9.94	98.8036	3.15278	9.96995
9.95	99.0025	3.15436	9.97497
9.96	99.2016	3.15595	9.97998
9.97	99.4009	3.15753	9.98499
9.98	99.6004	3.15911	9.98999
9.99	99.8001	3.16070	9.99500
10.00	100.000	3.16228	10.0000

APPENDIX M

Sample Computer Programs for Analysis of Variance

The following two computer programs demonstrate how the step-by-step approach of this book can be applied to the use of FORTRAN IV for computer programing. These particular programs were written from the computational examples presented in Sections 2.9 and 2.10 and were verified on an IBM Model 360 computer. They should prove especially useful to the beginning programer and also to the experienced programer who is not familiar with analysis of variance. They not only demonstrate the basic computational steps which are part of all analyses of variance, but they also show the special steps that are needed when data must be retabled (Section 2.9) and where counterbalancing is required (Section 2.10). While it is recognized that more efficient and elaborate programs could be written, the two that are presented here do demonstrate solutions to nearly every type of problem which might be encountered in the programing of such analyses of variance.

Appendix M. Program for Three-Factor Mixed Design: Repeated Measures on Two Factors (Section 2.9)

```
C      PROGRAM LANGUAGE = FORTRAN IV
C     PROGRAM WRITTEN FOR IBM 360 OS-E LEVEL
C
C  MAXIMUM CAPABILITIES ARE-
C UP TO 4 GROUPS
C NUMBER OF GROUPS READ IN AS NGROUP ON PARAMETER CARD
C UP TO 30 SS PER GROUP
C  NUMBER OF SS READ AS NN ON PARAMETER CARD
C UP TO 5 LEVELS OF EXPERIMENTAL TREATMENT 1
C NUMBER OF LEVELS OF TR. 1 IS READ AS KK ON PARAMETER CARD
C UP TO 5 LEVELS OF EXPERIMENTAL TREATMENT 2
C NUMBER OF LEVELS OF TR. 2 IS READ AS LL ON PARAMETER CARD
C NUMBER OF LEVELS OF EACH TREATMENT MAY BE INCREASED IF NECESSARY BY CHANGING
C DIMENSIONS
C
C
C    SUBROUTINES FMTM AND FMTC PERMIT CHANGING OF DATA READ IN FORMATS
C
C
C
C SUBMIT CARDS IN FOLLOWING ORDER
C
C     PROGRAM CARDS
C
C  OUTPUT   IDENTIFICATION CARD - - - THIS CARD CONTAINS THE FOLLOWING PUNCHES -
C STARTING IN COL. 1  TOTLBSUBGRPSERBTWTNSTR 1TR 2GRX1GRX21X2 GX23E1  E2  E3
C THIS CARD SETS UP THE FORM OF THE SUMMARY TABLE
C THIS CARD MUST BE INCLUDED AS THE FIRST DATA CARD
C
C
C THE NEXT CARD IS YOUR IDENTIFICATION CARD.  USE ANY OF THE 80 COLS. FOR
C INFORMATION THAT YOU WISH TO APPEAR AS A HEADING TO YOUR OUTPUT.
C
C THE NEXT CARD IS THE PARAMETER CARD AND SHOULD BE PUNCHED IN THE
C FOLLOWING MANNER ---
C COL. 1-2 THE NUMBER OF SUBJECTS PER GROUP (UP TO 30)
C COL. 4-5  THE NUMBER OF LEVELS OF TREATMENT 1 (UP TO 5)
C COL. 7-8 THE NUMBER OF LEVELS OF TREATMENT 2 (UP TO 5)
C COL. 10-11 LEAVE BLANK IF USING STANDARD FORMAT.  PUNCH 01 IF USING FMTM AND
C FMTC SUBROUTINES TO CHANGE DATA INPUT FORMAT
C COL 13-14 LEAVE BLANK IF ONLY 1 RUN--PUT 1 IN COL 14 IF HAVE ADDITIONAL RUNS
C COL 16-17 NUMBER OF COPIES OF OUTPUT DESIRED
C COL 19-20 NUMBER OF GROUPS (UP TO 4)
C
C NEXT CARD IS FORMAT CARD IF USING FMTM AND FMTC SUBROUTINES
C
C   DATA CARDS
C
C REPEAT BEGINNING WITH YOUR IDENTIFICATION CARD IF ADDITIONAL RUNS
C
      DIMENSION V(30,5,5), W(30,5,5), X(30,5,5),Y(30,5,5),SAV(30),
     1SAX(30),SAY(30),SVD(5,5),SWD(5,5),SXD(5,5),SYD(5,5),SS1(5),SS2(5)
      DIMENSION AVE1(30,5),AVE2(30,5)
      DIMENSION SAW(30)
      DIMENSION FMT(20),FMTIN(20)
      DIMENSION AWE1 (30,5),AXE1(30,5), AYE1(30,5), SIG1(5),AA(14)
      DIMENSION AWE2 (30,5),AXE2(30,5), AYE2(30,5), SIG2(5)
      DIMENSION SIG3(5), SIG4(5)
      DIMENSION IDENT(20)
   10 FORMAT (20F4.0)
  389 FORMAT (6(F6.2,2X))
      READ (1,4132) (AA(KKK),KKK=1,14)
 9000 READ(1,4132) (IDENT(M),M=1,20)
 4132 FORMAT (20A4)
C  READ PARAMETER CARD
      READ (1,1) NN,KK,LL,NFT,NRUN,NCOPY,NGROUP
    1 FORMAT (7(I2,1X))
      WRITE (3,666) NN,KK,LL,NGROUP
  666 FORMAT ('  PARAMETER CARD CHECK'/' NUMBER PER GROUP =',I4/'LEVELS
     1 OF TR-1=',I4/' LEVELS OF TR-2=',I4/'  NUMBER OF GROUPS =',I4///)
```

SOURCE: This program was written by Ronald E. Mihalick and is reproduced here by his permission.

```
C  ZERO EVERYTHING
      DO 2 N=1,NN
      DO 2 K=1,KK
      DO 2 L=1,LL
      V(N,K,L)=0
      W(N,K,L)=0
      X(N,K,L)=0
      Y(N,K,L)=0
      SAV(N)=0
      SAW(N)=0
      SAX(N)=0
      SAY(N)=0
      SVD(K,L)=0
      SWD(K,L)=0
      SXD(K,L)=0
      SYD(K,L)=0
      GS1=0
      GS2=0
      SIG3(K)=0
      SIG4(K)=0
      AVE2(N,K)=0
      SS1(K)=0
      SS2(L)=0
      AWE1(N,K)=0
      AXE1(N,K)=0
      AYE1(N,K)=0
      AWE2(N,L)=0
      AXE2(N,L)=0
      AYE2(N,L)=0
      AVE1(N,K)=0
      SIG1(K)=0
      SIG2(K)=0
    2 AVE2(N,L)=0
      GSUM = 0
      GSA=0
      GSUMSQ=0
      SUMYSQ=0
      SUMY=0
      SUMXSQ=0
      SUMX=0
      SUMWSQ=0
      SUMW=0
      SUMVSQ=0
      SUMV=0
      GSIG1=0
      GSIG2=0
      GSS12=0
      GS123=0
      GAE1=0
      GAE2=0
C
C
C TEST TO DETERMINE IF USING FMTC AND FMTM TO CHANGE DATA READ IN FORMATS
C
C
      IF (NFT) 1000,1000,2000
 2000 READ (1,3000) FMT
      CALL FMTC (FMT,FMTIN)
 3000 FORMAT (20A4)
C
C
C COMPUTED GO TO SENDS PROGRAM TO PROPER PLACE BASED ON NUMBER OF GRPS. (NGROUP)
C
C
      GO TO (7,6000,5000,4000),NGROUP
 1000 GO TO (7,6,5,4),NGROUP
 4000 CALL FMTM(FMTIN)
    4 READ (1,10) (((V(N,K,L),N=1,NN),K=1,KK),L=1,LL)
      IF (NFT) 5,5,5000
 5000 CALL FMTM (FMTIN)
    5 READ (1,10) (((W(N,K,L),N=1,NN),K=1,KK),L=1,LL)
      IF (NFT) 6,6,6000
 6000 CALL FMTM (FMTIN)
    6 READ (1,10) (((X(N,K,L),N=1,NN),K=1,KK),L=1,LL)
      IF (NFT) 8000,8000,8001
 8001 CALL FMTM (FMTIN)
 8000 READ (1,10) (((Y(N,K,L),N=1,NN),K=1,KK),L=1,LL)
      GO TO 555
    7 WRITE (3,8)
    8 FORMAT ('  ERROR ON PARAMETER CARD, NUMBER OF GROUPS = 1')
      GO TO 9001
```

```
C
C
C  STEP 2
C  STEP 3
C
  555 GO TO (7,13,12,11),NGROUP
   11 DO 14 N=1,NN
      DO 14 K=1,KK
      DO 14 L=1,LL
      SUMV=V(N,K,L)+SUMV
   14 SUMVSQ=V(N,K,L)**2+SUMVSQ
   12 DO 15 N=1,NN
      DO 15 K=1,KK
      DO 15 L=1,LL
      SUMW=W(N,K,L)+SUMW
   15 SUMWSQ=W(N,K,L)**2+SUMWSQ
   13 DO 16 N=1,NN
      DO 16 K=1,KK
      DO 16 L=1,LL
      SUMX=X(N,K,L)+SUMX
      SUMXSQ=SUMXSQ+X(N,K,L)**2
      SUMY=Y(N,K,L)+SUMY
   16 SUMYSQ=Y(N,K,L)**2+SUMYSQ
C
C  STEP 4
C  GET GSUM AND GSUMSQ
      GO TO  (7,19,18,17),NGROUP
   17 GSUM=GSUM+SUMV+SUMW+SUMX+SUMY
      GSUMSQ=GSUMSQ+SUMVSQ+SUMWSQ+SUMXSQ+SUMYSQ
      GO TO 30
   18 GSUM=GSUM+SUMW+SUMX+SUMY
      GSUMSQ=SUMWSQ+SUMXSQ+SUMYSQ+GSUMSQ
      GO TO 30
   19 GSUM=GSUM+SUMX+SUMY
      GSUMSQ=GSUMSQ+SUMXSQ+SUMYSQ
C
C  STEP 5
C GET CORRECTION TERM
   30 CTERM=GSUM**2/(NN*KK*LL*NGROUP)
C  STEP 6
C  GET SUM OF SQUARES TOTAL
      SST=GSUMSQ-CTERM
C  STEP 7
C  GET SUM OF SQUARES GROUPS
      GO TO (7,22,21,20),NGROUP
   20 GV=SUMV**2/(NN*KK*LL)
   21 GW=SUMW**2/(NN*KK*LL)
   22 GX=SUMX**2/(NN*KK*LL)
      GY=SUMY**2/(NN*KK*LL)
      GO TO (7,25,24,23),NGROUP
   23 GG=GV+GW+GX+GY
      GO TO 26
   24 GG=GW+GX+GY
      GO TO 26
   25 GG=GX+GY
   26 SSGRPS=GG-CTERM
C
C  STEP 8
C GET SUM OF SQUARES BETWEEN SUBJECTS
      DO 31 N=1,NN
      DO 31 K=1,KK
      DO 31 L=1,LL
      GO TO (7,29,28,27),NGROUP
   27 SAV(N)=V(N,K,L)+SAV(N)
   28 SAW(N)=W(N,K,L)+SAW(N)
   29 SAX(N)=X(N,K,L)+SAX(N)
   31 SAY(N)=Y(N,K,L)+SAY(N)
      DO 32 N=1,NN
      GO TO (7,35,34,33),NGROUP
   33 GSA=SAV(N)**2+SAW(N)**2+SAX(N)**2+SAY(N)**2+GSA
      GO TO 32
   34 GSA=SAW(N)**2+SAX(N)**2+SAY(N)**2+GSA
      GO TO 32
   35 GSA=SAX(N)**2+SAY(N)**2+GSA
   32 CONTINUE
      GGSA=GSA/(KK*LL)
      SSBTSB=GGSA-CTERM
C
C  STEP 9
C  BETWEEN SUBJECTS ERROR TERM
      SSEBS=SSBTSB-SSGRPS
```

```
C
C  STEP 10
C  WITHIN SUBJECTS SS
      SSWS=SST-SSBTSB
C GET SS DOWN FOR EACH GROUP
      DO 40 K=1,KK
      DO 40 L=1,LL
      DO 40 N=1,NN
      GO TO (7,43,42,41),NGROUP
   41 SVD(K,L)=V(N,K,L)+SVD(K,L)
   42 SWD(K,L)=W(N,K,L)+SWD(K,L)
   43 SXD(K,L)=X(N,K,L)+SXD(K,L)
      SYD(K,L)=Y(N,K,L)+SYD(K,L)
   40 CONTINUE
C  STEP 11
C GET SS FIRST WITHIN S FACTOR
      DO 50 K=1,KK
      DO 50 L=1,LL
      GO TO (7,53,52,51),NGROUP
   51 SS1(K)=SVD(K,L)+SWD(K,L)+SXD(K,L)+SYD(K,L)+SS1(K)
      GO TO 50
   52 SS1(K)=SWD(K,L)+SXD(K,L)+SYD(K,L)+SS1(K)
      GO TO 50
   53 SS1(K)=SXD(K,L)+SYD(K,L)+SS1(K)
   50 CONTINUE
      DO 60 K=1,KK
   60 GS1=SS1(K)**2/(NN*LL*NGROUP)+GS1
      S1=GS1-CTERM
C  STEP 12
C GET SS FOR SECOND WITHIN S FACTOR
      DO 70 L=1,LL
      DO 70 K=1,KK
      GO TO (7,73,72,71),NGROUP
   71 SS2(L)=SVD(K,L)+SWD(K,L)+SXD(K,L)+SYD(K,L)+SS2(L)
      GO TO 70
   72 SS2(L)=SWD(K,L)+SXD(K,L)+SYD(K,L)+SS2(L)
      GO TO 70
   73 SS2(L)=SXD(K,L)+SYD(K,L)+SS2(L)
   70 CONTINUE
      DO 74 L=1,LL
   74 GS2=SS2(L)**2/(NN*KK*NGROUP)+GS2
      S2=GS2-CTERM
C
C  STEP 13
C SS INTERACTION GROUPS WITH FIRST FACTOR
      DO 80 K=1,KK
      DO 80 L=1,LL
      GO TO (7,79,78,77),NGROUP
   77 SIG1(K)=SVD(K,L)+SIG1(K)
   78 SIG2(K)=SWD(K,L)+SIG2(K)
   79 SIG3(K)=SXD(K,L)+SIG3(K)
      SIG4(K)=SYD(K,L)+SIG4(K)
   80 CONTINUE
      DO 81 K=1,KK
      GO TO (7,9503,9502,9501),NGROUP
 9501 GSIG1=SIG4(K)**2/(NN*LL)+SIG3(K)**2/(NN*LL)+SIG2(K)**2/(NN*LL)+
     1 SIG1(K)**2/(NN*LL)+GSIG1
      GO TO 81
 9502 GSIG1=SIG4(K)**2/(NN*LL)+SIG3(K)**2/(NN*LL)+SIG2(K)**2/(NN*LL)
     1+GSIG1
      GO TO 81
 9503 GSIG1=SIG4(K)**2/(NN*LL)+SIG3(K)**2/(NN*LL)
     1+GSIG1
      GO TO 81
   81 CONTINUE
      SSIGW1=GSIG1-CTERM-SSGRPS-S1
C
C  STEP 14
C SS INTERACTION GROUPS WITH SECOND FACTOR
      DO 9509 K=1,KK
      SIG1(K)=0
      SIG2(K)=0
      SIG3(K)=0
 9509 SIG4(K)=0
      DO 90 L=1,LL
      DO 90 K=1,KK
      GO TO (7,89,88,87),NGROUP
   87 SIG1(L)=SVD(K,L)+SIG1(L)
   88 SIG2(L)=SWD(K,L)+SIG2(L)
   89 SIG3(L)=SXD(K,L)+SIG3(L)
      SIG4(L)=SYD(K,L)+SIG4(L)
   90 CONTINUE
      DO 91 L=1,LL
      GO TO (7,9508,9507,9506),NGROUP
 9506 GSIG2=SIG4(L)**2/(NN*KK)+SIG3(L)**2/(NN*KK)+SIG2(L)**2/(NN*KK)+
     1SIG1(L)**2/(NN*KK)+GSIG2
      GO TO 91
 9507 GSIG2=SIG4(L)**2/(NN*KK)+SIG3(L)**2/(NN*KK)+SIG2(L)**2/(NN*KK)+
     1GSIG2
      GO TO 91
 9508 GSIG2=SIG4(L)**2/(NN*KK)+SIG3(L)**2/(NN*KK)
     1+GSIG2
   91 CONTINUE
      SSIGW2=GSIG2-CTERM-SSGRPS-S2
C  STEP 15
C GET INTERACTION BETWEEN FIRST AND SECOND FACTOR
      DO 100 K=1,KK
      DO 100 L=1,LL
      GO TO (7,103,102,101),NGROUP
  101 GSS12=(SVD(K,L)+SWD(K,L)+SXD(K,L)+SYD(K,L))**2/(NN*NGROUP)+GSS12
      GO TO 100
  102 GSS12=(SXD(K,L)+SYD(K,L))**2/(NN*NGROUP)+GSS12
      GO TO 100
  103 GSS12=(SXD(K,L)+SYD(K,L))**2/(NN*NGROUP)+GSS12
  100 CONTINUE
      SSI12=GSS12-CTERM-S1-S2
C
C  STEP 16
C INTERACTION OF GROUPS X FIRST FACTOR X SECOND FACTOR
      DO 110 K=1,KK
      DO 110 L=1,LL
      GO TO (7,113,112,111),NGROUP
  111 GS123=SVD(K,L)**2/NN+SWD(K,L)**2/NN+SXD(K,L)**2/NN+SYD(K,L)**2/NN+
     1GS123
      GO TO 110
  112 GS123=SWD(K,L)**2/NN+SXD(K,L)**2/NN+SYD(K,L)**2/NN+GS123
      GO TO 110
  113 GS123=SXD(K,L)**2/NN+SYD(K,L)**2/NN+GS123
  110 CONTINUE
      SSI123=GS123-CTERM-SSGRPS-S1-S2-SSIGW1-SSIGW2-SSI12
C
C  STEP 17
C TOTAL ERROR TERM WITHIN SUBJECTS
      SEWS=SSWS-S1-S2-SSIGW1-SSIGW2-SSI12-SSI123
C  STEP 18
C GET ERROR TERM FOR S1 AND SIG1--SE1GW1
      DO 120 N=1,NN
      DO 120 K=1,KK
      DO 120 L=1,LL
      GO TO (7,116,115,114),NGROUP
  114 AVE1(N,K)=V(N,K,L)+AVE1(N,K)
  115 AWE1(N,K)=W(N,K,L)+AWE1(N,K)
  116 AXE1(N,K)=X(N,K,L)+AXE1(N,K)
      AYE1(N,K)=Y(N,K,L)+AYE1(N,K)
  120 CONTINUE
      DO 130 N=1,NN
      DO 130 K=1,KK
      L=LL
      GO TO (7,129,128,127),NGROUP
  127 GAE1=AVE1(N,K)**2/L+AWE1(N,K)**2/L+AXE1(N,K)**2/L+AYE1(N,K)**2/L+
     1GAE1
      GO TO 130
  128 GAE1=AWE1(N,K)**2/L+AXE1(N,K)**2/L+AYE1(N,K)**2/L+GAE1
      GO TO 130
  129 GAE1=AXE1(N,K)**2/L+AYE1(N,K)**2/L+GAE1
  130 CONTINUE
      SE1GW1=GAE1-CTERM-SSBTSB-S1-SSIGW1
C
C  STEP 19
C GET ERROR TERM FOR S2 AND SIG2 ---SE2GW2
      DO 140 N=1,NN
      DO 140 L=1,LL
      DO 140 K=1,KK
      GO TO (7,146,145,144),NGROUP
  144 AVE2(N,L)=V(N,K,L)+AVE2(N,L)
  145 AWE2(N,L)=W(N,K,L)+AWE2(N,L)
  146 AXE2(N,L)=X(N,K,L)+AXE2%N,L<
      AYE2(N,L)=Y(N,K,L)+AYE2(N,L)
  140 CONTINUE
      DO 150 N=1,NN
      DO 150 L=1,LL
      K=KK
      GO TO (7,159,158,157),NGROUP
  157 GAE2=AVE2(N,L)**2/K+AWE2(N,L)**2/K+AXE2(N,L)**2/K+AYE2(N,L)**2/K+
     1GAE2
      GO TO 150
  158 GAE2=AWE2(N,L)**2/K+AXE2(N,L)**2/K+AYE2(N,L)**2/K+GAE2
      GO TO 150
  159 GAE2=AXE2(N,L)**2/K+AYE2(N,L)**2/K+GAE2
  150 CONTINUE
      SE2GW2=GAE2-CTERM-SSBTSB-S2-SSIGW2
C
C  STEP 20
C E12 AND E 123 ---- E12123
      E12123=SEWS-SE1GW1-SE2GW2
C  STEP 21
C SET DEGREES OF FREEDOM
      DFTOT=NN*NGROUP*LL*KK-1
      DFSUB=NN*NGROUP-1
      DFGPS=NGROUP-1
      DFEBT=DFSUB-DFGPS
      DFWS=DFTOT-DFSUB
      DF1=KK-1
      DF2=LL-1
      DFG1=DFGPS*DF1
      DFG2=DFGPS*DF2
      DF12=DF1*DF2
      DF123=DFGPS*DF1*DF2
      DFE1=DF1*(NN-1)*NGROUP
      DFE2=DF2*(NN-1)*NGROUP
      DFE3=DF1*DF2*(NN-1)*NGROUP
C  STEP 22
C MEAN SQUARES
      XMSG=SSGRPS/DFGPS
      XMSEB=SSEBS/DFEBT
      XMS1=S1/DF1
      XMSE1=SE1GW1/DFE1
      XMS2=S2/DF2
      XMSE2=SE2GW2/DFE2
      XMSGX1=SSIGW1/DFG1
      XMSGX2=SSIGW2/DFG2
      XMS1X2=SSI12/DF12
      XMS123=SSI123/DF123
      XMSE3=E12123/DFE3
C
C  STEP 23
C DIFFERENCE BETWEEN GROUPS
      F1=XMSG/XMSEB
C TREATMENT 1
      F2=XMS1/XMSE1
C TR-I
      F3=XMS2/XMSE2
C GX1
      F4=XMSGX1/XMSE2
C GX2
      F5=XMSGX2/XMSE2
C 1X2
      F6=XMS1X2/XMSE3
C 1X2X3
       F7=XMS123/XMSE3
C
C  STEP 24
C OUTPUT
      WRITE (3,4132) (IDENT(M),M=1,20)
 9009 WRITE (3,200)
  200 FORMAT ('   TYPE VI  ANOV      R. MIHALICK  7/10/66'////)
      WRITE (3,201)
  201 FORMAT ('  SOURCE        SS           DF            MS', 12X,'F'//)
      KKK=1
C
C
C  AA(KKK) ALLOWS OUTPUT OF SUMMARY TABLE HEADINGS AS READ IN ON OUTPUT IDENT CD
C
C
      WRITE (3,202) AA(KKK),SST,DFTOT
      KKK=KKK+1
  202 FORMAT (2X,A4,3X,F13.3,3X,F5.0,5X,F13.3,5X,F12.4//)
      WRITE (3,202) AA(KKK),SSBTSB,DFSUB
      KKK=KKK+1
      WRITE (3,202) AA(KKK),SSGRPS,DFGPS,XMSG,F1
      KKK=KKK+1
      WRITE (3,202) AA(KKK),SSEBS,DFEBT,XMSEB
      KKK=KKK+1
      WRITE (3,202) AA(KKK),SSWS,DFWS
      KKK=KKK+1
      WRITE (3,202) AA(KKK),S1,DF1,XMS1,F2
      KKK=KKK+1
      WRITE (3,202) AA(KKK),S2,DF2,XMS2,F3
      KKK=KKK+1
      WRITE (3,202) AA(KKK),SSIGW1 ,DFG1,XMSGX1,F4
      KKK=KKK+1
      WRITE (3,202) AA(KKK), SSIGW2,DFG2,XMSGX2,F5
      KKK=KKK+1
      WRITE (3,202) AA(KKK), SSI12,DF12,XMS1X2,F6
      KKK=KKK+1
      WRITE (3,202) AA(KKK), SSI123,DF123,XMS123,F7
      KKK=KKK+1
      WRITE (3,202) AA(KKK), SE1GW1,DFE1,XMSE1
      KKK=KKK+1
      WRITE (3,202) AA(KKK), SE2GW2,DFE2,XMSE2
      KKK=KKK+1
      WRITE (3,202) AA(KKK), E12123,DFE3,XMSE3
      NCOPY=NCOPY-1
C  TEST TO DETERMINE THE NUMBER OF PRINTOUTS DESIRED
      IF (NCOPY) 9005,9005,9006
 9006 WRITE (3,9007)
 9007 FORMAT (////////'           DUPLICATE COPY'///)
      GO TO 9009
C  TEST TO DETERMINE IF HAVE MORE THAN ONE SET OF DATA TO RUN
 9005 IF (NRUN) 9001,9001,9002
 9002 WRITE (3,9003)
 9003 FORMAT (/////'   END OF FIRST RUN--BEGIN SECOND PROBLEM'//////)
 9004 GO TO 9000
 9001 STOP
      END
         SIZE OF COMMON  00000    PROGRAM  35358
```

Appendix M. Program for Latin Square Design: Simple (Section 2.10)

```
C             PROGRAM LANGUAGE = FORTRAN IV
C             PROGRAM WRITTEN FOR IBM 360 OS-E LEVEL
C
C             J = THE NUMBER OF TREATMENTS; FROM 1 TO NB TREATMENTS.
C             K = THE NUMBER OF GROUPS;  FROM 1 TO NA GROUPS.
C             I = THE NUMBER OF SUBJECTS PER CELL;  FROM 1 TO NSPS S'S EACH.
C             THE TOTAL NUMBER OF S'S = NA * NSPS.
C             NO. = THE NUMBER OF ORDERS.
C             THE "FIRST", "SECOND", & "THIRD" GIVE THE ORDER OF
C             PRESENTATION OF THE TREATMENTS.
C
C             THE DATA IS PUNCHED WITH ONE SUBJECT'S DATA PER CARD.
C
C        THIS DESIGN REQUIRES EQUAL N'S. THE NUMBER OF SUBJECTS PER CELL MUST
C             BE THE SAME, AND THE NUMBER OF GROUPS MUST BE EQUAL TO THE NUMBER
C             OF TREATMENTS.  SINCE THE NUMBER OF GROUPS = THE NUMBER OF
C             TREATMENTS = THE NUMBER OF ORDERS, ALL OF THESE WILL BE REPLACED
C             BY THE COMMON VALUE "F" (FREQUENCY).
C
C   VALUES ON THE PARAMETER CARD ARE AS FOLLOWS:
C          R = THE NUMBER OF THE READ STATEMENT FOR YOUR MACHINE.
C                     ****  NOTE  ****
C                       THIS READ NUMBER MUST ALSO APPEAR IN THE FIRST READ
C                       STATEMENT IN ORDER TO READ THE PARAMETERS!
C          W = THE NUMBER OF THE WRITE COMMAND ON YOUR MACHINE.
C          F = THE FREQUENCY, AS REFERRED TO ABOVE.
C          NS = THE TOTAL NUMBER OF SUBJECTS (INTEGER FORM).
C          FNS = THE TOTAL NUMBER OF SUBJECTS (REAL FORM - TO AVOID MIXED MODE).
C          NSPS = THE NUMBER OF SUBJECTS PER CELL.
C
C    R, W, F, & NS ARE IN INTEGER FORM, FNS & NSPS ARE IN REAL FORM.
C
C   THE PARAMETER CARD IS TO BE PUNCHED IN THE FOLLOWING ORDER,
C          SIX SPACES, "R", SIX SPACES, "W", SIX SPACES, "F",   SIX
C          SPACES, "NS", SIX SPACES, "FNS", SIX SPACES, "NSPS".
C          SEE BELOW FOR EXAMPLE.   MAKE SURE THE NUMBERS ARE IN THE
C          RIGHT PLACE; A PRINT OUT CHECK OF THE PARAMETERS IS INCLUDED.
C          CHECK TO SEE THAT THEY HAVE BEEN READ IN PROPERLY.
C
C
C      R       W       F       NS       FNS        NSPS
C
C     00.     00      00      000      +000.      +000.
C
C      1       3       3       21      + 21.      +  7.
C
C     THE PLUS NEED NOT BE PRINTED, IT IS ONLY THERE TO REMIND YOU TO
C          LEAVE A SPACE FOR IT IN THE FORMAT.
C
C   PUT PARAMETER CARD ON TOP OF DATA DECK, JUST IN BACK OF  /DATA CARD.
C
C    MAKE SURE ALL OF THE DATA CARDS ARE IN THE DECK, IF ONE IS MISSING,
C    THE OS-E LEVEL 360 WILL PRINT AN ERROR MESSAGE, AND NOT RUN DATA.
C
C   CHECK FORMAT 1000  THE NUMBER OF F6.2 PLACES MUST EQUAL THE NUMBER OF GROUPS
C     EXAMPLE: 3 GROUP FORMAT:(3X, 3F6.2)  5 GROUP  FORMAT:(3X, 5F6.2)
C
C
      DIMENSION SR(10), SC(10), SCL(10,10), SX(50),  X(50,50)
      REAL MSG,MSEB,MSTR,MSO,MSBO,MSEW, NSPS
      INTEGER R, W, F
      ALPHA = .05
C  **NOTF** MAKE SURE THE READ NUMBER OF YOUR SYSTEM IS BELOW
      READ (1, 9001) R, W, F, NS, FNS, NSPS
 9001 FURMAT (6X, I2, 6X, I2, 6X,  I2,  6X, I3, 6X, F5.0, 6X, F5.0)
C  CLEARING ARRAYS TO ZERO
      DO 7 K=1,F
    7 SR(K)=0.0
      DO 8 J=1,F
    8 SC(J)=0.0
      DO 9 I=1,NS
    9 SX(I)=0.0
      SXX = 0.0
      DO 10 K=1,F
      DO 10 J=1,F
```

SOURCE: This program was written by Morris K. Morgret and is reproduced by his permission.

```
   10 SCL(K,J)=0.0
C  READING DATA & DATA CHECK
      DO 11  I=1,NS
   11 READ (R, 1000) (X(I,J), J=1,F )
 1000 FORMAT (3X, 5F6.2)
      WRITE (W, 1111)
      WRITE (W, 1001) (X(I,J), J=1,F )
 1001 FORMAT (3X, 'THE LAST DATA CARD, CHECK IT!', 5F6.2)
C STEP 2  SUMMING CELLS
C STEP 3  SUMMING ROWS
      DO 2 K=1,F
      DO 2 J=1,F
      L1 = 1 + (NSPS * (K-1))
      L2 = L1 + (NSPS-1)
      DO 2 I=L1, L2
      SR(K)=SR(K) + X(I,J)
    2 SCL(K,J)=SCL(K,J)+ X(I,J)
C  SUMMING COLUMNS
      DO 3 J=1, F
      DO 3 I=1, NS
    3 SC(J)=SC(J) + X(I,J)
C STEP 4 SUMMING SUBJECTS
C STEP 5 SUM OF SUBJECTS SQUARED
      DO 4 I=1, NS
      DO 4 J=1, F
      SX(I)=SX(I) + X(I,J)
    4 SXX = SXX + X(I,J) **2
C STEP 6 OBTAINING GRAND SUM
C        CHECK OF GRAND SUM
      G1=0.0
      G=0.0
      DO 20 J=1,F
   20 G=G + SC(J)
      DO 21 K=1, F
   21 G1=G1 + SR(K)
C CHECKING ROW SUMS WITH COLUMN SUMS
      IF (G-G1)30,31,30
   30 WRITE (3, 2001)
 2001 FORMAT (2X, 'ERROR, ROW SUM DOES NOT EQUAL COLUMN SUM')
   31 CONTINUE
C    THE TOTAL NUMBER OF MEASURES (FM) = THE NUMBER OF S'S TIMES THE TOTAL
C    NUMBER OF MEASURES PER SUBJECT.
      FM = NSPS *(F **2)
C STEP 7 OBTAINING CORRECTION TERM (CORR)
      CORR = G ** 2 / FM
C STEP 8 TOTAL SUM OF SQUARES (SST)
      SST = SXX - CORR
C STEP 9 SUM OF SQUARES BETWEEN SUBJECTS (SSB)
      SX2 = 0.0
      DO 100 I = 1, NS
  100 SX2 = SX2 + SX   (I) ** 2
      SSB = ( SX2 / F   ) - CORR
C STEP 10 GROUPS EFFECT (SSG)
      SR2 = 0.0
      DO 101 K = 1, F
  101 SR2 = SR2 + ((SR(K) ** 2 ) / (FNS))
      SSG = SR2 - CORR
C STEP 11 BETWEEN S'S ERROR TERM (SSEB)
      SSEB = SSB - SSG
C STEP 12 WITHIN S'S SUM OF SQUARES (SSW)
      SSW = SST - SSB
C STEP 13 TREATMENTS EFFECT (SSTR)
      SC2 = 0.0
      DO 102  J = 1, F
  102 SC2 = SC2 + (SC(J) ** 2 ) / (FNS)
      SSTR = SC2 - CORR
C STEP 14 ORDER EFFECTS (SSO)
C             SUM CELL TOTALS FOR ORDER OF PRESENTATION  IE: SUM CELLS
C             RECEIVING TREATMENT FIRST   CALL IT: "FIRST"  ETC.
      FIRST = 0.0
      SECOND = 0.0
      THIRD = 0.0
      FOURTH = 0.0
      FIFTH = 0.0
      IF(F -4) 60, 61, 62
C   ORDER EFFECTS FOR A FOUR GROUP DESIGN
   60 FIRST = FIRST + (SCL(1,1) + SCL (2,2) + SCL (3,3))
      SECOND = SECOND + (SCL (1,2) + SCL (2,3) + SCL (3,1))
      THIRD = THIRD + (SCL (1,3) + SCL (2,1) + SCL ( 3,2))
      ORDER = ((FIRST ** 2) + (SECOND ** 2 ) + (THIRD ** 2)) / (FNS)
      GO TO 63
```

```
C   ORDER EFFECTS FOR A FOUR GROUP DESIGN
   61 FIRST = FIRST +  (SCL(1,1) + SCL(2,2) + SCL(3,3) + SCL(4,4))
      SECOND = SECOND + (SCL(1,2) + SCL(2,3)+ SCL(3,4) + SCL(4,1))
      THIRD = THIRD + (SCL(1,3) + SCL(2,4) + SCL(3,1) + SCL(4,2))
      FOURTH = FOURTH + (SCL(1,4) + SCL(2,1) + SCL(3,2) + SCL(4,3))
      ORDER = (FIRST **2  + SECOND**2 + THIRD**2 + FOURTH**2 ) / FNS
      GO TO 63
C   ORDER EFFECTS FOR FIVE GROUPS DESIGN
   62 FIRST=FIRST + SCL(1,1)+SCL(2,2)+SCL(3,3)+SCL(4,4)+SCL(5,5)
      SECOND=SECOND+ SCL(1,2)+SCL(2,3)+SCL(3,4)+SCL(4,5)+SCL(5,1)
      THIRD=THIRD + SCL(1,3)+SCL(2,4)+SCL(3,5)+SCL(4,1)+SCL(5,2)
      FOURTH=FOURTH + SCL(1,4)+SCL(2,5)+SCL(3,1)+SCL(4,2)+SCL(5,3)
      FIFTH=FIFTH + SCL(1,5)+SCL(2,1)+SCL(3,2)+SCL(4,3)+SCL(5,4)
      ORDER = FIRST**2 + SECOND**2 + THIRD**2 + FOURTH**2 + FIFTH**2/FNS
   63 SSO = ORDER - CORR
C STEP 15 SUM OF SQUARES B * ORDER INTERACTION (SSBO)
      CELL2 = 0.0
      DO 103 K = 1, F
      DO 103 J = 1, F
  103 CELL2 = CELL2 + (SCL (K,J) ** 2) / NSPS
      SSBO = CELL2 - CORR - SSG - SSTR - SSO
C STEP 16 ERROR WITHIN (SSEW)
      SSEW = SSW - ( SSTR + SSO + SSBO)
C STEP 17 COMPUTATION OF DEGREES OF FREEDOM
      DFSST = FM - 1
      DFSSB = (NSPS * F  ) -1
      DFSSG = F  -1
      DFSSEB = DFSSB - DFSSG
      DFSSW = DFSST - DFSSB
      DFSSTR = F  -1
      DFSSO = F -1
      DFSSBO = (F -2) * (F -1)
      DFSSEW = DFSSW - (DFSSTR + DFSSO + DFSSBO)
C STEP 18 COMPUTATION OF MEAN SQUARES
      MSG = SSG / DFSSG
      MSEB = SSEB / DFSSEB
      MSTR = SSTR / DFSSTR
      MSO = SSO / DFSSO
      MSBO = SSBO / DFSSBO
      MSEW = SSEW / DFSSEW
C STEP 19 COMPUTATION OF F RATIOS AND PROBABILITIES
      FGPS = MSG / MSEB
      FTR = MSTR / MSEW
      FO = MSO / MSEW
      FBOI = MSBO / MSEW
C               CALLING SUBROUTINE "PLEVEL" TO COMPUTE PROBABILITIES
      CALL PLEVEL ( DFSSG, DFSSEB, FGPS, PGPS)
      CALL PLEVEL (DFSSTR, DFSSEW, FTR, PTR)
      CALL PLEVEL (DFSSO, DFSSEW, FO, PO)
      CALL PLEVEL (DFSSBO, DFSSEW, FBOI, PBOI)
C
C  AOV PRINT OUT PART OF PROGRAM
C  WRITE OUTS OF ROW AND COLUMN SUMS, SUMS OF SCORES, SUM OF SCORES
C         SQUARED, AND GRAND SUM
      WRITE (W, 1111)
 1111 FORMAT ('1')
      DO 37 K=1, F
   37 WRITE (W, 3001)  SR(K)
 3001 FORMAT(2X, 'SUM OF ROW(K)=', F10.2)
      WRITE (W, 2222)
 2222 FORMAT ('0', //)
      DO 38 J=1, F
   38 WRITE(W, 3002)  SC(J)
 3002 FORMAT (2X, 'SUM OF COLUMNS (J)=', F10.2)
      WRITE (W, 2232)
 2232 FORMAT ('0', ///)
      DO 39 I=1,  NS
   39 WRITE (W, 4001) SX(I)
 4001 FORMAT(2X, 'SUM OF X=' F10.2)
      WRITE (W, 2242)
 2242 FORMAT ('0', ////)
      DO 40 K=1, F
      DO 40 J=1, F
   40 WRITE(W, 5001)  SCL(K,J)
 5001 FORMAT (2X, 'THE SUM OF CELLS(K,J) =' ,  9F8.2)
      WRITE (W, 6001)  SXX
 6001 FORMAT (20X, 'SUM OF SCORES SQUARED=', F12.2,/)
      WRITE (W, 6002)  G
 6002 FORMAT (20X, 'THE GRAND SUM=', F12.2,//)
      WRITE(W, 7002)  SST
 7002 FORMAT(20X, 'THE TOTAL SUM OF SQUARES =', F12.2,//)
```

```
      WRITE (W, 7001)  CORR
 7001 FORMAT (20X, 'THE CORRECTIUN TERM=', F12.2,//)
      WRITE (W, 5553)  CELL2
 5553 FORMAT (20X, 'CELL2=', F10.2, /)
      WRITE (W, 5552)  SC2
 5552 FORMAT (20X, 'SC2=', F8.2, /)
      WRITE (W, 5551) SX2
 5551 FORMAT  (20X, 'SX2=', F8.2)
      WRITE (W, 7777)  R, W, F, NS, FNS, NSPS
 7777 FORMAT (20X, 'R=',I2, 5X, 'W=', I2, 5X, 'F=', I2, 5X, 'NS=', I3,
     1  5X, 'FNS=', F5.2, 5X, 'NSPS=', F5.2)
      WRITE (W, 1111)
      WRITE (W, 1011)
 1011 FORMAT (3X, 'SOURCE', 20X, 'SS',  9X, 'DF',  6X, 'MS', 10X, 'F',
     1 10X, 'P')
      WRITE (W, 1012)
 1012 FORMAT ('0')
      WRITE (W, 1013) SST, DFSST
 1013 FORMAT (3X, 'TOTAL', 19X, F7.2, 5X, F6.2,/ )
      WRITE (W, 1014) SSB, DFSSB
 1014 FORMAT (5X, 'BETWEEN', 15X, F7.2, 5X, F6.2 ,/ )
      WRITE (W, 1015) SSG, DFSSG, MSG, FGPS, PGPS
 1015 FORMAT (7X, 'GROUPS', 14X, F7.2, 5X, F6.2, 3X, F6.2, 4X, F5.2,
     1 4X, F5.3,/ )
      WRITE (W, 1016) SSEB, DFSSEB, MSEB
 1016 FORMAT (7X, 'ERROR', 15X, F7.2, 5X, F6.2, 3X, F6.2,/ )
      WRITE (W, 1012)
      WRITE (W, 1017) SSW, DFSSW
 1017 FORMAT (3X, 'WITHIN', 18X, F7.2, 5X, F6.2,/ )
      WRITE (W, 1018) SSTR, DFSSTR, MSTR, FTR, PTR
 1018 FORMAT (7X, 'TREATMENT', 11X, F7.2, 5X, F6.2, 3X, F6.2, 4X, F5.2,
     1 4X, F5.3,/ )
      WRITE(W, 1019) SSO, DFSSO, MSO, FO, PO
 1019 FORMAT (7X, 'ORDER', 15X, F7.2, 5X, F6.2, 3X, F6.2, 4X, F5.2,
     1 4X, F5.3 ,/ )
      WRITE (W, 1020)  SSBO, DFSSBO, MSBO, FBOI, PBOI
 1020 FORMAT (7X, 'B X ORDER', 11X, F7.2, 5X, F6.2, 3X, F6.2, 4X, F5.2,
     1 4X, F5.3 ,/ )
      WRITE (W, 1021)  SSEW, DFSSEW, MSEW
 1021 FORMAT (7X,  'ERROR', 15X, F7.2, 5X, F6.2, 3X, F6.2, ////)
      CALL FSTAT  (DFSSG, DFSSEB, ALPHA, FGP)
      WRITE (W, 6661) FGP
 6661 FORMAT (30X,'THE F REQUIRED FOR SIG. GROUPS EFFECT=', F8.2,/)
      CALL FSTAT (DFSSTR, DFSSEW, ALPHA, FT)
      WRITE (W, 6662)  FT
 6662 FORMAT (30X,'THE F REQUIRED FOR  SIG. TRIALS EFFECT=', F8.2,/)
      CALL FSTAT( DFSSO, DFSSEW, ALPHA, FOR)
      WRITE (W, 6663)  FOR
 6663 FORMAT (30X, 'THE F REQUIRED FOR SIG. ORDER EFFECT=', F8.2, /)
      CALL FSTAT (DFSSBO, DFSSEW, ALPHA, FBO)
      WRITE (W, 6664)   FBO
 6664 FORMAT (30X,'THE F REQUIRED FOR SIG. B BY ORDER INTERACTION=',
     1 F8.2, ///)
      STOP
      END
```

INDEX

Analysis of variance
  designs for, 16–105
    completely randomized, 17, 22–25
    discussion of, 16–21
    factorial
      three factors, 17–18, 30–37
      two factors, 17, 25–30
    Latin square
      complex, 21, 92–105
      simple, 20–21, 84–91
        computer program for, 262
    mixed
      three factors (repeated measures on one factor), 19–20, 61–72
      three factors (repeated measures on two factors), 20, 72–83
        computer program for, 255
      two factors (repeated measures on one factor), 19, 54–61
    repeated-measures, 18, 43–47
    treatments-by-levels, 18, 38–43
    treatments-by-subjects, 18, 43–47
    treatments-by-treatments-by-subjects, 18–19, 47–54
  supplemental computations for, 106–149
    differences among several independent variances, 106, 110–111
    differences between variances of two independent samples, 106, 107–108
    differences between variances of two related samples, 106, 109–110
    discussion of, 106–107
    Duncan's multiple-range test for nearly equal *ns*, 107, 115–117
    F-maximum test for homogeneity of variances, 106, 110–111
    F-tests for simple effects, 107, 117–123
    homogeneity of independent variances, 106, 107–108
    homogeneity of related variances, 106, 109–110
    orthogonal components in tests for trend, 107, 123–149
    t-test for differences among several means, 107, 112–114

Blocking and tabling of data, 2–4

Chi-square
  complex, 196, 209–213
  simple, 196, 207–209
  table of centile values for, 220–221
Computer programs
  for simple Latin square design, 262
  for three-factor mixed design (repeated measures on two factors), 255
Contingency coefficient (C), 196, 209–213
Correlation, 150–194
  correlation ratio (*eta*), 151, 166–168
  covariance, *see* Covariance, analysis of
  discussion of, 150–152
  Fisher's z transformation of r, 191
    table of, 226–227
  Kendall rank-order correlation coefficient (*tau*), 151, 159–163, 170–171
  multiple correlation (three variables), 151, 171–173
  partial correlation (three variables), 151, 168–169
    using Kendall's *tau*, 170–171
  Pearson product-moment correlation coefficient (r), 150, 152–155
    table of critical values of, 228–229
  point-biserial correlation, 151, 163–166
  rank-order correlation
    Kendall coefficient (*tau*), 151, 159–163
    partial, using Kendall's *tau*, 151, 170–171
    Spearman coefficient (*rho*), 151, 156–159
  reliability of measurement tests
    test as individual items (Kuder-Richardson and Hoyt), 151, 188–191
    test as a whole (test-retest, parallel forms, split halves), 151, 187–188
  significance tests of
    between dependent correlations, 152, 193–194
    between independent correlations, 152, 191–192
    See also the end of each section presenting a particular correlation.

Covariance, analysis of
  complex, 151, 177–186
  simple, 151, 173–177

Data
  blocking and tabling of, 2–4
  organization of, discussion, 1–2
Duncan's multiple-range test for nearly equal *ns*, 107, 115–117
  table of significant ranges for, 236–241

F-distribution table, 222–225
F-maximum test for homogeneity of variances, 106, 110–111
  table of critical values for, 234–235
F-tests for simple effects, 107, 117–123
Factorial analysis of covariance, 151, 177–186
Factorial designs for analysis of variance
  two factors, 17, 25–30
  three factors, 17–18, 30–37
Fisher's *z* transformation of *r*, 191
  table of, 226–227

Hoyt's test for reliability of measurement, 151, 188–191

Kendall rank-order correlation coefficient (*tau*), 151, 159–163
  in partial correlation, 151, 170–171
Kuder-Richardson test for reliability of measurement, 151, 188–191

Latin square design for analysis of variance
  complex, 21, 92–105
  simple, 20–21, 84–91
    computer program for, 262
Levels design for analysis of variance, 18, 38–43

Mann-Whitney *U*-test, 196, 201–204
  table of critical *U* values, 230–233
Mean, standard error of the, 6–7
Multiple correlation (three variables), 151, 171–173

Nonparametric tests
  chi-square
    complex, and the contingency coefficient (*C*), 196, 209–213
    simple, and the *phi* coefficient, 196, 207–209
  for differences between independent samples (Mann-Whitney *U*-test), 196, 201–204
  for differences between related samples (Wilcoxon sign test), 196, 205–206
  for significance of difference between two proportions, 195–196, 199–201
  for significance of a proportion, 195–196, 197–198
Normal-curve areas, table of, 216–217

Organizing data
  blocking and tabling, 2–4
  range and standard deviation, 4–6
  standard error of the mean, 6
Orthogonal tests for trend, 107, 123–149
  table of orthogonal coefficients, 243

Parallel-forms reliability measure, 151, 187–188
Partial correlation
  using Kendall's *tau*, 151, 170–171
  with three variables, 151, 168–169
Pearson product-moment correlation coefficient (*r*), 150, 152–155
  Fisher's *z* transformation of *r*, 191
    table of, 226–227
  table of critical values of, 228–229
Phi coefficient, 196, 207–209
Point-biserial correlation, 151, 163–166
Proportions
  significance of differences between two, 195–196, 199–201
  significance of one, 195–196, 197–198

Range, 4–6
Rank-order correlations
  Kendall's coefficient (*tau*), 151, 159–163
  partial
    using Kendall's *tau*, 151, 170–171
    with three variables, 151, 168–169
  Spearman's coefficient (*rho*), 151, 156–159
Reliability-of-measurement tests
  test as individual items (Kuder-Richardson and Hoyt), 151, 188–191
  test as a whole (test-retest, parallel forms, split halves), 151, 187–188

Sign test (Wilcoxon), 196, 205–206
  table of critical values of, 242
Significance tests, *see* each section covering a particular topic
Spearman rank-order correlation (*rho*), 151, 156–159
Split-halves reliability measure, 151, 187–188
Squares and square roots, table of, 244–253
Standard deviation, 4–6
Standard error of the mean, 6
Student's *t*, *see* *t*-tests

t table (critical values), 218–219
t-tests
  for differences between sample mean and population mean, 7–9
  for differences among several means, 107, 112–114
  for differences between two independent means, 9–12
  for differences between two related measures, 12–15
Tabling and blocking of data, 1–4
Test for difference between variances of two independent samples, 106, 107–108
Test for difference between variances of two related samples, 106, 109–110
Test for differences among several independent variances, 106, 110–111
Test for significance of difference between two proportions, 195–196, 199–201
Test for significance of a proportion, 195–196, 197–198
Test-retest reliability measure, 151, 187–188
Tests for trend, 107, 123–149
Treatments-by-levels design for analysis of variance, 18, 38–43
Treatments-by-subjects design for analysis of variance, 18, 43–47
Treatments-by-treatments-by-subjects design for analysis of variance, 18–19, 47–54

U-test of Mann-Whitney, 196, 201–204
  table of critical values of, 230–233

Variance, analysis of, *see* Analysis of variance

Wilcoxon sign test, 196, 205–206
  table of critical values of, 242

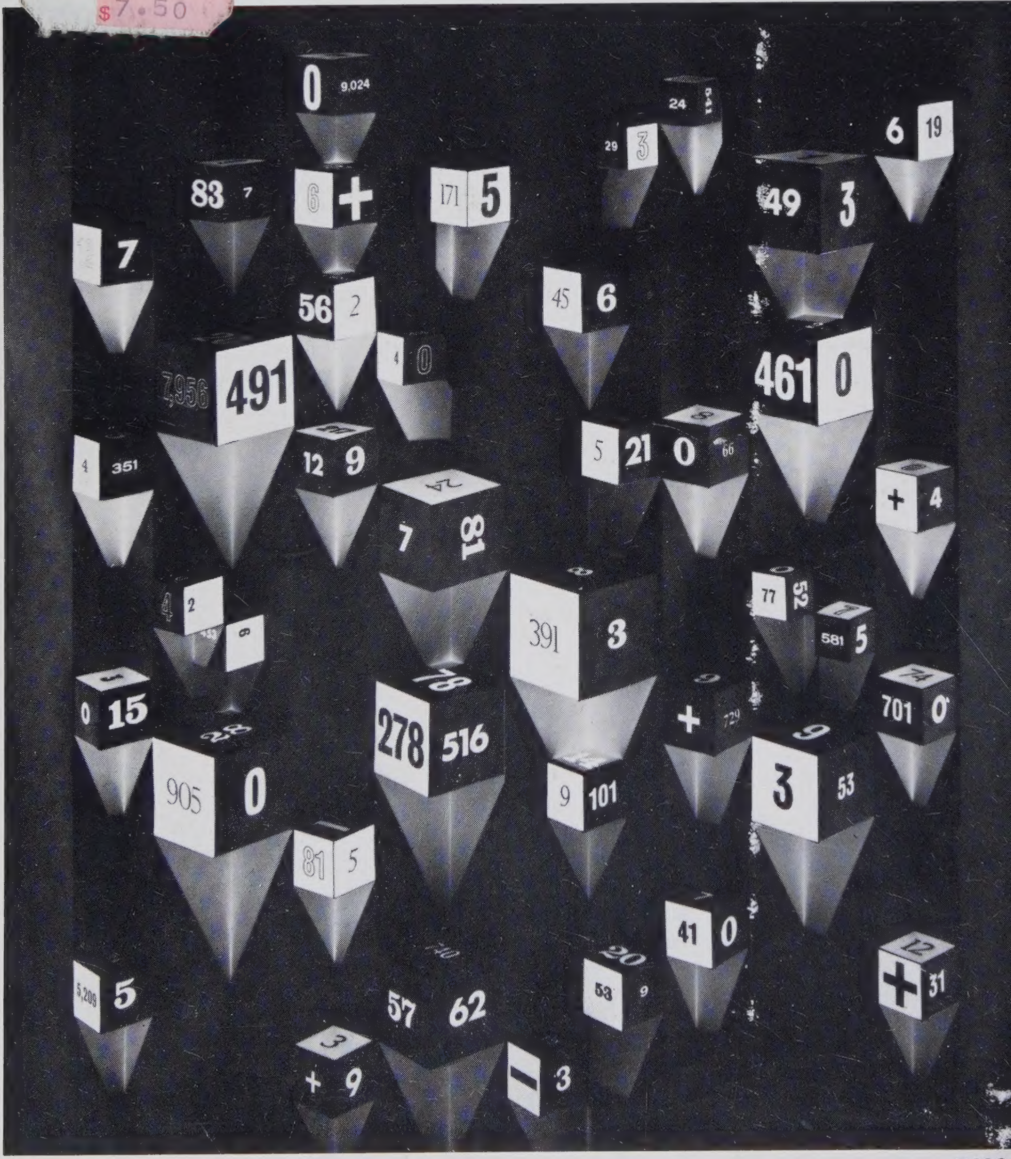
z (standard deviates), table of, 216–217
z transformation (Fisher's) of *r*, 191
  table of, 226–227

7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 74 73

Als

Intr , Frank W. Carlborg, Northern Illinois University.
Em concepts underlying many of the most widely used statistical methods.
Th ductory probability theory, testing hypotheses, and estimating
pa ing on multiple regression and the analysis of variance, the book is
int ader identify a statistical problem in his work and interpret a statistical
an n.
Cc ages.

BOOK AND SUPPLY STORE
DATE 0974
DEPT. 501
CLASS 0
PRICE $7.50



549